# Efficient Computation of Range Aggregates against Uncertain Location-Based Queries

Ying Zhang, Xuemin Lin, Yufei Tao, Wenjie Zhang, and Haixun Wang

**Abstract**—In many applications, including location-based services, queries may not be precise. In this paper, we study the problem of efficiently computing range aggregates in a multidimensional space when the query location is uncertain. Specifically, for a query point $Q$ whose location is uncertain and a set $S$ of points in a multidimensional space, we want to calculate the aggregate (e.g., *count*, *average* and *sum*) over the subset $S'$ of $S$ such that for each $p \in S'$, $Q$ has at least probability $\theta$ within the distance $\gamma$ to $p$. We propose novel, efficient techniques to solve the problem following the *filtering-and-verification* paradigm. In particular, two novel filtering techniques are proposed to effectively and efficiently remove data points from verification. Our comprehensive experiments based on both real and synthetic data demonstrate the efficiency and scalability of our techniques.

**Index Terms**—Uncertainty, index, range aggregate query.

## 1 Introduction

Query imprecision or uncertainty may be often caused by the nature of many applications, including location-based services. The existing techniques for processing location-based spatial queries regarding certain query points and data points are not applicable or inefficient when uncertain queries are involved. In this paper, we investigate the problem of efficiently computing distance-based range aggregates over certain data points and uncertain query points as described in the abstract. In general, an *uncertain query* $Q$ is a multidimensional point that might appear at any location $x$ following a probabilistic density function $pdf(x)$ within a region $Q.region$. There is a number of applications where a query point may be uncertain. Below are two sample applications.

**Motivating Application 1.** A blast warhead carried by a missile may destroy things by blast pressure waves in its *lethal area* where the lethal area is typically a circular area centered at the point of explosion (blast point) with radius $\gamma$ [24] and $\gamma$ depends on the explosive used. While firing such a missile, even the most advanced laser-guided missile cannot exactly hit the aiming point with 100 percent guarantee. The actual falling point (blast point) of a missile blast warhead regarding a target point usually follows some probability density functions ($PDF$s); different $PDF$s have been studied in [24] where *bivariate normal* distribution is the simplest and the most common one [24]. In military applications, firing such a missile may not only destroy military targets but may also damage civilian objects. Therefore, it is important to avoid the civilian casualties by estimating the likelihood of damaging civilian objects once the aiming point of a blast missile is determined. As depicted in Fig. 1, points $\{p_i\}$ for $1 \leq i \leq 7$ represent some civilian objects (e.g., residential buildings, public facilities ). If $q_1$ in Fig. 1 is the actual falling point of the missile, then objects $p_1$ and $p_5$ will be destroyed. Similarly, objects $p_2$, $p_3$, and $p_6$ will be destroyed if the actual falling point is $q_2$. In this application, the risk of civilian casualties may be measured by the total number $n$ of civilian objects which are within $\gamma$ distance away from a possible blast point with at least $\theta$ probability. Note that the probabilistic threshold is set by the commander based on the levels of tradeoff that she wants to make between the risk of civilian damages and the effectiveness of military attacks; for instance, it is unlikely to cause civilian casualties if $n = 0$ with a small $\theta$. Moreover, different weight values may be assigned to these target points and hence the aggregate can be conducted based on the *sum* of the values.

**Motivating Application 2.** Similarly, we can also estimate the effectiveness of a police vehicle patrol route using range aggregate against uncertain location-based query $Q$. For example, $Q$ in Fig. 1 now corresponds to the possible locations of a police patrol vehicle in a patrol route. A spot (e.g., restaurant, hotel, residential property), represented by a point in $\{p_1, p_2, \ldots, p_7\}$ in Fig. 1, is likely under reliable *police patrol coverage* [11] if it has at least $\theta$ probability within $\gamma$ distance to a moving patrol vehicle, where $\gamma$ and $\theta$ are set by domain experts. The number of spots under reliable *police patrol coverage* is often deployed to evaluate the effectiveness of the police patrol route.

Motivated by the above applications, in the paper we study the problem of aggregate computation against the data points which have at least probability $\theta$ to be within distance $\gamma$ regarding an uncertain location-based query.

**Challenges.** A naive way to solve this problem is that for each data point $p \in S$, we calculate the probability, namely *falling probability*, of $Q$ within $\gamma$ distance to $p$, select $p$ against a

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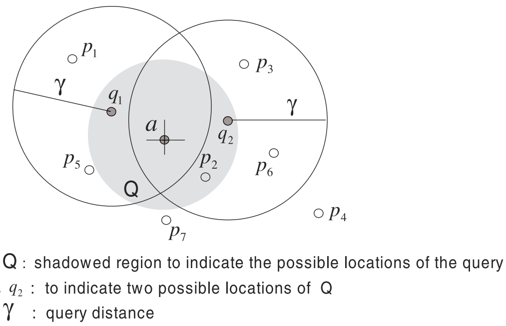



Fig. 1. Missile example.

given probability threshold, and then conduct the aggregate. This involves the calculation of an integral regarding each $p$ and $Q.pdf$ for each $p \in S$; unless $Q.pdf$ has a very simple distribution (e.g., uniform distributions), such a calculation may often be very expensive and the naive method may be computationally prohibitive when a large number of data points is involved. In the paper, we target the problem of efficiently computing range aggregates against an uncertain $Q$ for arbitrary $Q.pdf$ and $Q.region$. Note that when $Q.pdf$ is a uniform distribution within a circular region $Q.region$, a circular "window" can be immediately obtained according to $\gamma$ and $Q.region$ so that the computation of range aggregates can be conducted via the window aggregates [27] over $S$.

**Contributions.** Our techniques are developed based on the standard *filtering-and-verification* paradigm. We first discuss how to apply the existing probabilistically constrained regions (*PCR*) technique [26] to our problem. Then, we propose two novel distance-based filtering techniques, statistical filtering (*STF*) and anchor point filtering (*APF*), respectively, to address the inherent limits of the *PCR* technique. The basic idea of the *STF* technique is to bound the falling probability of the points by applying some well-known statistical inequalities where only a small amount of statistic information about the uncertain location-based query $Q$ is required. The *STF* technique is simple and space efficient (only $d + 2$ float numbers required where $d$ denotes the dimensionality), and experiments show that it is effective. For the scenarios where a considerable "large" space is available, we propose a view-based filter which consists of a set of *anchor points*. An *anchor point* may reside at any location and its falling probability regarding $Q$ is precomputed for several $\gamma$ values. Then, many data points might be effectively filtered based on their distances to the *anchor points*. For a given space budget, we investigate how to construct the anchor points and their accessing orders.

To the best of our knowledge, we are the first to identify the problem of computing range aggregates against uncertain location-based query. In this paper, we investigate the problem regarding both *continuous* and *discrete* $Q$. Our principle contributions can be summarized as follows:

- We propose two novel filtering techniques, *STF* and *APF*, respectively. The *STF* technique has a decent filtering power and only requires the storage of very limited precomputed information. *APF* provides the flexibility to significantly enhance the filtering power by demanding more precomputed information to be stored. Both of them can be applied to *continuous* case and *discrete* case.

TABLE 1
The Summary of Notations

| Notation | Definition |
|---|---|
| $Q$ | uncertain location based query |
| $S$ | a set of points |
| $q$ | instance of an uncertain query $Q$ |
| $d$ | dimensionality |
| $P_q$ | the probability of the $q$ to appear |
| $\theta$ and $\gamma$ | probabilistic threshold and query distance |
| $P_{fall}(Q, p, \gamma)$ | the falling probability of $p$ regarding $Q$ and $\gamma$ |
| $Q_{\theta,\gamma}(S)$ | $\{p \vert p \in S \wedge P_{fall}(Q, p, \gamma) \geq \theta\}$ |
| $p, x, y, b(S)$ | point (a set of data points) |
| $e$ | $R$ tree entry |
| $C_{p,r}$ | a circle(sphere) centred at $p$ with radius $r$ |
| $\delta(x, y)$ | the *distance* between $x$ and $y$ |
| $\delta_{max(min)}(r_1, r_2)$ | the maximal(minimal) distance between two rectangular regions |
| $g_Q$ | *mean* of $Q$ |
| $\eta_Q$ | weighted average distance of $Q$ |
| $\sigma_Q$ | *variance* of $Q$ |
| $\epsilon$ | arbitrarily small positive constant value |
| $a$ | *anchor point* |
| $n_{ap}$ | the number of *anchor points* |
| $LP_{fall}(p, \gamma)$ | lower bound of the $P_{fall}(p, \gamma)$ |
| $UP_{fall}(p, \gamma)$ | upper bound of the $P_{fall}(p, \gamma)$ |
| $n_d$ | the number of different distances pre-computed for each *anchor point* |
| $D_a$ | a set of distance values used by *anchor point* $a$ |

- Extensive experiments are conducted to demonstrate the efficiency of our techniques.
- While we focus on the problem of range *counting* for uncertain location-based queries in the paper, our techniques can be immediately extended to other range aggregates.

The remainder of the paper is organized as follows: Section 2 formally defines the problem and presents preliminaries. In Section 3, following the *filtering-and-verification* framework, we propose three filtering techniques. Section 4 evaluates the proposed techniques with extensive experiments. Then, some possible extensions of our techniques are discussed in Section 5. This is followed by related work in Section 6. Section 7 concludes the paper.

## 2 Background Information

We first formally define the problem in Section 2.1, then Section 2.2 presents the *PCR* technique [26] which is employed in the filtering technique proposed in Section 3.3. Table 1 summarizes the notations used throughout the paper.

### 2.1 Problem Definition

In the paper, $S$ is a set of points in a $d$-dimensional numerical space. The distance between two points $x$ and $y$ is denoted by $\delta(x, y)$. Note that techniques developed in the paper can be applied to any *distance* metrics [5]. In the examples and experiments, the *euclidean* distance is used. For two rectangular regions $r_1$ and $r_2$, we have $\delta_{max}(r_1, r_2) = \max_{\forall x \in r_1, y \in r_2} \delta(x, y)$ and

$$\delta_{min}(r_1, r_2) = \begin{cases} 0 & \text{if } r_1 \cap r_2 \neq \emptyset \\ \min_{\forall x \in r_1, y \in r_2} \delta(x, y) & \text{otherwise.} \end{cases} \quad (1)$$

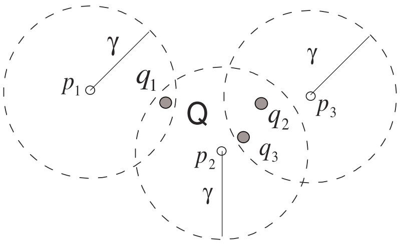



Fig. 2. Example of $P_{fall}(Q, p, \gamma)$.

An uncertain (location based) query $Q$ may be described by a *continuous* or a *discrete* distribution as follows:

**Definition 1 (Continuous Distribution).** *An uncertain query $Q$ is described by a probabilistic density function $Q.pdf$. Let $Q.region$ represent the region where $Q$ might appear, then $\int_{x \in Q.region} Q.pdf(x)dx = 1$;*

**Definition 2 (Discrete Distribution).** *An uncertain query $Q$ consists of a set of instances (points) $\{q_1, q_2, \ldots, q_n\}$ in a d-dimensional numerical space where $q_i$ appears with probability $P_{q_i}$, and $\sum_{q \in Q} P_q = 1$;*

Note that, in Section 5 we also cover the applications where $Q$ can have a nonzero probability to be absent; that is, $\int_{x \in Q.region} Q.pdf(x)dx = c$ or $\sum_{q \in Q} P_q = c$ for a $c < 1$.

For a point $p$, we use $P_{fall}(Q, p, \gamma)$ to represent the probability of $Q$ within $\gamma$ distance to $p$, called *falling* probability of $p$ regarding $Q$ and $\gamma$. It is formally defined below.

For *continuous* cases

$$P_{fall}(Q, p, \gamma) = \int_{x \in Q.region \wedge \delta(x,p) \leq \gamma} Q.pdf(x)dx. \qquad (2)$$

For *discrete* cases

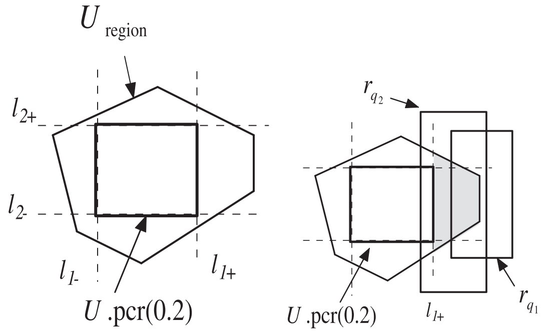
$$P_{fall}(Q, p, \gamma) = \sum_{q \in Q \wedge \delta(q,p) \leq \gamma} P_q. \qquad (3)$$

In the paper hereafter, $P_{fall}(Q, p, \gamma)$ is abbreviated to $P_{fall}(p, \gamma)$, and $Q.region$ and $Q.pdf$ are abbreviated to $Q$ and $pdf$, respectively, whenever there is no ambiguity. It is immediate that $P_{fall}(p, \gamma)$ is a monotonically increasing function with respect to distance $\gamma$.

**Problem Statement.** In many applications, users are only interested in the points with falling probabilities exceeding a given probabilistic threshold regarding $Q$ and $\gamma$. In this paper, we investigate the problem of probabilistic threshold based uncertain location range aggregate query on points data; it is formally defined below.

**Definition 3 (Uncertain Range Aggregate Query).** *Given a set $S$ of points, an uncertain query $Q$, a query distance $\gamma$, and a probabilistic threshold $\theta$, we want to compute an aggregate function (e.g., count, avg, and sum) against points $p \in Q_{\theta,\gamma}(S)$, where $Q_{\theta,\gamma}(S)$ denotes a subset of points $\{p\} \subseteq S$ such that $P_{fall}(p, \gamma) \geq \theta$.*

In this paper, our techniques will be presented based on the aggregate *count*. Nevertheless, they can be immediately extended to cover other aggregates, such as $min$, $max$, $sum$, $avg$, etc., over some nonlocational attributes (e.g., weight value of the object in missile example).




(a) *PCR* (b) *PCR* based filtering

Fig. 3. A 2d probabilistically constrained region (*PCR* (0.2)).

**Example 1.** In Fig. 2, $S = \{p_1, p_2, p_3\}$ and $Q = \{q_1, q_2, q_3\}$ where $P_{q_1} = 0.4$, $P_{q_2} = 0.3$, and $P_{q_3} = 0.3$. According to Definition 3, for the given $\gamma$, we have $P_{fall}(p_1, \gamma) = 0.4$, $P_{fall}(p_2, \gamma) = 1$, and $P_{fall}(p_3, \gamma) = 0.6$. Therefore, $Q_{\theta,\gamma}(S) = \{p_2, p_3\}$ if $\theta$ is set to 0.5, and hence $|Q_{\theta,\gamma}(S)| = 2$.

## 2.2 Probabilistically Constrained Regions

In [26], Tao et al. study the problem of range query on uncertain objects, in which the query is a rectangular window and the location of each object is uncertain. Although the problem studied in [26] is different with the one in this paper, in Section 3.3 we show how to modify the techniques developed in [26] to support uncertain location-based query.

In the following part, we briefly introduce the *PCR* technique developed in [26]. Same as the uncertain location-based query, an uncertain object $U$ is modeled by a probability density function $U.pdf(x)$ and an uncertain region $U.region$. The probability that the uncertain object $U$ falls in the rectangular window query $r_q$, denoted by $P_{fall}(U, r_q)$, is defined as $\int_{x \in U.region \cap r_q} U.pdf(x)dx$. In [26], the probabilistically constrained region of the uncertain object $U$ regarding probability $\theta$ ($0 \leq \theta \leq 0.5$), denoted by $U.pcr(\theta)$, is employed in the filtering technique. Particularly, $U.pcr(\theta)$ is a rectangular region constructed as follows:

For each dimension $i$, the projection of $U.pcr(\theta)$ is denoted by $[U.pcr_{i-}(\theta), U.pcr_{i+}(\theta)]$ where

$$\int_{x \in U.region \& x_i \leq U.pcr_{i-}(\theta)} U.pdf(x)dx = \theta$$

and $\int_{x \in U.region \& x_i \geq U.pcr_{i+}(\theta)} U.pdf(x)dx = \theta$. Note that $x_i$ represents the coordinate value of the point $x$ on $i$th dimension. Then, $U.pcr(\theta)$ corresponds to a rectangular region $[U.pcr_-(\theta), U.pcr_+(\theta)]$ where $U.pcr_-(\theta)$ ($U.pcr_+(\theta)$) is the lower (upper) corner and the coordinate value of $U.pcr_-(\theta)$ ($U.pcr_+(\theta)$) on $i$th dimension is $U.pcr_{i-}(\theta)$ ($U.pcr_{i+}(\theta)$). Fig. 3a illustrates the $U.pcr(0.2)$ of the uncertain object $U$ in 2D space. Therefore, the probability mass of $U$ on the left (right) side of $l_{1-}$ ($l_{1+}$) is 0.2 and the probability mass of $U$ below (above) the $l_{2-}$ ($l_{2+}$) is 0.2 as well. Following is a motivating example of how to derive the lower and upper bounds of the falling probability based on *PCR*.

**Example 2.** According to the definition of *PCR*, in Fig. 3b the probabilistic mass of $U$ in the shaded area is 0.2, i.e., $\int_{x \in U.region \& x_1 \geq U.pcr_{1+}(\theta)} U.pdf(x)dx = 0.2$. Then, it is immediate that $P_{fall}(U, r_{q_1}) < 0.2$ because $r_{q_1}$ does not intersect $U.pcr(0.2)$. Similarly, we have $P_{fall}(U, r_{q_2}) \geq 0.2$ because the shaded area is enclosed by $r_{q_2}$.

The following theorem [26] formally introduces how to *prune* or *validate* an uncertain object $U$ based on $U.pcr(\theta)$ or $U.pcr(1-\theta)$. Note that we say an uncertain object is *pruned* (*validated*) if we can claim $P_{fall}(U, r_q) < \theta$ ($P_{fall}(U, r_q) \geq \theta$) based on the $PCR$.

**Theorem 1.** *Given an uncertain object $U$, a range query $r_q$ ($r_q$ is a rectangular window) and a probabilistic threshold $\theta$.*

1. *For $\theta > 0.5$, $U$ can be* pruned *if $r_q$ does not fully contain $U.pcr(1-\theta)$;*
2. *For $\theta \leq 0.5$, the pruning condition is that $r_q$ does not intersect $U.pcr(\theta)$;*
3. *For $\theta > 0.5$, the validating criterion is that $r_q$ completely contains the part of $U_{mbb}$ on the right (left) of plane $U.pcr_{i-}(1-\theta)$ ($U.pcr_{i+}(1-\theta)$) for some $i \in [1, d]$, where $U_{mbb}$ is the minimal bounding box of uncertain region $U.region$;*
4. *For $\theta \leq 0.5$ the validating criterion is that $r_q$ completely contains the part of $U_{mbb}$ on the left (right) of plane $U.pcr_{i-}(\theta)$ ($U.pcr_{i+}(\theta)$) for some $i \in [1, d]$.*

## 3 *FILTERING-AND-VERIFICATION* ALGORITHM

According to the definition of *falling probability* (i.e., $P_{fall}(p, \gamma)$) in (2), the computation involves integral calculation, which may be expensive in terms of CPU cost. Based on Definition 3, we only need to know whether or not the *falling probability* of a particular point regarding $Q$ and $\gamma$ exceeds the probabilistic threshold for the uncertain aggregate range query. This motivates us to follow the *filtering-and-verification* paradigm for the uncertain aggregate query computation. Particularly, in the *filtering phase*, effective and efficient filtering techniques will be applied to *prune* or *validate* the points. We say a point $p$ is *pruned* (*validated*) regarding the uncertain query $Q$, distance $\gamma$ and probabilistic threshold $\theta$ if we can claim that $P_{fall}(p, \gamma) < \theta$ ($P_{fall}(p, \gamma) \geq \theta$) based on the filtering techniques without explicitly computing the $P_{fall}(p, \gamma)$. The points that cannot be *pruned* or *validated* will be *verified* in the *verification* phase in which their *falling probabilities* are calculated. Therefore, it is desirable to develop effective and efficient filtering techniques to *prune* or *validate* points such that the number of points being *verified* can be significantly reduced.

In this section, we first present a general framework for the *filtering-and-verification* Algorithm based on filtering techniques in Section 3.1. Then, a set of filtering techniques are proposed. Particularly, Section 3.2 proposes the statistical filtering technique. Then, we investigate how to apply the *PCR*-based filtering technique in Section 3.3. Section 3.4 presents the *anchor point*-based filtering technique.

For presentation simplicity, we consider the *continuous* case of the uncertain query in this section. Section 3.5 shows that techniques proposed in this section can be immediately applied to the *discrete* case.

### 3.1 A Framework for *Filtering-and-Verification* Algorithm

In this section, following the *filtering-and-verification* paradigm we present a general framework to support uncertain range aggregate query based on the filtering technique. To facilitate the aggregate query computation, we assume a set $S$ of points is organized by an aggregate $R$-Tree [22], denoted by $R_S$. Note that an entry $e$ of $R_S$ might be a data entry or an intermediate entry where a data entry corresponds to a point in $S$ and an intermediate entry groups a set of data entries or child intermediate entries. Assume a filter, denoted by $F$, is available to *prune* or *validate* a data entry (i.e., a point) or an intermediate entry (i.e., a set of points).

Algorithm 1 illustrates the framework of the *filtering-and-verification* Algorithm. Note that details of the filtering techniques will be introduced in the following sections. The algorithm consists of two phases. In the *filtering* phase (Line 3-16), for each entry $e$ of $R_S$ to be processed, we do not need to further process $e$ if it is *pruned* or *validated* by the filter $F$. We say an entry $e$ is *pruned* (*validated*) if the filter can claim $P_{fall}(p, \gamma) < \theta$ ($P_{fall}(p, \gamma) \geq \theta$) for any point $p$ within $e_{mbb}$. The counter $cn$ is increased by $|e|$ (Line 6) if $e$ is *validated* where $|e|$ denotes the aggregate value of $e$ (i.e., the number of data points in $e$). Otherwise, the point $p$ associated with $e$ is a candidate point if $e$ corresponds to a data entry (Line 10), and all child entries of $e$ are put into the queue for further processing if $e$ is an intermediate entry (Line 12). The *filtering phase* terminates when the queue is empty. In the *verification* phase (Line 17-21), candidate points are *verified* by the integral calculations according to (2).

**Algorithm 1.** Filtering-and-Verification($R_S$, $Q$, $F$, $\gamma$, $\theta$)
**Input:** $R_S$: an aggregate $R$ tree on data set $S$,
$Q$: uncertain query, $F$: Filter, $\gamma$: query distance,
$\theta$: probabilistic threshold.
**Output:** $|Q_{\theta,\gamma}(S)|$
**Description:**
1: $Queue := \emptyset$; $cn := 0$; $C := \emptyset$;
2: Insert root of $R_S$ into $Queue$;
3: **while** $Queue \neq \emptyset$ **do**
4:   $e \leftarrow$ dequeue from the $Queue$;
5:   **if** $e$ is *validated* by the filter $F$ **then**
6:     $cn := cn + |e|$;
7:   **else**
8:     **if** $e$ is not *pruned* by the filter $F$ **then**
9:       **if** $e$ is data entry **then**
10:         $C := C \cup p$ where $p$ is the data point $e$ represented;
11:       **else**
12:         put all child entries of $e$ into $Queue$;
13:       **end if**
14:     **end if**
15:   **end if**
16: **end while**
17: **for** each point $p \in C$ **do**
18:   **if** $P_{fall}(Q, p, \gamma) \geq \theta$ **then**
19:     $cn := cn + 1$;
20:   **end if**
21: **end for**
22: Return $cn$

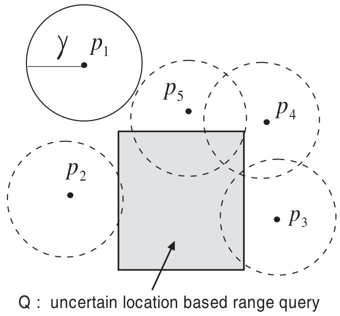



Fig. 4. Running example.

**Cost Analysis.** The total time cost of Algorithm 1 is as follows:

$$Cost = N_f \times C_f + N_{io} \times C_{io} + N_{ca} \times C_{vf}. \quad (4)$$

Particularly, $N_f$ represents the number of entries being tested by the filter on Line 5 and $C_f$ is the time cost for each test. $N_{io}$ denotes the number of nodes (pages) accessed (Line 13) and $C_{io}$ corresponds to the delay of each node (page) access of $R_S$. $N_{ca}$ represents the size of candidate set $C$ and $C_{vf}$ is the computation cost for each verification (Line 15) in which numerical integral computation is required. With a reasonable filtering time cost (i.e., $C_{vf}$), the dominant cost of Algorithm 1 is determined by $N_{io}$ and $N_{ca}$ because $C_{io}$ and $C_{vf}$ might be expensive. Therefore, in the paper we aim to develop effective and efficient filtering techniques to reduce $N_{ca}$ and $N_{io}$.

**Filtering.** Suppose there is no filter $F$ in Algorithm 1, all points in $S$ will be *verified*. Regarding the example in Fig. 4, 5 points $p_1, p_2, p_3, p_4$, and $p_5$ will be *verified*. A straightforward filtering technique is based on the minimal and maximal distances between the minimal bounding boxes(MBBs) of an entry and the uncertain query. Clearly, for any $\theta$ we can safely *prune* an entry if $\delta_{min}(Q_{mbb}, e_{mbb}) > \gamma$ or *validate* it if $\delta_{max}(Q_{mbb}, e_{mbb}) \leq \gamma$. We refer this as *maximal/minimal distance*-based filtering technique, namely *MMD*. *MMD* technique is time efficient as it takes only $O(d)$ time to compute the minimal and maximal distances between $Q_{mbb}$ and $e_{mbb}$. Recall that $Q_{mbb}$ is the minimal bounding box of $Q.region$.

**Example 3.** As shown in Fig. 4, suppose the *MMD* filtering technique is applied in Algorithm 1, then $p_1$ is *pruned* and the other four points $p_2$, $p_3$, $p_4$, and $p_5$ will be verified.

Although the *MMD* technique is very time efficient, its filtering capacity is limited because it does not make use of the distribution information of the uncertain query $Q$ and the probabilistic threshold $\theta$. This motivates us to develop more effective filtering techniques based on some precomputations on the uncertain query $Q$ such that the number of entries (i.e., points) being *pruned* or *validated* in Algorithm 1 is significantly increased. In the following sections, we present three filtering techniques, named *STF*, *PCR*, and *APF*, respectively, which can significantly enhance the filtering capability of the filter.

### 3.2 Statistical Filter

In this section, we propose a statistical filtering technique, namely *STF*. After introducing the motivation of the technique, we present some important statistic information of the uncertain query and then show how to derive the lower and upper bounds of the *falling probability* of a point

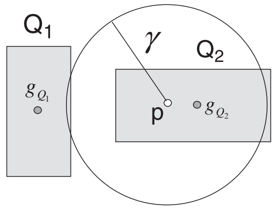



Fig. 5. Motivation example.

regarding an uncertain query $Q$, distance $\gamma$, and probabilistic threshold $\theta$.

**Motivation.** As shown in Fig. 5, given an uncertain query $Q_1$ and $\gamma$ we cannot *prune* point $p$ based on the *MMD* technique, regardless of the value of $\theta$, although intuitively the falling probability of $p$ regarding $Q_1$ is likely to be small. Similarly, we cannot *validate* $p$ for $Q_2$. This motivates us to develop a new filtering technique which is as simple as *MMD*, but can exploit $\theta$ to enhance the filtering capability. In the following part, we show that lower and upper bounds of $P_{fall}(p, \gamma)$ can be derived based on some statistics of the uncertain query. Then, a point may be immediately *pruned* (*validated*) based on the upper(lower) bound of $P_{fall}(p, \gamma)$, denoted by $UP_{fall}(p, \gamma)$ ($LP_{fall}(p, \gamma)$).

**Example 4.** In Fig. 5 suppose $\theta = 0.5$ and we have $UP_{fall}(Q_1, p, \gamma) = 0.4$ ($LP_{fall}(Q_2, p, \gamma) = 0.6$) based on the statistical bounds, then $p$ can be safely *pruned* (*validated*) without explicitly computing its falling probability regarding $Q_1$ ($Q_2$). Regarding the running example in Fig. 4, suppose $\theta = 0.2$ and we have $UP_{fall}(p_2, \gamma) = 0.15$ while $UP_{fall}(p_i, \gamma) \geq 0.2$ for $3 \leq i \leq 5$, then $p_2$ is *pruned*. Therefore, three points ($p_3$, $p_4$, and $p_5$) are verified in Algorithm 1 when *MMD* and statistical filtering techniques are applied.

**Statistics of the uncertain query.** To apply the statistical filtering technique, the following statistics of the uncertain query $Q$ are precomputed.

**Definition 4 (mean ($g_Q$)).** $g_Q = \int_{x \in Q} x \times Q.pdf(x)dx.$

**Definition 5 (weighted average distance ($\eta_Q$)).** *$\eta_Q$ equals* $\int_{x \in Q} \delta(x, g_Q) \times Q.pdf(x)dx.$

**Definition 6 (variance ($\sigma_Q$)).** *$\sigma_Q$ equals* $\int_{x \in Q} \delta(x, g_Q)^2 \times Q.pdf(x)dx.$

**Derive lower and upper bounds of $P_{fall}(p, \gamma)$.** For a point $p \in S$, the following theorem shows how to derive the lower and upper bounds of $P_{fall}(p, \gamma)$ based on above statistics of $Q$. Then, without explicitly computing $P_{fall}(p, \gamma)$, we may *prune* or *validate* the point $p$ based on $UP_{fall}(p, \gamma)$ and $LP_{fall}(p, \gamma)$ derived based on the statistics of $Q$.

**Theorem 2.** *Given an uncertain query $Q$ and a distance $\gamma$, and suppose the* mean $g_Q$, weighted average distance $\eta_Q$, *and* variance $\sigma_Q$ *of $Q$ are available. Then, for a point $p$, we have*

1. *If* $\gamma > \mu_1$, $P_{fall}(p, \gamma) \geq 1 - \frac{1}{1 + \frac{(\gamma - \mu_1)^2}{\sigma_1^2}}$, *where* $\mu_1 = \delta(g_Q, p) + \eta_Q$ *and*

$$\sigma_1^2 = \sigma_Q - \eta_Q^2 + 4\eta_Q \times \delta(g_Q, p).$$

2. *If* $\gamma < \delta(g_Q, p) - \eta_Q - \epsilon$, $P_{fall}(p,\gamma) \leq \frac{1}{1+\frac{(\gamma'-\mu_2)^2}{\sigma_2^2}}$, *where*
$$\mu_2^2 = \Delta + \eta_Q, \sigma_2^2 = \sigma_Q - \eta_Q^2 + 4\eta_Q \times \Delta,$$
$$\Delta = \gamma + \gamma' + \epsilon - \delta(p, g_Q)$$
*and* $\gamma' > 0$. *The* $\epsilon$ *represents an arbitrarily small positive constant value.*

Before the proof of Theorem 2, we first introduce the *Cantelli's Inequality* [19] described by Lemma 1 which is one-sided version of the *Chebyshev Inequality*.

**Lemma 1.** *Let* $X$ *be an* univariate *random variable with the expected value* $\mu$ *and the finite variance* $\sigma^2$. *Then, for any* $C > 0$, $Pr(X - \mu \geq C \times \sigma) \leq \frac{1}{1+C^2}$.

Following is the proof of Theorem 2.

**Proof.**

**Intuition of the Proof.** For a given point $p \in S$, its distance to $Q$ can be regarded as an *univariate* random variable $Y$, and we have $P_{fall}(p,\gamma) = Pr(Y \leq \gamma)$. Given $\gamma$, we can derive the lower and upper bounds of $Pr(Y \leq \gamma)$ ($P_{fall}(p,\gamma)$) based on the statistical inequality in Lemma 1 if the *expectation* ($E(Y)$) and *variance*($Var(Y)$) of the random variable $Y$ are available. Although $E(Y)$ and $Var(Y)$ take different values regarding different points, we show that the upper bounds of $E(Y)$ and $Var(Y)$ can be derived based on *mean*($g_Q$), *weighted average distance* ($\eta_Q$), and *variance*($\sigma_Q$) of the query $Q$. Then, the correctness of the theorem follows.

**Details of the Proof.** The uncertain query $Q$ is a random variable which equals $x \in Q.region$ with probability $Q.pdf(x)$. For a given point $p$, let $Y$ denote the distance distribution between $p$ and $Q$; that is, $Y$ is an *univariate* random variable and
$$Y.pdf(l) = \int_{x \in Q.region and \delta(x,p)=l} Q.pdf(x)dx$$
for any $l \geq 0$. Consequently, we have $P_{fall}(p,\gamma) = Pr(Y \leq \gamma)$ according to (2). Let $\mu = E(Y)$, $\sigma^2 = Var(Y)$, and $C = \frac{\gamma - \mu}{\sigma}$, then based on lemma 1, if $\gamma > \mu$ we have
$$Pr(Y \geq \gamma) = Pr(Y - \mu \geq C \times \sigma) \leq \frac{1}{1 + (\frac{\gamma-\mu}{\sigma})^2}.$$
Then, it is immediate that
$$Pr(Y \leq \gamma) \geq 1 - Pr(Y \geq \gamma) \geq 1 - \frac{1}{1 + \frac{(\gamma-\mu)^2}{\sigma^2}}. \quad (5)$$

According to Inequation 5 we can derive the lower bound of $P_{fall}(p,\gamma)$. Next, we show how to derive upper bound of $P_{fall}(p,\gamma)$. As illustrated in Fig. 6, let $p'$ denote a dummy point on the line $\overline{pg_Q}$ with $\delta(p', p) = \gamma + \gamma' + \epsilon$ where $\epsilon$ is an arbitrarily small positive constant value. Similar to the definition of $Y$, let $Y'$ be the distance distribution between $p'$ and $Q$; that is, $Y'$ is an *univariate* random variable where
$$Y'.pdf(l) = \int_{x \in Q.region \text{ and } \delta(x,p')=l} Q.pdf(x)dx$$
for any $l \geq 0$. Then, as shown in Fig. 6, for any point $x \in Q$ with $\delta(x, p') \leq \gamma'$(shaded area), we have

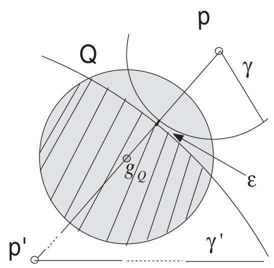



Fig. 6. Proof of upper bound.

$\delta(x,p) > \gamma$. This implies that $P(Y \leq \gamma) \leq P(Y' \geq \gamma')$. Let $\mu' = E(Y')$ and $\sigma' = Var(Y')$, according to Lemma 1 when $\gamma' > \mu'$ we have
$$Pr(Y \leq \gamma) \leq P(Y' \geq \gamma') \leq \frac{1}{1 + \frac{(\gamma'-\mu')^2}{\sigma'^2}}. \quad (6)$$

Because values of $\mu$, $\sigma^2$, $\mu'$, and $\sigma'^2$ may change regarding different point $p \in S$, we cannot precompute them. Nevertheless, in the following part we show that their upper bounds can be derived based on the statistic information of the $Q$, which can be precomputed based on the probabilistic distribution of $Q$.

Based on the **triangle inequality**, for any $x \in Q$ we have $\delta(x,p) \leq \delta(x, g_Q) + \delta(p, g_Q)$ and $\delta(x,p) \geq |\delta(x,g_Q) - \delta(p,g_Q)|$ for any $x \in Q$. Then, we have
$$\begin{aligned}\mu &= \int_{y \in Y} y \times Y.pdf(y)dy = \int_{x \in Q} \delta(x,p) \times pdf(x)dx \\ &\leq \int_{x \in Q} (\delta(p,g_Q) + \delta(x,g_Q)) \times pdf(x)dx \\ &\leq \delta(g_Q,p) + \eta_Q = \mu_1,\end{aligned}$$
and
$$\begin{aligned}\sigma^2 &= E(Y^2) - E^2(Y) \\ &\leq \int_{x \in Q} (\delta(g_Q,p) + \delta(x,g_Q))^2 pdf(x)dx \\ &\quad - (\delta(g_Q,p) - \eta_Q)^2 \\ &= 2\int_{x \in Q} \delta(g_Q,p) \times \delta(x,g_Q) \times pdf(x)dx \\ &\quad + \int_{x \in Q} \delta(x,g_Q)^2 \times pdf(x)dx \\ &\quad + 2 \times \delta(g_Q,p) \times \eta_Q - \eta_Q^2 \\ &= \sigma_Q - \eta_Q^2 + 4\eta_Q \times \delta(g_Q,p) = \sigma_1^2.\end{aligned}$$

Together with Inequality 5, we have
$$Pr(Y \leq \gamma) \geq 1 - \frac{1}{1 + \frac{(\gamma-\mu)^2}{\sigma^2}} \geq 1 - \frac{1}{1 + \frac{(\gamma-\mu_1)^2}{\sigma_1^2}}$$
if $\mu_1 < \gamma$. With similar rationale, let $\Delta = \delta(g_Q, p') = \gamma + \gamma' + \epsilon - \delta(p, g_Q)$ we have $\mu' \geq \Delta + \eta_Q = \mu_2$ and $\sigma'^2 \leq \sigma_Q - \eta_Q^2 + 4\eta_Q \times \Delta = \sigma_2^2$. Based on Inequality 6, we have
$$Pr(Y \leq \gamma) \leq \frac{1}{1 + \frac{(\gamma'-\mu')^2}{\sigma'^2}} \leq \frac{1}{1 + \frac{(\gamma'-\mu_2)^2}{\sigma_2^2}}$$

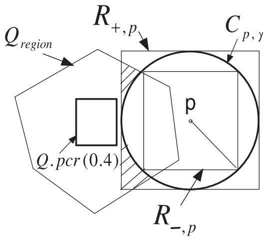



Fig. 7. Transform query.

if $\gamma < \delta(g_Q, p) - \eta_Q - \epsilon$. Therefore, the correctness of the theorem follows. □

The following extension is immediate based on the similar rationale of Theorem 2.

**Extension 1.** Suppose $r$ is a rectangular region, we can use $\delta_{min}(r, g_Q)$ and $\delta_{max}(r, g_Q)$ to replace $\delta(p, g_Q)$ in Theorem 2 for lower and upper probabilistic bounds computation, respectively.

Based on Extension 1, we can compute the upper and lower bounds of $P_{fall}(e_{mbb}, \gamma)$ where $e_{mbb}$ is the minimal bounding box of the entry $e$, and hence *prune* or *validate* $e$ in Algorithm 1. Since $g_Q, \eta_Q$, and $\sigma_Q$ are precomputed, the dominant cost in filtering phase is the distance computation between $e_{mbb}$ and $g_Q$ which is $O(d)$.

### 3.3 *PCR*-Based Filter

**Motivation.** Although the statistical filtering technique can significantly reduce the candidate size in Algorithm 1, the filtering capacity is inherently limited because only a small amount of statistics are employed. This motivates us to develop more sophisticated filtering techniques to further improve the filtering capacity; that is, we aim to improve the filtering capacity with more precomputations (i.e., more information kept for the filter). In this section, the *PCR* technique proposed in [26] will be modified for this purpose.

***PCR*-based Filtering technique.** The *PCR* technique proposed in [26] cannot be directly applied for filtering in Algorithm 1 because the range query studied in [26] is a rectangular window and objects are uncertain. Nevertheless we can adapt the *PCR* technique as follows: As shown in Fig. 7, let $C_{p,\gamma}$ represent the circle (sphere) centered at $p$ with radius $\gamma$. Then, we can regard the uncertain query $Q$ and $C_{p,\gamma}$ as an uncertain object and the range query, respectively. As suggested in [28], we can use $R_{+,p}$ (mbb of $C_{p,\gamma}$) and $R_{-,p}$ (inner box) as shown in Fig. 7 to *prune* and *validate* the point $p$ based on the $PCR$s of $Q$, respectively. For instance, if $\theta = 0.4$ the point $p$ in Fig. 7 can be *pruned* according to case 2 of Theorem 1 because $R_1$ does not intersect $Q.pcr(0.4)$. Note that similar transformation can be applied for the intermediate entries as well.

**Example 5.** Regarding the running example in Fig. 8, suppose $Q.pcr(0.2)$ is precomputed, then $p_1$, $p_2$, and $p_4$ are *pruned* because $R_{+,p_1}$, $R_{+,p_2}$, and $R_{+,p_4}$ do not overlap $Q.pcr(0.2)$ according to Theorem 1 in Section 2.2. Consequently, only $p_3$ and $p_5$ go to the verification phase when $Q.pcr(0.2)$ is available.

Same as [26], [28], a finite number of $PCR$s are precomputed for the uncertain query $Q$ regarding different probability values. For a given $\theta$ at query time, if the $Q.pcr(\theta)$ is not precomputed we can choose two precomputed $PCR$s$Q.pcr(\theta_1)$ and $Q.pcr(\theta_2)$ where $\theta_1$ ($\theta_2$) is the largest (smallest) existing probability value smaller (larger) than $\theta$. We can apply the modified *PCR* technique as the filter in Algorithm 1, and the filtering time regarding each entry tested is $O(m + \log(m))$ in the worst case , where $m$ is the number of $PCR$s precomputed by the filter.

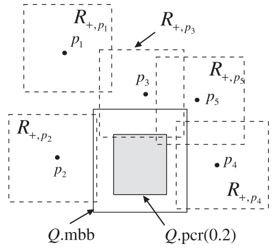



Fig. 8. Running example.

The *PCR* technique can significantly enhance the filtering capacity when a particular number of *PCR* s are precomputed. The key of the *PCR* filtering technique is to partition the uncertain query along each dimension. This may inherently limit the filtering capacity of the *PCR*-based filtering technique. As shown in Fig. 7, we have to use two rectangular regions for *pruning* and *validation* purpose, and hence the $C_{p,\gamma}$ is enlarged (shrunk) during the computation. As illustrated in Fig. 7, all instances of $Q$ in the striped area is counted for $P_{fall}(p, \gamma)$ regarding $R_{+,p}$, while all of them have distances larger than $\gamma$. Similar observation goes to $R_{-,p}$. This limitation is caused by the transformation, and cannot be remedied by increasing the number of $PCR$s. Our experiments also confirm that the *PCR* technique cannot take advantage of the large index space. This motivates us to develop new filtering technique to find a better tradeoff between the filtering capacity and precomputation cost (i.e., index size).

### 3.4 Anchor Points-Based Filter

The *anchor* (*pivot*) *point* technique is widely employed in various applications, which aims to reduce the query computation cost based on some precomputed *anchor* (*pivot*) *points*. In this section, we investigate how to apply *anchor* point technique to effectively and efficiently reduce the candidate set size. Following is a motivating example for the *anchor point*-based filtering technique.

**Motivating Example.** Regarding our running example, in Fig. 9 the shaded area, denoted by $C_{o,d}$, is the circle centered at $o$ with radius $d$. Suppose the probabilistic mass of $Q$ in $C_{o,d}$ is 0.8, then when $\theta = 0.2$ we can safely prune $p_1$,

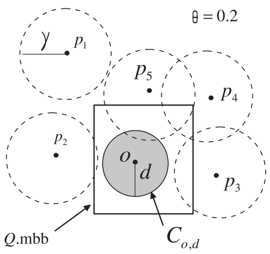
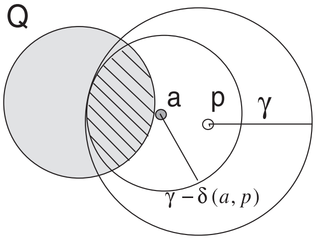
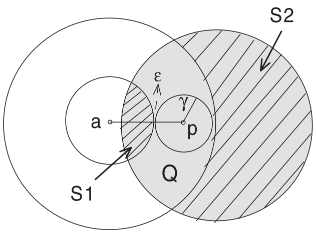



Fig. 9. Running example regarding the *anchor* point.




(a) Lower Bound




(b) Upper Bound

Fig. 10. Lower and upper bound.

$p_2$, $p_3$, and $p_4$ because $C_{p_i,\gamma}$ does not intersect $C_{o,d}$ for $i = 1, 2,$ 3, and 4.

In the paper, an anchor point $a$ regarding the uncertain query $Q$ is a point in multidimensional space whose falling probability against different $\gamma$ values are precomputed. We can *prune* or *validate* a point based on its distance to the *anchor point*. For better filtering capability, a set of *anchor points* will be employed.

In the following part, Section 3.4.1 presents the *anchor point* filtering technique. In Section 3.4.2, we investigate how to construct *anchor points* for a given space budget, followed by a time efficient filtering algorithm in Section 3.4.3.

### 3.4.1 Anchor Point Filtering Technique

For a given *anchor point* $a$ regarding the uncertain query $Q$, suppose $P_{fall}(a, l)$ is precomputed for arbitrary distance $l$. Lemma 2 provides lower and upper bounds of $P_{fall}(p, \gamma)$ for any point $p$ based on the triangle inequality. This implies we can *prune* or *validate* a point based on its distance to an *anchor point*.

**Lemma 2.** *Let $a$ denote an anchor point regarding the uncertain query $Q$. For any point $p \in S$ and a distance $\gamma$, we have*

1. *If $\gamma > \delta(a,p)$, $P_{fall}(p,\gamma) \geq P_{fall}(a, \gamma - \delta(a,p))$.*
2. *$P_{fall}(p,\gamma) \leq P_{fall}(a, \delta(a,p) + \gamma) - P_{fall}(a, \delta(a,p) - \gamma - \epsilon)$ where $\epsilon$ is an arbitrarily small positive value.*[1]

**Proof.** Suppose $\gamma > \delta(a,p)$, then according to the triangle inequality for any $x \in Q$ with $\delta(x,a) \leq \gamma - \delta(a,p)$, we have

$$\delta(x,p) \leq \delta(a,p) + \delta(x,a) \leq \delta(a,p) + (\gamma - \delta(a,p)) = \gamma$$

This implies that $P_{fall}(p,\gamma) \geq P_{fall}(a, \gamma - \delta(a,p))$ according to (2). Fig. 10a illustrates an example of the proof in 2D space. In Fig. 10a, we have $C_{a,\gamma-\delta(a,p)} \subseteq C_{p,\gamma}$ if $\gamma > \delta(a,p)$. Let $S$ denote the striped area which is the intersection of $C_{a,\gamma-\delta(a,p)}$ and $Q$. Clearly, we have $P_{fall}(a, \gamma - \delta(a,p)) = \int_{x \in S} Q.pdf(x)dx$ and $\delta(x,p) \leq \gamma$ for any $x \in S$. Consequently, $P_{fall}(p,\gamma) \geq P_{fall}(a, \gamma - \delta(a,p))$ holds.

With similar rationale, for any $x \in Q$ we have $\delta(x,a) \leq \delta(a,p) + \gamma$ if $\delta(x,p) \leq \gamma$. This implies that $P_{fall}(p,\gamma) \leq P_{fall}(a, \delta(a,p) + \gamma)$. Moreover, for any $x \in Q$ with $\delta(x,a) \leq \delta(a,p) - \gamma - \epsilon$, we have $\delta(x,a) > \gamma$. Recall that $\epsilon$ represents an arbitrarily small constant value. This implies that $x$ does not contribute to $P_{fall}(p,\gamma)$ if $\delta(x,a) \leq \delta(a,p) - \gamma - \epsilon$. Consequently, $P_{fall}(p,\gamma) \leq P_{fall}(a, \delta(a,p) + \gamma) - P_{fall}(a, \delta(a,p) - \gamma - \epsilon)$ holds. As shown in Fig. 10b, we have $P_{fall}(p,\gamma) \leq P_{fall}(a, \delta(a,p) + \gamma)$ because $C_{p,\gamma} \subseteq C_{a,\delta(a,p)+\gamma}$. Since $C_{a,\delta(a,p)-\gamma-\epsilon} \subseteq C_{a,\delta(a,p)+\gamma}$ and $C_{p,\gamma} \cap C_{p,\delta(a,p)-\gamma-\epsilon} = \emptyset$, this implies that $P_{fall}(p,\gamma) \leq P_{fall}(a, \delta(a,p) + \gamma) - P_{fall}(a, \delta(a,p) - \gamma - \epsilon)$. □

1. We have $P_{fall}(a, \delta(a,p) - \gamma - \epsilon) = 0$ if $\delta(a,p) \leq \gamma$.

Let $LP_{fall}(p,\gamma)$ and $UP_{fall}(p,\gamma)$ denote the lower and upper bounds derived from Lemma 2 regarding $P_{fall}(p,\gamma)$. Then, we can immediately *validate* a point $p$ if $LP_{fall}(p,\gamma) \geq \theta$, or *prune* $p$ if $UP_{fall}(p,\gamma) < \theta$.

Clearly, it is infeasible to keep $P_{fall}(a,l)$ for arbitrary $l \geq 0$. Since $P_{fall}(a,l)$ is a monotonic function with respect to $l$, we keep a set $D_a = \{l_i\}$ with size $n_d$ for each *anchor point* such that $P_{fall}(a, l_i) = \frac{i}{n_d}$ for $1 \leq i \leq n_d$. Then, for any $l > 0$, we use $UP_{fall}(a,l)$ and $LP_{fall}(a,l)$ to represent the upper and lower bound of $P_{fall}(a,l)$, respectively. Particularly, $UP_{fall}(a,l) = P_{fall}(a,l_i)$ where $l_i$ is the smallest $l_i \in D_a$ such that $l_i \geq l$. Similarly, $LP_{fall}(a,l) = P_{fall}(a,l_j)$ where $l_j$ is the largest $l_j \in D_a$ such that $l_j \leq l$. Then, we have the following theorem by rewriting Lemma 2 in a conservative way.

**Theorem 3.** *Given an uncertain query $Q$ and an anchor point $a$, for any rectangular region $r$ and distance $\gamma$, we have*

1. *If $\gamma > \delta_{max}(a,r)$, $P_{fall}(r,\gamma) \geq LP_{fall}(a, \gamma - \delta_{max}(a,r))$.*
2. $$P_{fall}(r,\gamma) \leq UP_{fall}(a, \delta_{max}(a,r) + \gamma) - LP_{fall}(a, \delta_{min}(a,r) - \gamma - \epsilon)$$

*where $\epsilon$ is an arbitrarily small positive value.*

Let $LP_{fall}(r,\gamma)$ and $UP_{fall}(r,\gamma)$ represent the lower and upper bounds of the falling probability derived from Theorem 3. We can safely *prune* (*validate*) an entry $e$ if $UP_{fall}(e_{mbb},\gamma) < \theta$ ($LP_{fall}(e_{mbb},\gamma) \geq \theta$). Recall that $e_{mbb}$ represents the minimal bounding box of $e$. It takes $O(d)$ time to compute $\delta_{max}(a, e_{mbb})$ and $\delta_{min}(a, e_{mbb})$. Meanwhile, the computation of $LP_{fall}(a,l)$ and $UP_{fall}(a,l)$ for any $l > 0$ costs $O(\log n_d)$ time because precomputed distance values in $D_a$ are sorted. Therefore, the filtering time of each entry is $O(d + \log n_d)$ for each *anchor point*.

### 3.4.2 Heuristic with a Finite Number of Anchor Points

Let *AP* denote a set of *anchor points* for the uncertain query $Q$. We do not need to further process an entry $e$ in Algorithm 1 if it is filtered by **any** anchor point $a \in$ AP. Intuitively, the more *anchor points* employed by $Q$, the more powerful the filter will be. However, we cannot employ a large number of *anchor points* due to the space and filtering time limitations. Therefore, it is important to investigate how to choose a limited number of *anchor points* such that the filter can work effectively.

***Anchor points* construction.** We first investigate how to evaluate the "goodness" of an *anchor point* regarding the computation of $LP_{fall}(p,\gamma)$. Suppose all *anchor points* have the same falling probability functions; that is $P_{fall}(a_i,l) = P_{fall}(a_j,l)$ for any two *anchor points* $a_i$ and $a_j$. Then, the closest *anchor point* regarding $p$ will provide the largest $LP_{fall}(p,\gamma)$. Since there is no a priori knowledge about the distribution of the points, we assume they follow the uniform distribution. Therefore, *anchor points* should be uniformly distributed. If falling probabilistic functions of the *anchor points* are

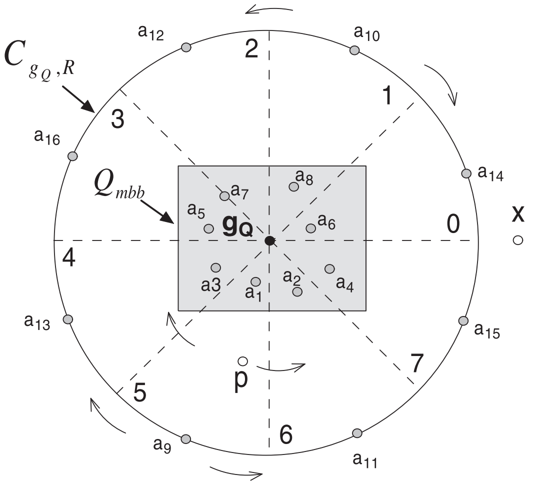



Fig. 11. *Anchor points* construction.

different, besides the distance a point $p$ prefers the *anchor point* whose falling probability grows rapidly with respect to distance $l$. However, since it is costly to evaluate the falling probability functions for all possible *anchor points*, we only consider that *anchor points* should be evenly distributed.

As to $UP_{fall}(p,\gamma)$, observations above do not hold. According to Lemma 2, $UP_{fall}(p,\gamma)$ equals $P_{fall}(a,\delta(a,p)+\gamma) - P_{fall}(a,\delta(a,p)-\gamma-\epsilon)$. Clearly $P_{fall}(a,\delta(a,p)+\gamma)$ prefers a small $\delta(a,p)$, while $P_{fall}(a,\delta(a,p)-\gamma-\epsilon)$ is in favor of large $\delta(a,p)$. Therefore, for $UP_{fall}(p,\gamma)$ we evaluate the "goodness" of an anchor point from another point of view. As shown in Fig. 10b, let $R$ denote the shaded area which is the intersection of $S$ and $Q$, where $S$ represents $C_{a,\delta(a,p)+\gamma} - C_{a,\delta(a,p)-\gamma-\epsilon}$. Intuitively $R$ with smaller area is more likely to lead to a tighter upper bound. Consequently the anchor point should be far from $Q$.

Considering the different preferences of $LP_{fall}$ and $UP_{fall}$, we employ two kinds of *anchor points*, namely *inner anchor points* (*IAPs*) and *outer anchor points* (*OAPs*), respectively, where *inner anchor points* and *outer anchor points* are chosen in favor of *validating* and *pruning*, respectively. Let $k_i$ and $k_o$ denote the number of *IAPs* and *OAPs*, respectively, and the total number of anchor points is denoted by $n_{ap}$ where $n_{ap} = k_i + k_o$. We assume $k_i = k_o$ in the paper. Note that the construction time of each anchor point is $O(n_d \times C_I)$ where $C_I$ is the cost to identify each $l_i \in D_a$ such that $P_{fall}(a,l_i) = \frac{i}{n_d}$.

Fig. 11 illustrates how to construct *anchor points* on 2D data with $n_{ap} = 16$. Considering that points far from $Q$ have less chance to be *validated*, we choose 8 *IAPs* such that their locations are uniformly distributed within $Q_{mbb}$. They are $a_1, a_2, \ldots, a_8$ in Fig. 11. Then another 8 *OAPs* ($a_9, a_{10}, \ldots, a_{16}$ in Fig. 11) are evenly located along $C_{g_Q,R}$ where $R$ is set to $\frac{L}{2}$ in our experiment and $L$ is the domain size. For each anchor point $a$, $P_{fall}(a,l)$ is computed for each $l \in D_a$. This construction approach can be easily extended to higher dimensions.

### 3.4.3 Time Efficient Filtering Algorithm

Based on Theorem 3, we can compute $LP_{fall}$ and $UP_{fall}$ for each entry $e$ against the *anchor points* as discussed in Section 3.4.1, and then try to *prune* or *validate* $e$. Nevertheless, we show that the filtering time cost can be improved by the following techniques.

**1. Quick-Filtering Algorithm.** Since in Algorithm 1 we only need to check whether an entry $e$ can be filtered, the explicit computation of $LP_{fall}$ and $UP_{fall}$ regarding $e$ may be avoided. Let $UD(\theta)$ represent the smallest $l_i \in D_a$ such that $P_{fall}(a,l_i) \geq \theta$ and $LD(\theta)$ be the largest $l_i \in D_a$ with $P_{fall}(a,l_i) \leq \theta$. Clearly, for any $l \leq LD(\theta)$, we have $UP_{fall}(a,l) \leq \theta$ and $LP_{fall}(a,l) \geq \theta$ for any $l \geq UP(\theta)$.

Algorithm 2 illustrates how to efficiently filter an entry based on an *anchor point*. In case 1 of Algorithm 2 (Line 2-8), $\gamma - \delta(a,e_{mbb}) \geq UD(\theta)$ implies $P_{fall}(e_{mbb},\gamma) \geq \theta$ according to Theorem 3 and the definition of $UD(\theta)$. Therefore the entry $e$ can be safely *validated*.

**Algorithm 2.** *Quick-Filtering* ($e$, $a$, $\gamma$, $\theta$)
**Input:** $e$: entry of $R_S$, $a$: *anchor point*
$\gamma$: query distance, $\theta$: probabilistic threshold
**Output:** $status : \{validated, pruned, unknown\}$
**Description:**
1: Compute $\delta_{max}(a,e_{mbb})$;
2: **if** $\gamma - \delta_{max}(a,e_{mbb}) \geq UD(\theta)$ **then**
3: **if** $\delta_{min}(a,e_{mbb}) + \gamma + \epsilon > UD(1-\theta)$ **then**
4: Return *pruned*;
5: **else**
6: Return *unknown*;
7: **end if**
8: **end if**
9: **if** $\delta_{min}(a,e_{mbb}) - \gamma - \epsilon \leq LD(0)$ **then**
10: **if** $\delta_{max}(a,e_{mbb}) < LD(\theta) - \gamma$ **then**
11: Return *pruned*;
12: **else**
13: Return *unknown*;
14: **end if**
15: **end if**
16: **if** $UP_{fall}(e_{mbb},\gamma) < \theta$ **then**
17: Return *pruned*;
18: **else**
19: Return *unknown*;
20: **end if**

For case 2 (Line 9-15), $\delta_{max}(a,e_{mbb}) + \gamma \geq UD(1)$ implies $UP_{fall}(a,\delta_{max}(a,e_{mbb})+\gamma) = 1.0$.[2] We can *prune* $e$ if $\delta_{min}(a,e_{mbb}) + \gamma + \epsilon > UD(1-\theta)$ because this implies $UP_{fall}(e_{mbb},\gamma) < \theta$. With similar rationale, the correctness of case 3 (Line 16-20) immediately follows:[3]

Once $\theta$ and $\gamma$ are given for the query, we can precompute $UD(\theta)$, $LD(\theta)$, $LD(0)$, and $UD(1-\theta)$ for each *anchor point*. Therefore, the time cost for Line 1-14 in Algorithm 2 is $O(d)$. We need to further apply Theorem 3 to compute $UP_{fall}(e_{mbb},\gamma)$ in Line 15 if none of above three cases occurs. Consequently, the filtering time complexity of Algorithm 2 remains $O(d + \log n_d)$ for each *anchor point* in the worst case. Nevertheless, our experiment shows that new technique significantly improves the filtering time.

**2. Accessing order.** In Algorithm 1, for a given entry $e$, we can apply Algorithm 2 for each *anchor point* $a \in AP$ until $e$ is filtered by **any** *anchor point* or all *anchor points* fail to

2. Note that in case 2, we have $S2 = \emptyset$ in Fig. 10b where $S2 = \{x | x \in Q$ and $\delta(x,a) > \delta(a,p) + \gamma\}$. This implies that $P_{fall}(p,\gamma) \leq 1.0 - P_{fall}(a,\delta(a,p)-\gamma-\epsilon)$.

3. Note that in case 3, we have $S1 = \emptyset$ in Fig. 10b where $S1 = \{x | x \in Q$ and $\delta(x,a) < \delta(a,p) - \gamma - \epsilon\}$. This implies that $P_{fall}(p,\gamma) \leq P_{fall}(a,\delta(a,p)+\gamma) - 0$.

filter $e$. Intuitively, we should choose a good accessing order of the *anchor points* such that the entry $e$ could be eliminated as early as possible. The ranking criteria for *validating* is simple, and *anchor points* close to $e$ have high priority. As to the *pruning*, we prefer anchor points which are close to or far from $e$. This is because Line 6 of Algorithm 2 prefers the large $\delta_{min}(a, e_{mbb})$ while Line 11 is in favor of small $\delta_{max}(a, e_{mbb})$. Our experiment shows that a good accessing order can significantly reduce the filtering cost.

**3. Sector-based heuristic.** Since we can immediately apply Algorithm 2 once the minimal and maximal distances between $e$ and an *anchor point* are computed, it is less efficient to decide the accessing order after computing distances between all *anchor points* and $e$. Therefore, we employ a simple heuristic to order *anchor points* without distance computations. We first consider the 2D case, and the heuristic can be easily extended to higher dimensions.

As shown in Fig. 11, suppose $C_{g_Q,R}$ is evenly partitioned into $n_s = 8$ sectors, labeled from 0 to 7, respectively. Recall that $g_Q$ represents the *mean* of the uncertain query $Q$. Given a point $p$, we can find its corresponding sector by computing the angle $\alpha$ between vectors $\overrightarrow{g_Q p}$ and $\overrightarrow{g_Q x}$. Then, $p$ locates at $\frac{\alpha \times k_o}{2\pi}$ th sector which is 5 in the example. The other sectors are sorted and the $i$th sector accessed has subindex $s + (-1)^i \frac{i}{2} + n_s \ mod \ n_s$ for $2 \leq i \leq n_s$ where $s$ indicates the subindex of the first sector visited, which is 5 in the example. Therefore, the accessing order of the sectors in the example is 5, 6, 4, 7, 3, 0, 2, and 1. Intuitively, anchor points in the sectors accessed earlier are likely to be closer to the point $p$. In the example, we visit *inner anchor points* with the following order: $a_1$, $a_2$, $a_3$, $a_4$, $a_5$, $a_6$, $a_7$, and $a_8$.

As to *outer anchor points*, eight sectors will be ordered exactly same as previous one. Nevertheless they will be accessed from head and tail at same time as shown in Fig. 11 because *anchor points* which are close to or far from $p$ are preferred. Consequently, the accessing order of the *outer anchor points* is $a_9$, $a_{10}$, $a_{11}$, $a_{12}$, $a_{13}$, $a_{14}$, $a_{15}$, and $a_{16}$ in the example. Our experiments confirm the effectiveness of this heuristic.

### 3.5 Discrete Case

Techniques proposed in this section can be immediately applied to *discrete* case. We abuse the notation of $pdf$ and also use $pdf$ to describe the uncertain query $Q$ in *discrete* case; that is, $Q.pdf(x) = P_x$ if $x \in Q$ and $Q.pdf(x) = 0$ if $x \notin Q$. $Q.region$ corresponds to the minimal bounding box of all instances of $Q$. Moreover, the statistic information of the uncertain query $Q$ in *discrete* case is defined as follows: $g_Q = \sum_{q \in Q} q \times P_q$, $\eta_Q = \sum_{q \in Q} \delta(q, g_Q) \times P_q$, and $\sigma_Q = \sum_{q \in Q} \delta(q, g_Q)^2 \times P_q$.

Then, the techniques proposed in this section can be applied to *discrete* case.

## 4 Experiment

We present results of a comprehensive performance study to evaluate the efficiency and scalability of the techniques proposed in the paper. Following the frame work of Algorithm 1 in Section 3, four different filtering techniques have been implemented and evaluated.

- *MMD.* The *maximal/minimal distance* filtering technique is employed as a benchmark to evaluate the efficiency of other filtering techniques.
- *STF.* The *statistical filtering* technique proposed in Section 3.2.
- *PCR.* The *PCR* technique discussed in Section 3.3. For the fairness of the comparison, we always choose a number of *PCR*s for the uncertain query such that space usage of *PCR* is same as that of *APF*.
- *APF.* The *anchor point* filtering technique proposed in Section 3.4.

The *euclidean* distance is used in the experiments. All algorithms are implemented in C++ and compiled by GNU GCC. Experiments are conducted on PCs with Intel P4 2.8 GZ CPU and 2 G memory under Debian Linux.

Two real spatial data sets, *LB* and *US*, are employed as target data sets which contain $53k$ and $1M$ 2D points representing locations in the Long Beach country and the United State, respectively.[4] All of the dimensions are normalized to domain $[0, 10000]$. To evaluate the performance of the algorithms, we also generate synthetic data set *Uniform* with dimensionality 2 and 3, respectively, in which points are uniformly distributed. The domain size is $[0, 10000]$ for each dimension, and the number of the points varies from 1 M to 5 M. All of the data sets are indexed by aggregate $R$ trees with page size 4,096 bytes. *US* data set is employed as the default data set.

A workload consists of 200 uncertain queries in our experiment. The uncertain regions of the uncertain queries in our experiment are circles or spheres with radius $q_r$. $q_r$ varies from 200 to 1,000 with default value 600 which is 6 percent of the domain size. The centers of the queries are randomly generated within the domain. Recall that the domain of each dimension is normalized to $[0, 10000]$ for all data sets in our experiment. Two popular distributions, *normal*and *Uniform*, are employed in our experiment to describe the PDF of the uncertain queries, where *normal* serves as default distribution. Specifically, we use the constrained normal distribution such that the possible location of the uncertain query are restricted in the uncertain region of query $Q$. The standard deviation $\sigma$ is set to $\frac{q_r}{2}$ by default. 10 k sampled locations are chosen from the distributions.[5] For an uncertain query $Q$, instances of the *normal* have the same appearance probabilities. Half of the instances of the *Uniform* have appearance probabilities 1/40,000, and another half have appearance probabilities 3/40,000. We assume instances of each query are maintained by an in memory aggregate $R$ tree with fan-out 8 where the aggregate probability of the child instances is kept for each entry. The query distance $\gamma$ varies from 600 to 2,000 with default value 1,200. Note that queries in a workload share the same system parameters. In order to avoid favoring a particular $\theta$ value, we randomly choose the probabilistic threshold between 0 and 1 for each uncertain query.

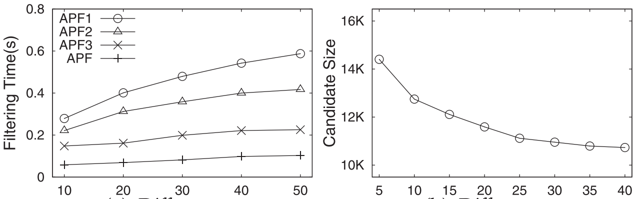
According to the analysis of Section 3.1, the dominant cost of Algorithm 1 comes from the IO operation and verification. Therefore, we measure the performance of the

4. Available at http://www.census.gov/geo/www/tiger.

5. As shown in [26], [28], the samples of the $Q.pdf$ are employed for the integral calculations. Therefore, we use the samples of the uncertain query to evaluate the performance of the algorithm for both *continuous* case and *discrete* case.

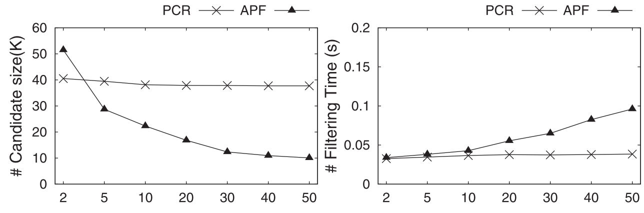
TABLE 2
System Parameters

| Notation | Definition (Default Values) |
|---|---|
| $q_r$ | the radius of the uncertain region (600) |
| $\sigma$ | standard deviation for *Normal* distribution (300) |
| $n_{ap}$ | the number of *anchor points* in *APF* (30) |
| $n_d$ | the size of $D_a$ for each anchor point (30) |
| $\gamma$ | query distance (1200) |
| $\theta$ | probabilistic threshold ($\in [0,1]$) |
| $n$ | the number of data points in $P$ (1m) |



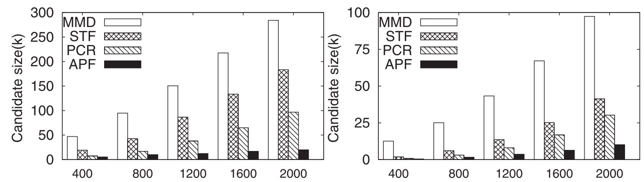

(a) *Diff.* $n_{ap}$ (b) *Diff.* $n_{ap}$

Fig. 12. Filter Performance Evaluation.




(a) *Diff.* $n_{ap}$ (b) *Diff.* $n_{ap}$

Fig. 13. Filter performance versus space usage.




(a) *US* (b) *3d uniform points*

Fig. 14. Candidate size versus $\gamma$.

filtering techniques by means of IO cost and candidate size during the computation. The IO cost is the number of pages visited from $R_P$, and the candidate size is the number of points which need verification. In addition, we also evaluate the filtering time and the query response time.

In each experiment, we issue 200 uncertain queries and use the average candidate size, number of nodes accessed, filtering time, and query response time to evaluate the performance of the techniques.

Table 2 lists parameters which may potentially have an impact on our performance study. In our experiments, all parameters use default values unless otherwise specified.

### 4.1 Evaluate *Anchor Point* Filtering Technique

In the first set of experiments, we evaluate the three techniques proposed for the *anchor point* filtering technique. All system parameters are set to default values. Let $APF1$ represent the algorithm which computes the lower and upper probabilistic bounds for each entry based on Theorem 3. Algorithm 2 is applied in algorithm $APF2$, $APF3$, and $APF$. $APF2$ always accesses *anchor points* in a random order, while $APF3$ chooses a "good" order based on the distances between the entry and *anchor points*. The *sector*-based heuristic is employed in *APF*. As expected, Fig. 12a demonstrates the effectiveness of the *APF*, which outperforms others and its filtering cost grows slowly against the number of *anchor points*. This implies the number of *anchor points* involved in the filtering process is less sensitive to $n_{ap}$ for *APF*. Consequently, *APF* is used to represent the *anchor point* filtering technique in the rest of the experiments.

In the second set of experiments, we evaluate the impact of $n_d$ (i.e., the number of different $\gamma$ values precomputed for each *anchor point*) against the performance of the *APF* technique based on *US* data set. As expected, Fig. 12b shows that the candidate size decreases against the growth of $n_d$. We set $n_d = 30$ in the rest of the experiments unless otherwise specified.

Fig. 13 evaluates the effect of $n_{ap}$ (i.e., the number of *anchor points*) against the filtering capacity of *APF* and *PCR*. Fig. 13a shows that *APF* outperforms *PCR* when there are not less than 5 *anchor points*. Moreover, the performance of the *APF* is significantly improved when the number of *anchor points* grows. Recall that we always choose a number of $PCRs$ such that *PCR* uses the same amount of space as *APF*. However, there is only a small improvement for *PCR* when more space is available. Although the filtering cost of *APF* increases with the number of *anchor points* and it is always outperformed by *PCR* in terms of filtering cost as shown in Fig. 13b, the cost is clearly paid off by the significant reduction of candidate size. This implies that *APF* can make a better tradeoff between the filtering capacity and space usage. We set $n_{ap} = 30$ in the rest of the experiments unless otherwise specified.

### 4.2 Performance Evaluation

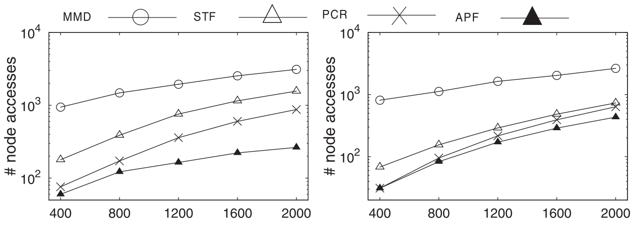
In this section, we conduct comprehensive experiments to evaluate the effectiveness and efficiency of our filtering techniques proposed in the paper.

In the first set of experiments, we evaluate the impact of query distance $\gamma$ on the performance of the filtering techniques in terms of candidate size, IO cost, query response time, and filtering cost. All evaluations are conducted against *US* and 3d *Uniform* data sets.

Fig. 14 reports the candidate size of *MMD*, *STF*, *PCR*, and *APF* when query distance $\gamma$ grows from 400 to 2,000. We set $d_{ap}$ to 30 and 60 for *US* and 3d *Uniform* data sets, respectively. As expected, larger $\gamma$ value results in more candidate data points in the *verification* phase. It is interesting to note that with only a small amount of statistic information, *STF* can significantly reduce the candidate size compared with *MMD*. *PCR* can further reduce the candidate size while *APF* significantly outperforms others especially for the large $\gamma$: only 19,650 data points need to be further verified for *APF* when $\gamma = 2{,}000$ on *US* data set, which is 96,769, 1,83,387 and 284,136 for *PCR*, *STF*, and *MMD*, respectively.

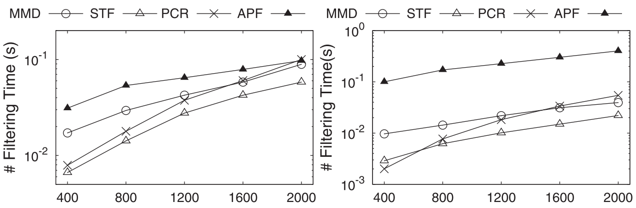
Fig. 15 reports the IO cost of the techniques. As expected, *APF* is more IO efficient than other filtering techniques on both data sets.

Fig. 16 reports the filtering time of four techniques against different query distance $\gamma$. It is shown that *STF* outperforms *MMD* under all settings. This is because both of them need at most two distance computations (i.e.,



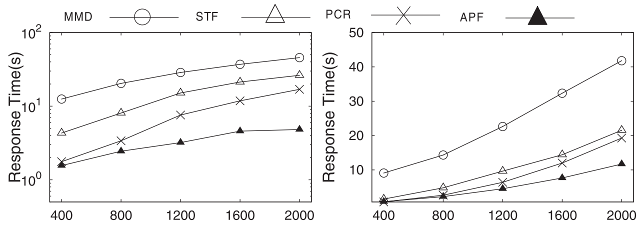

(a) *US* (b) *3d uniform points*

Fig. 15. # node accesses versus $\gamma$.




(a) *US* (b) *3d uniform points*

Fig. 16. Filtering time versus $\gamma$.




(a) *US* (b) *3d uniform points*

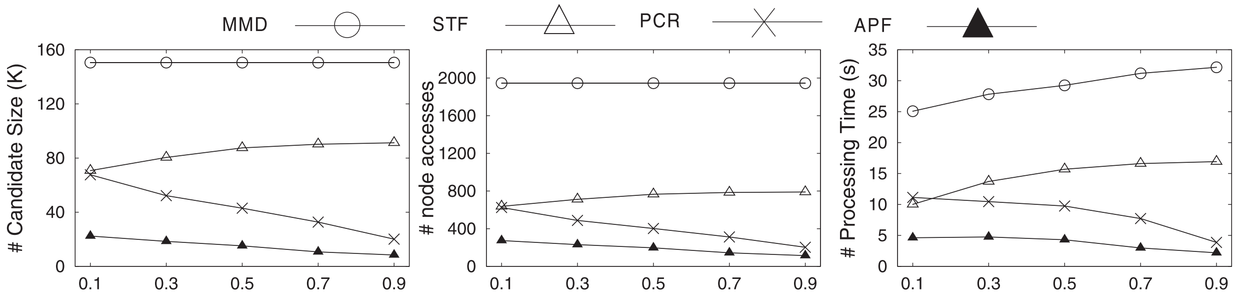
Fig. 17. Query response time versus $\gamma$.

maximal and minimal distance) for each entry, while the filtering capability of *STF* is much better than that of *MMD*. As expected, *APF* takes more time for filtering especially on 3D *Uniform* data set because more distance computations are required in *APF* and each costs $O(d)$ time. Nevertheless, it only takes less than 0.4 second for 3d *Uniform* data set with $\gamma = 2{,}000$, which is only a small portion of the computation cost as shown in Fig. 17.

In Fig. 17, we report the average processing time of 200 uncertain queries. It is not surprising to see that *APF* outperforms other filtering techniques on the query response time evaluation because with reasonable filtering cost, *APF* has best filtering capability regarding candidate size and IO cost.

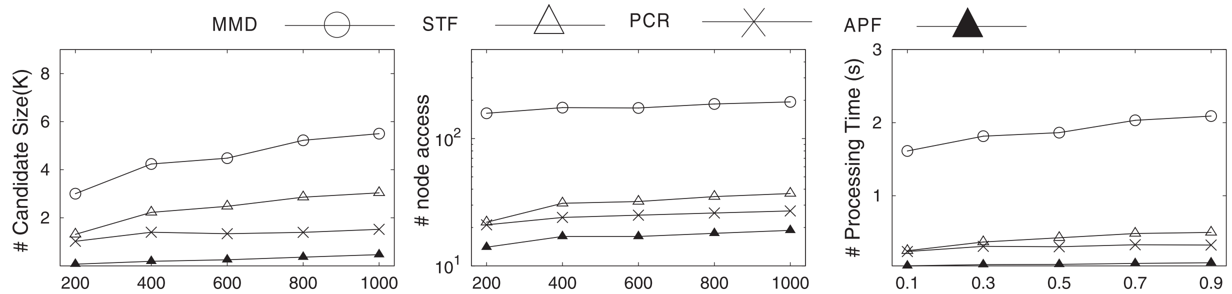
The second set of experiments depicted in Fig. 18 is conducted to investigate the impact of probabilistic threshold $\theta$ against the performance of the filtering techniques, where candidate size, the number of nodes assessed, and the average query response time are recorded. The performance of *MMD* does not change with $\theta$ value on the candidate size and number of nodes accessed because the probabilistic threshold is not considered in *MMD* technique. Intuitively, the larger the threshold, there should be less entries which have a chance to get involved in the computation. This is confirmed by *PCR* and *APF* whose performances are better when $\theta$ is large, especially for *PCR*. It is interesting that *STF* has better performance when $\theta$ is small, even as good as *PCR* when $\theta = 0.1$.

We investigate the impact of $q_r$ against the performance of the techniques in the third set of experiments. *LB* data set is employed and uncertain queries follow *Uniform* distribution, while other system parameters are set to default values. Clearly, the increment of $q_r$ degrades the performance of all filtering techniques because more objects are



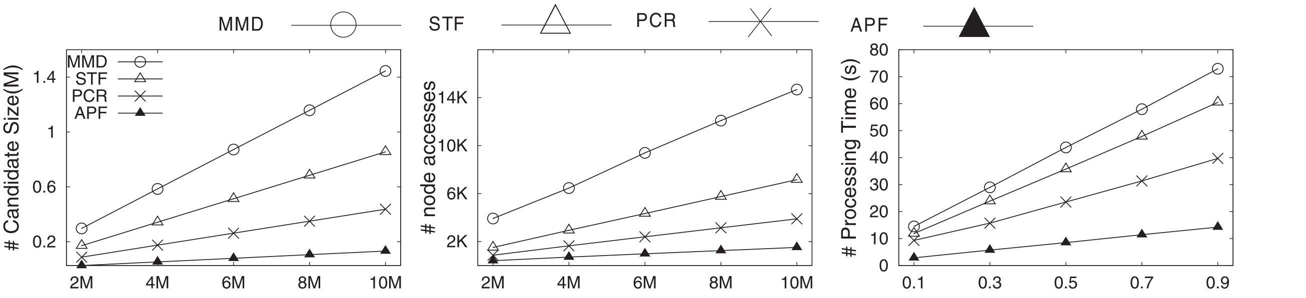

(a) Candidate Size (b) # Node Accesses (c) # Response Time

Fig. 18. Performance versus Diff. $\theta$.




(a) Candidate Size (b) # Node Accesses (c) # Response Time

Fig. 19. Performance versus $q_r$ (*LB*).




(a) Candidate Size (b) # Node Accesses (c) # Response Time

Fig. 20. Performance versus $n$.

involved in the computation. Nevertheless, as illustrated in Fig. 19 *PCR* and *APF* techniques are less sensitive to $q_r$.

In the last set of experiments, we study the scalability of the techniques against 2d *Uniform* data set with size varying from 2M to 10M. As shown in Fig. 20, the performance of *PCR* and *APF* are quite scalable toward the size of data set.

### 4.3 Summary

It has a much better performance than *MMD* with the help of a small amount of precomputed statistic information. When more space is available, *PCR* and *APF* have better performance than *MMD* and *STF*. Compared with *PCR*, our experiments show *APF* can achieve a better tradeoff between the filtering capability and the space usage.

## 5 Discussions

**Sum, min, max, and average.** Since the aggregate $R$ tree is employed to organize the data points, instead of the number of descendant data points, other aggregate values like *sum*, *max*, and *min* can be kept to calculate corresponding aggregate values. Note that for *average* we need to keep *count* and *sum* for each entry.

**Uncertain query with nonzero absent probability.** In some applications, the probability sum of the uncertain query might be less than 1 because the query might have a particular probability of being absent; that is, $\int_{x \in Q.region} Q.pdf(x)dx = c$ or $\sum_{q \in Q} P_q = c$ where $0 \leq c \leq 1$. The current techniques can be immediately applied by normalizing probabilities of the instances and the probabilistic threshold. For instances, suppose the uncertain query $Q$ has two instances with appearance probability 0.3 and 0.2, respectively, and the probability threshold $\theta$ is set to 0.2. Then, we can set the appearance probability of two instances to 0.6 and 0.4, respectively, and $\theta$ is set to 0.4.

**Computing expected value.** In some applications, instead of finding the number objects whose falling probabilities exceed a given probabilistic threshold, users may want to compute the expected number of objects falling in $Q$ regarding $\gamma$, denoted by $n_{avg}$; that is

$$n_{avg} = \sum_{p \in P} P_{fall}(p, \gamma). \tag{7}$$

Clearly, a point $p \in P$ can be immediately *pruned* (*validated*) if we can claim $P_{fall}(p, \gamma) = 0$ ($P_{fall}(p, \gamma) = 1$). Then, we need to compute the exact falling probabilities for all points survived, i.e., points not being *pruned* or *validated*. As a future direction, it is interesting to develop efficient algorithm to compute $n_{avg}$ such that the computational cost can be significantly reduced.

**Computing range aggregate against mutual exclusion.** In the motivating application 1 of the paper, the actual number of civilian objects destroyed may be less than $|Q_{\theta,\gamma}(P)|$ because the events of objects to be destroyed may be mutually exclusive; that is, once the falling position of the missile is determined, two objects $p_1$ and $p_2$ cannot be destroyed at same time (e.g., when $\delta(p_1, p_2) > 2\gamma$). Therefore, in addition to counting the number of civilian objects which are at risk, i.e., with falling probability at least $\theta$, it is also interesting to find out the maximal possible number of objects being destroyed with probability at least $\theta$. Although the algorithm in the paper can provide an upper bound for this problem, the techniques developed cannot be trivially applied for the exact solution because it is computational expensive to check the mutually exclusive property for each pair of objects in the candidate set. Therefore, as a future direction, it is desirable to develop novel techniques to efficiently solve this problem.

## 6 Related Work

To the best of our knowledge, this is the first work on the problem of computing range aggregates against uncertain location-based query. Our study is related to the previous work on querying uncertain objects [13], [21], [30], [8], [10], [1], [26], [28], [6], [29], [12].

Cheng et al. present a broad classification of probabilistic queries over 1d uncertain data as well as techniques for evaluating probabilistic queries [8]. There are four types in all, and the problem we studied in the paper belongs to the value-based aggregates. In recent years, *probability-thresholding range queries* has attracted much attention of the literature because of the importance of the problem. For the given region $r_q$ and probabilistic threshold $\theta$, such a query returns all objects which appear in $r_q$ with likelihood at least $\theta$. Based on the notation of *x-bounds*, a novel index, *probability threshold index* ($PTI$), is proposed in [10] to efficiently process the query against one dimension uncertain objects. Recently, Agarwal et al. [1] develop novel indexing technique for one dimension uncertain object to support range query with theoretical guarantee.

As a general version of the *x-bounds*, the concept of *probabilistic constrained region*(*PCR*) is introduced by Tao et al. [26] to support the range query on uncertain objects in a multidimensional space where the pdf of the uncertain object might be an arbitrary function. In order to prune or validate a set of uncertain objects at the same time, the *U-Tree* technique is proposed to index the uncertain objects based on *PCR* technique. Tao et al. further improve the *PCR* technique in [28] and they also study another range query problem in which locations of query and objects are uncertain. A similar problem is studied by Chen and Cheng [6] where the *PCR* technique is also applied to efficiently *prune* or *validate* data points or uncertain objects. Recently, Yang et al. [29] investigate the problem of range aggregate query processing over uncertain data in which two sampling approaches are proposed to estimate the aggregate values for the range query.

In [12], Dai et al. investigate the probabilistic range query on existentially uncertain data in which each object exists with a particular probability. Bohm et al. [3], [4] study range queries with the constraint that instances of uncertain objects follow *Gaussian* distribution. Results are ranked according to the probability of satisfying range queries. A more recent work addressing indexing high dimensional uncertain data is [2].

Besides the range query over uncertain data, many other conventional queries are studied in the context of uncertain data such as clustering [17], [20], similarity join [16], [9], top-k queries [25], [14], nearest neighbor queries [7], and skyline query [23], [18]. Among the previous work on uncertain data, [28], [6] and [15] are the most closely related to the problem we studied. However, the target objects considered in [28] are uncertain objects which are organized by $U$-tree, so their algorithms cannot be directly applied to the problem studied in the paper. Our problem definition is similar with the *C-IPQ* problem studied in [6], but their query search region is rectangle which might be less interesting to the applications mentioned in our paper. Moreover, we investigate the range aggregates computation in our paper which is different from [28], [6]. We also show that although the *PCR* technique employed by Tao et al. [28], [6] can be modified to our problem as discussed in the paper, it is less efficient compared with *anchor point* filtering technique proposed in the paper. Although Ishikawa et al. [15] study the range query in which the location of the query is imprecise, their techniques are based on the assumption that the possible locations of the query follow the *Gaussian* distribution, hence cannot support the general case. Therefore, their techniques cannot be applied to the problem studied in the paper.

## 7 Conclusion

In this paper, we formally define the problem of uncertain location-based range aggregate query in a multidimensional space; it covers a wide spectrum of applications. To efficiently process such a query, we propose a general filtering and verification framework and two novel filtering techniques, named *STF* and *APF*, respectively, such that the expensive computation and *IO* cost for verification can be significantly reduced. Our experiments convincingly demonstrate the effectiveness and efficiency of our techniques.

## Acknowledgments

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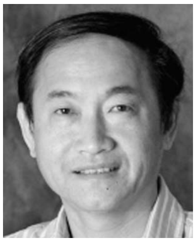

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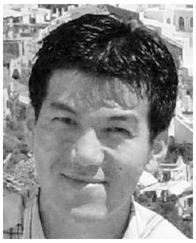

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