Challenges in Resource and Cost Allocation

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Abstract

Many models and mechanisms in resource and cost allocation have been developed that are simple and abstract. By means of two case studies, I argue that it is now timely to consider richer models for the fair division of resources and for the allocation of costs. Such models should have features like asynchronicity which reflect more of the true complexity of many fair division and cost allocation problems met in the real world. I suggest that computation can be used in such models to increase both efficiency and fairness of the allocations. As a result, we may be able to do more with fewer resources and greater fairness.

Introduction

Resource allocation considers how to share contested resources between agents. We focus here on fair division, the special case of resource allocation where the goal is simply to allocate resources fairly and money is not exchanged. For instance, we might want to share viewing slots on a large telescope, or CPUs in a cloud server within the university. We also consider cost allocation. This looks at how to divide costs between agents sharing some resource in a fair way. For instance, we might want to divide the costs of running a smart grid between its users, or the costs of a supply chain amongst all the customers to whom we are delivering goods.

Computation is a powerful weapon in solving fair division and cost allocation problems. It can, as we shall argue, help us deal with larger and more complex models than are possible to reason about manually. Computation can also be used to improve both fairness and efficiency. There are several notions of efficiency and fairness in the fair division literature. These include envy freeness (no agent would prefer another's allocation), equitability (every agent assigns the same utility to their allocation), proportionality (each of the n agents assigns at least $\frac{1}{n}$ of their total utility to their allocation), Pareto optimality (there is no other allocation in which one agent is better off and all the other agents are not worst off), as well as welfare notions like utilitarian and egalitarian optimality. As many of these notions can conflict (e.g. giving all items to one agent is Pareto optimal but not equitable, whilst throwing away all the items is equitable but not Pareto optimal), an interesting direction is to use computation to explore the trade-off between the different notions.

Another reason to apply a computational lens to such problems is that performance in practice may not be characterised very well by a purely theoretical worst-case analysis. The worst-case may be met very rarely. One tool that we have used to good effect in a related area of social choice is identifying computational phase transitions (Walsh 2009; 2010; 2011b). Hard problems may be restricted to a small part of the parameter space. This approach has proved very useful in a number of domains (e.g. (Cheeseman, Kanefsky, and Taylor 1991; Mitchell, Selman, and Levesque 1992; Gent and Walsh 1994; 1996b; 1996a)). Another, related tool that has proved useful in several closely related domains like scheduling (allocating time slots) is parameterized complexity (e.g. (Bessière et al. 2008; Walsh 2008; Chu et al. 2013)). The worst-case may require, say, many agents or the number of items to be unrealistically large. If we can bound parameters describing the problem like the number of agents, problems may be tractable.

The research proposed here lies at an exciting interface which is opening up between optimisation, social choice and game theory. In the past, researchers have consciously simplified models of fair division and cost allocation as a means of making progress. By contrast, we propose here that more complex models should be tackled head on. We have the advantage that we can now throw significant computational resources at these problems. In addition, we have the advantage of being able to design mechanisms for new (computational) markets. Many of these markets are emerging in the internet and in mobile settings where we can (and in some cases must) use computational agents to do the allocation.

Much of the motivation driving this proposed research comes from the not-for-profit and public sector. In these domains, criteria like fairness are often very important, and the transfer of money may not be possible for legal, ethical and other reasons. Indeed, fairness is looking increasingly likely to be a major driver of political and economic reform over the next few decades. It is therefore very timely to explore how computation can be used to increase fairness and efficiency. Note that there are other areas of resource allocation like combinatorial auctions where complex and more realistic models have already been under development for some time but fairness is not necessarily the primary concern.

Models for fair division and cost allocation

Fair division has been categorised along several orthogonal dimensions: divisible or indivisible goods, centralised or decentralised mechanisms, cardinal or ordinal preferences, etc (Chevaleyre et al. 2006). As we describe shortly, such categories are not able to capture the full richness of many practical fair division problems. As an example of the sort of abstract model found in the literature, one of the most studied fair division problems is "cake cutting" in which we have a single resource that is infinitely divisible and agents with additive utility functions (Brams and Taylor 1996). As a second example, a simple model for fair division of indivisible goods that has been studied recently in the AI literature supposes we have m goods to allocate to n agents, and agents have additive utility functions that are based on Borda scores (Bouveret and Lang 2011; Kalinowski et al. 2013; Kalinowski, Narodytska, and Walsh 2013). As a third example, a simple model for cost allocation in cooperative game theory supposes we can assign a cost to each subset of agents (Winter 2002).

As we argue shortly, abstract models like these ignore the richness and structure of problems actually met in practice. For example, fair division problems are often repeated. The problem we meet today is likely to be similar to the one we will meet tomorrow. As a second example, fair division problems are often online. We must start allocating items before all the data is available. As a third example, cost functions are often complex, and dependent on time and other features of the problem. Such real world features offer both a challenge and an opportunity. For instance, by exploiting the repeated nature of a fair division problem, we may be able to increase fairness without decreasing efficiency. On the other hand, the online nature of an fair division problem makes it harder both to be efficient and fair.

Two case-studies

We discuss two case-studies (Walsh 2014) which illustrate some of the real world features met in problems in practice.

Case study #1: FoodBank Local

FoodBank Local is a startup coming out of the University of New South Wales that is working with Food Bank Australia and NICTA to improve the efficiency of the charity's operations. FoodBank Local were finalists in the worldwide Microsoft Imagine Cup for their novel and innovative approach to using technology for social good. The food bank allocates and distributes donated food to charities who themselves then distribute the food to those in need. This involves repeated fair division problems in which the donated food is allocated to the different charities in a fair way according to their different preferences. The food bank wants to be fair in allocating food so that it does not alienate charities, and so that every sector of the population receives at least some food. The food bank also wants to be efficient as wasting food puts off future donors, and as one of their primary goals is to reduce hunger. In addition, the charities have different preferences over the donated food. For example, some charities can cook the food whilst others cannot.

This fair division problem has several dimensions rarely considered in the literature. It is online (we cannot wait till the end of the day before starting to allocate food), repeated (we have a very similar though not identical fair division problem every day), contains items that don't have to be allocated (but can be stored), as well as items that must be allocated before their expiry date. In addition, there are unequal entitlements (as the different charities have different sizes), complex preferences (the different charities have non-linear preferences over bundles of items), and both divisible and indivisible goods. Finally, each fair division problem induces a vehicle routing problem to distribute the food. This means that we really have a combined fair division and routing problem. We need both to ensure a fair division and to minimize distribution costs.

To reason about such issues, we need to develop more complex and realistic models of fair division. Based on this case study, we propose the following challenges.

Challenge 1 (Complex fair division) *To develop and analyse models and mechanisms for fair division problems that are simultaneously online, repeated, combinatorial and constrained, as well as for subsets of these feaures.*

Challenge 2 (Mixed fair division and optimisation) *To develop and analyse models and mechanisms for combined fair division and distribution problems.*

Challenge 3 (Divisible and indivisible fair division) *To develop and analyse models and mechanisms for fair division problems that simultaneously involve both divisible and indivisible goods.*

Once we have richer models and mechanisms for fair division that address these challenges, we need to consider how agents will *actually* behave when using them. For example, consider a simple mechanism for fair division like sequential allocation (Brams and Taylor 1996; Bouveret and Lang 2011). In the sequential allocation mechanism, agents simply take turns to pick the item that they prefer most. This can be viewed as a repeated game in which agents may act strategically. Brams and Straffin (1979) argued that *"no algorithm is known which will produce optimal play [in this repeated game] more efficiently than by checking many branches of the game tree"*. In fact, we have proved that computing optimal play is PSPACE-hard (Kalinowski et al. 2013). Is it reasonable then to suppose agents will play optimally? In addition, behavioural game theory has identified a number of behaviours that are observed in practice like loss aversion and reciprocity. These take us away from idealized assumptions of agents playing optimally. Can we design mechanisms to take advantage both of agents (perhaps limited) computational resoruces and of their actual behaviours? This suggests two further challenges.

Challenge 4 (Behavioural analysis of mechanisms) *To analyse mechanisms for fair division problems based on how people actually behave including computational limits in their responses.*

Challenge 5 (Behaviourally optimised mechanisms) *To develop mechanisms for fair division problems that exploit how people actually behave to improve efficiency and fairness.*

Up till now, behavioural game theory has been largely a descriptive theory, attempting to model how humans behave in competitive situations by means of experimental observation (e.g. (Kahneman and Tversky 1979; Camerer 1997; 2003)). Our thesis is that we can use computation to turn behavioural game theory around into a normative theory that predicts how to build new mechanisms that work well in practice and exploits people's actual behaviours. Ultimately, we believe this will lay the foundations of a new field, which we call "*mechanism engineering*" where we identify the principles behind building mechanisms that work well in practice, taking into account how people actually behave. Mechanism design (Hurwicz and Reiter 2008) provides many of the theoretical foundations of how to build mechanisms with good properties. However, it does not consider the **engineering principles** of building mechanisms that work well **in practice** in large, complex, messy real world games. How, for example, can we build mechanisms that are robust? How do we build mechanisms that will scale well? And how should we test new mechanisms before releasing them in the wild?

Case study #2: cost allocation

We have come across similar rich features in a partner's real world cost allocation problem. Consider delivering goods from a depot to locations on a road network. At each location there is a customer, e.g. a vending machine or shop, that has requested some goods, e.g. milk, bread, or soda. One challenge in such a setting is deciding the cost to serve each location. More precisely, we must divide the costs of transportation to each location in a fair and efficient manner. The results of such a "cost to serve" analysis can be used in several ways. We could, of course, simply charge locations their portion of the transportation costs. More realistically, we can use the cost allocation when (re-)negotiating contracts with customers. We may also use the cost allocation to deciding when to change distribution channels or distribution frequency to a given location.

A naive method to allocate costs is simply to use the marginal cost of each customer. Unfortunately marginal costs tend to under-estimate the actual cost. Consider just two customers a long distance from the depot. Each has a small marginal cost to visit since we are already visiting the other. However, their actual cost to the business is half the total cost. Fortunately, more principled methods to allocate costs exist like the Shapley value, and these cope with such problems. The Shapley value equals the average marginal cost of a customer in *every* possible subset of customers. It has nice axiomatic properties like efficiency (it allocates the whole cost), anonymity (it treats all customers alike) and monotonicity (when the overall costs go up, no individual costs go down). Indeed, it is the *only* cost allocation mechanism that satisfies these three properties. However, we run into several complications when applying it to the our distribution problem.

One complication is that the Shapley value is computationally challenging to compute in general (Chalkiadakis, Elkind, and Wooldridge 2011). It involves summing an exponential number of terms (one for each possible subset of customers), and in our case each term requires solving to optimality a NP-hard routing problem. One response to the computational intractability of computing the Shapley value, is to look to approximate it. However, we have proved (Aziz et al. 2014) that even finding an approximation to the Shapley value of a customer is intractable in general.

Theorem 1 *Unless P=NP, there is no polynomial time α-approximation to the Shapley value of a customer in a TSP game for any constant factor $\alpha > 1$.*

We have therefore considered heuristic methods based on Monte Carlo sampling (Mann and Shapley 1960; Castro, Gomez, and Tejada 2009), and on approximating the cost of the optimal route. We have also considered simple proxies to the Shapley value like the depot distance (that is, allocating costs proportional to the distance between customer and depot), the shortcut distance (that is, allocating costs proportional to the reduction in distance if we simply skip the customer), as well as more complex proxies based on good heuristics like the Christofides and Held-Karp heuristics. Our experiments demonstrate that such proxies, especially the more complex ones can work well in practice.

There are many other complications in this cost allocation problem. These reflect the fact that the problem is much richer than imagined in such a simple abstract model based on the cost of each possible subset of customers. This richness raises several other issues besides computational complexity which are rarely considered in the literature. Several of these issues are similar to those encountered in the food bank fair division problem. For instance, the cost allocation problem is repeated (every day, we deliver to a similar set of customers), and constrained (since we must deliver to all supermarkets of one chain or none, we must constrain the subsets of customers to consider only those with all or none of the supermarkets in the chain). However, there are also several new issues to consider including the heterogeneity of customers (the Shapley value supposes customers are identical, which is not the case in our problem as different customers order different amounts of product), the complexity of the cost function (the Shapley value ignores the fact that costs are time dependent due to traffic), and the robustness of solutions (a small change to the customer base can have a large knock-on effect on the cost to serve).

Based on this case study, we propose the following challenges in cost allocation.

Challenge 6 (Complex cost allocation) *To adapt cost allocation mechanisms like the Shapley value to deal with multiple real world features like heterogenous customers, constrained coalitions and complex cost functions.*

Challenge 7 (Approximate cost allocation methods) *To develop and analyse approximations for cost allocation* *mechanisms like the Shapley value in complex vehicle routing games and other real life settings.*

Challenge 8 (Sensitivity analysis in cost allocation) *To develop and analyse tools for performing sensitivity analysis for cost allocation methods like the Shapley value in both simple games, as well as in more complex vehicle routing games and other real life settings.*

Related work

Richer models for fair division and cost allocation have been considered previously. For example, we have proposed an online version of the cake cutting problem in which agents arrive over time (Walsh 2011a). However, this model includes none of the other features that we have described here like repeated or constrained allocations. As a second example, Guo, Conitzer, and Reeves (2009) looked at repeated fair division problems with a single indivisible good. As a third example, Bouveret and Lang (2008) studied the fair division problems of indivisible goods when agents do not have completely ordered preferences over the goods, but instead have dichotomous and other succinctly specified types of preferences. Again, this model includes none of the other features that we have described here like repeated or constrained allocations. As a fourth example, Kash, Procaccia, and Shah (2013) set up a dynamic version of fair division, proposed some desirable axiomatic properties for such dynamic resource allocation, and designed two mechanisms that satisfy these properties. not. As a fifth example, Balan, Richards, and Luke (2011) studied a fairness in repeated game. However, a central assumption here was a centralized decision make who looked to maximize the leximin utility of the agents. As a sixth example, Engevall, Göthe-Lundgren, and Värbrand (2004) looked at cost allocation in a vehicle routing game with a heterogeneous fleet of vehicles. However, this study other features of the actual real world problem like the repeated nature of the delivery problems and the full complexity of the cost function.

We believe that putting together *multiple* real world features *simultaneously* presents a significant research challenge, and that computational power will help in many cases get good allocations. We do, however, note that there are several areas of resource allocation where more realistic models with multiple real world features have started to be analysed and fielded in practice including combinatorial auctions, kidney exchange, and course and school allocation problems (e.g. (Cramton, Shoham, and Steinberg 2006; Dickerson, Procaccia, and Sandholm 2012; Budish and Cantillon 2012)).

Conclusions

We have proposed some challenges in fair division and cost allocation focused on developing richer, more realistic computational models, and designing mechanisms for such models that work well both in theory and in practice. Many of the new applications of such models will be in distributed and asynchronous environments enabled by the internet and mobile technology. It is here that computational thinking and computational implementation is necessary, and is set to transform how we allocate costs and resources fairly and efficiently. Ultimately we need to design mechanisms that work well in practice for these richer models, even in the face of fundamental theoretical limitations that are likely to be discovered.

The motivation for these challenges comes out of two recent projects at NICTA involving fair division and cost allocation. Each is an allocation problem with several new dimensions, rarely considered in the literature. For example, our fair division problem is online, repeated, and constrained, whilst our cost allocation problem is also repeated and constrained, and additionally involves a complex cost function. These two case studies identify a number of features missing in many existing models. A number of research questions follow immediately from these case studies. For example, can we define new mechanisms for such complex models with good axiomatic properties? How do such mechanisms work in practice? Are there impossibility results that limit the properties that can be achieved with any mechanism in such complex models?

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