# Poster Abstract: Projection Matrix Optimisation for Compressive Sensing Based Applications in Embedded Systems

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## ABSTRACT

The information-preserving sampling properties of compressive sensing have found a number of successful applications, such as sensor scheduling, localisation and tracking to deal with the resource constraints of the embedded systems. In this paper, we investigate an approach to improve the performance of compressive sensing applications through a novel strategy for optimising the projection matrix. We formulate the projection matrix optimisation problem and apply greedy algorithm to solve the optimisation problem efficiently. We evaluate the proposed approach by an emerging background subtraction method designed specifically for the embedded systems and show the proposed approach outperforms existing approaches significantly with little overhead.

## Categories and Subject Descriptors

I.6 [**Simulation Modeling**]: Miscellaneous

## General Terms

Algorithm, Performance, Optimisation

## Keywords

Compressive Sensing, Background Subtraction, Greedy Search

## 1. INTRODUCTION

Compressive sensing (CS) applies to *sparse* signals. A discrete signal $x \in \mathbb{R}^N$ is said to be sparse if its representation in some transform domain $\mathcal{D} \in \mathbb{R}^{N \times N}$ has *few* non-zero elements. The representation of $x$ in $\mathcal{D}$ is the coefficient vector $\theta \in \mathbb{R}^N$ where $x = \mathcal{D}\theta$. The number of non-zero elements in $\theta$ is given by its $\ell_0$-"norm": $S = \|\theta\|_0$. Sparsity means that $S \ll N$.

CS employs a sampling method that can be expressed as the projection

$$y = \Phi x \tag{1}$$



where the the matrix $\Phi \in \mathbb{R}^{M \times N}$ is called the projection matrix ($M \ll N$) and $y \in \mathbb{R}^M$ contains the projected/sampled values. CS is designed to solve the following problem: given an unknown signal $x$ which is known to be sparse in transform domain $\mathcal{D}$, the problem is to recover $x$ from the projection values $y = \Phi x$ together with the knowledge of $\Phi$ and $\mathcal{D}$. The problem is underdetermined because the number of unknowns $N$ is larger than number of constraints $M$. However, by exploiting the sparsity of $x$, the theory of CS shows that it is possible to recover $x$, by the following $\ell_1$ minimisation problem

$$\hat{\theta} = \arg\min \|\theta\|_1 \quad \text{subject to } y = \Phi\mathcal{D}\theta \tag{2}$$

CS uses projection matrices to produce "compressed" samples. Random Gaussian and Bernoulli matrices are the projection matrices of choice in most CS based applications. However, The randomness of such matrices makes the performance vary substantially . In this paper we propose a novel strategy for optimising the projection matrix that offers superior performance to purely random strategies. Then we evaluate our optimised matrix on the efficient background subtraction method [2].

## 2. PROJECTION MATRIX OPTIMISATION

A sensing matrix $A \in \mathbb{R}^{M \times N}$ is defined as the product of projection matrix $\Phi$ and transform matrix $\mathcal{D}$

$$A = \Phi\mathcal{D} \tag{3}$$

There are two types of coherence in CS: the coherence between the columns (e.g. mutual coherence) and between the rows (row coherence) in the sensing matrix. It has been shown [1] in fact that *both* mutual coherence and row coherence affect the performance of CS. In this paper, we focus on the row coherence.

The row coherence is defined as:

$$\nu(A) = \max |A \cdot A^T - \text{diag}(A \cdot A^T)| \tag{4}$$

which is the maximum absolute value of cross-correlation between the rows of matrix $A$ which are normalised.

### 2.1 Optimising Row Coherence

Let us assume we have a finite (but still very large) set $\Omega$ of projection matrices stemming from either a random Gaussian or Bernoulli distribution. We know that any projection matrix $\Phi$ stemming from the set $\Omega$ will result in a sensing matrix $A$ that with high probability will have relatively low mutual and row coherence. The row-coherence of $A$ can be further minimised by choosing the sampling matrix $\Phi$ that minimises,

$$\arg\min_{\Phi} \nu(A) \quad \text{subject to } \Phi \subseteq \Omega \ . \tag{5}$$

It is clear that since the set $\Omega$ is finite, random is not convex making the objective in Equation (5) non-convex. Further, the search space for this combinatorial optimisation problem is enormous (even though $\Omega$ is finite) and is in fact NP-hard.

# 3. APPLICATIONS AND PERFORMANCE EVALUATION

In this section, we evaluate the performance of the optimised projection matrix derived from an efficient solution (greedy search) of Equation (5). We use G-mtx denote the optimised projection matrix from greedy algorithm, and compare it with the traditional pure random matrix: R-mtx.

In [2], the authors proposed a new MoG based background subtraction method, called CS-MoG. CS-MoG applies random Bernoulli matrix to reduce the dimensionality.

## 3.1 Coherence Analysis

Since there is no signal reconstruction in this application, the sparse transform domain $\mathcal{D}$ is not determined. However it is known that natural image are often sparse in DCT or wavelet domain. DCT or wavelet domain can be expressed as an orthonormal basis. We can assume there is a *hidden* basis in this application, which is either DCT, wavelet or other bases with similar properties. The comparison of the coherence of the R-mtx and G-mtx is shown in Figure 1. The results of R-mtx come from 30 independent trials. From the results, we can assert our proposed algorithm can achieve significantly better performance in minimising the row coherence of the projection matrix (up to 30%). The coherence of R-mtx shows large variance.

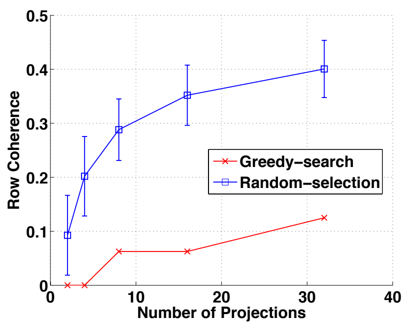


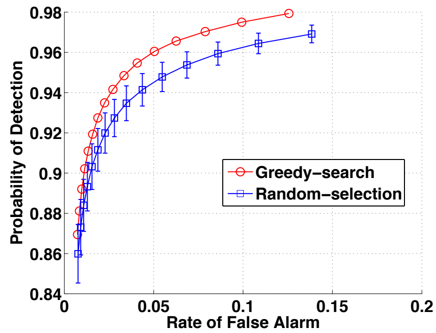

**Figure 1: Comparison of row coherence of the Bernoulli matrix under different number of projections**

## 3.2 Performance Evaluation

To further demonstrate the validity of the row coherence minimisation strategy, we evaluated the CS-MoG with R-mtx and G-mtx on two different datasets: PETS 2001 and VS-PETS' 2003 from `http://www.cvg.rdg.ac.uk/`.

In a similar fashion to [2], we regard a pixel in background (resp. foreground) as a negative (positive) event. Therefore, the accuracy of the background subtraction is analogous to


(a) PETS 2001




(b) PETS 2003

**Figure 2: ROC curves of different public datasets**

binary classification problem. The vertical axis of the ROC curve is the probability of detection ($P_D$) which is the total number of true positive detection divided by the number of the positive events in the ground truth. The horizontal axis of the ROC curve is the false alarm ($F_A$) which is the total number of false positives divided by the number of the negative events in the ground truth.

The ROC curves of the CS-MoG with different projection matrices are shown in Figure 3.2. We evaluate background subtraction performance using 1000 consecutive video frames from each dataset. Figure 3.2 demonstrates that G-mtx significantly outperforms R-mtx.

# 4. CONCLUSION

In this paper, we propose that by minimising the row coherence of the sensing matrix, we can find an optimal projection matrix that will significantly improve the performance of CS based background subtraction in embedded systems. The evaluation shows that the optimised projection matrix give consistently better performance than random projection matrix.

# 5. REFERENCES

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