The RANGE and ROOTS Constraints: some applications

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Let $X_1, \ldots, X_n$ be a set of integer variables each with domain $dom(X_i)$, and let $D = \bigcup_i dom(X_i)$. Let $X : \{1, \ldots n\} \rightarrow D$ be the assignment function from the set of variable indices to their domain set D.

The RANGE constraint constrains T to be the range of function X restricted to the variables whose index belong to S. When S is not used it is implicitly equal to $\{1, ..., n\}$.

$$\text{RANGE}([X_1, \ldots, X_n], S, T) \text{ iff } T = \bigcup_{i \in S} \{X_i\}$$

The ROOTS constraint constrains S to be the set of indices which are inverses of an element of T.

$$\text{ROOTS}([X_1, \ldots, X_n], S, T) \text{ iff } S = \bigcup_{j \in T} X^{-1}(j)$$

where $X^{-1}(j)$ is the set of indices whose image is j.

We now give an extensive catalog of global constraints that can be expressed in terms of ROOTS and RANGE.

ALLDIFFERENT:

Semantics: ALLDIFFERENT($[X_1, \ldots, X_n]$) holds iff all X_i take different values.
Decomposition: RANGE($[X_1, \ldots, X_n], \{1, \ldots, n\}, T$) $\wedge$ $|T| = n$.

ALLDIFFERENTEXCEPT0:

Semantics: ALLDIFFERENTEXCEPT0($[X_1, \ldots, X_n]$) holds iff all X_i take different values except those variables which are assigned to 0.
Decomposition: RANGE($[X_1, \ldots, X_n], \{1, \ldots, n\}, T$) $\wedge$ COUNT($[X_1, \ldots, X_n], 0, =, N$) $\wedge$ $|T - \{0\}| = n - N$.

ALLDIFFERENTINTERVAL:

Semantics: ALLDIFFERENTINTERVAL($[X_1, \ldots, X_n], k$) holds iff all X_i take values from different intervals. The intervals are defined by $i * k..i * k + k - 1$ where i is an integer.
Decomposition: $\forall i \in \{0, \ldots, max(D)/k\}$. AMONGINTERVAL($[X_1, \ldots, X_n], i*k, i*k+k-1, N$) $\wedge$ $0 \leq N \leq 1$.

ALLDIFFERENTMODULO:

Semantics: ALLDIFFERENTMODULO($[X_1, \ldots, X_n], m$) holds iff all X_i have distinct rest when divided by m.
Decomposition: $\forall i \in \{0, \ldots, m-1\}$. AMONGMODULO($[X_1, \ldots, X_n], i, m, N$) $\wedge$ $0 \leq N \leq 1$.

AllDifferentPartition:

Semantics: AllDifferentPartition $([X_1, \ldots, X_n], [p_1, \ldots, p_m])$ holds iff all X_i take values from differe
partitions where each p_i defines a partition.
Decomposition: $\forall i \in \{1, \ldots, m\}$. AmongLowUp$(0, 1, [X_1, \ldots, X_n], p_i)$.

Among:

Semantics: Among$([X_1, \ldots, X_n], [d_1, \ldots, d_m], N)$ holds iff the number of variables in $X_1, \ldots, X_n$ which ta
their value in $d_1, \ldots, d_m$ is N.
Decomposition: Roots$([X_1, \ldots, X_n], S, \{d_1, \ldots, d_m\}) \ \wedge \ |S| = N$.

AmongInterval:

Semantics: AmongInterval$([X_1, \ldots, X_n], low, up, N)$ holds iff the number of variables in $X_1, \ldots, X_n$ wh
take their value in the interval $low..up$ is N.
Decomposition: Among$([X_1, \ldots, X_n], [low, low + 1 \ldots, up - 1, up], N)$.

AmongLowUp:

Semantics: AmongLowUp$(low, up, [X_1, \ldots, X_n], [d_1, \ldots, d_m])$ holds iff the variables in $X_1, \ldots, X_n$ take
least low and at most up values in $d_1, \ldots, d_m$.
Decomposition: Among$([X_1, \ldots, X_n], [d_1, \ldots, d_m], N) \ \wedge \ low \leq N \leq up$.

AmongModulo:

Semantics: AmongModulo$([X_1, \ldots, X_n], rem, quot, N)$ holds iff the number of variables in $X_1, \ldots,$
which take their value as rem by modulo $quot$ is N.
Decomposition: Among$([X_1, \ldots, X_n], [rem, rem + quot \ldots, max], N)$ where max is the maximum value
D which is rem by modulo $quot$.

AmongSeq:

Semantics: AmongSeq$(low, up, seq, [X_1, \ldots, X_n], [d_1, \ldots, d_m])$ holds iff all sequences of seq variables
$X_1, \ldots, X_n$ take at least low and at most up values in $d_1, \ldots, d_m$.
Decomposition: $\forall i \in \{1, \ldots, n - seq\}$, AmongLowUp$(low, up, [X_i, \ldots, X_{i+seq-1}], [d_1, \ldots, d_m])$.

Apply:

Semantics: Apply(f, S, T) holds iff T is the set of values constructed by applying the function f to the
S.
Decomposition: Range$([X_1, \ldots, X_n], S, T)$ where $X_i = j$ iff $f(i) = j$.

Assign&Counts:

Semantics: Assign&Counts$(colours, [X_1, \ldots, X_n], [Y_1, \ldots, Y_n], op \in [\leq, \geq, <, >, \neq, =], N)$ holds iff by t
assignment of bin and colour to each of n items, the number of items d_i of colour $colour$ in each bin satisf
$d_i \ op \ N$.
Decomposition: $\forall i \in D$. Roots$([X_1, \ldots, X_n], S_i, \{i\}) \ \wedge$ Range$([Y_1, \ldots, Y_n], S_i, T_i) \ \wedge \ |T_i \cap colours| \ op$

ASSIGN&NVALUES:

Semantics: ASSIGN&NVALUES($[X_1, \dots, X_n], [Y_1, \dots, Y_n], op \in [\leq, \geq, <, >, \neq, =], N$) holds iff by the assignment of bin and value to each of n items, the number of distinct values d_i in each bin satisfies d_i *op* N.
Decomposition: $\forall i \in D$. ROOTS($[X_1, \dots, X_n], S_i, \{i\}$) $\wedge$ RANGE($[Y_1, \dots, Y_n], S_i, T_i$) $\wedge$ $|T_i|$ *op* N.

ATLEAST:

Semantics: ATLEAST($[X_1, \dots, X_n], v, N$) holds iff the number of variables in $X_1, \dots, X_n$ assigned to v is at least N.
Decomposition: ROOTS($[X_1, \dots, X_n], S, \{v\}$) $\wedge$ $|S| \geq N$.

ATLEASTNVALUE:

Semantics: ATLEASTNVALUE($[X_1, \dots, X_n], N$) holds iff the number of distinct values assigned to $X_1, \dots, X_n$ is at least N.
Decomposition: RANGE($[X_1, \dots, X_n], \{1, \dots, n\}, T$) $\wedge$ $|T| \geq N$.

ATMOST:

Semantics: ATMOST($[X_1, \dots, X_n], v, N$) holds iff the number of variables in $X_1, \dots, X_n$ assigned to v is at most N.
Decomposition: ROOTS($[X_1, \dots, X_n], S, \{v\}$) $\wedge$ $|S| \leq N$.

ATMOSTNVALUE:

Semantics: ATMOSTNVALUE($[X_1, \dots, X_n], N$) holds iff the number of distinct values assigned to $X_1, \dots, X_n$ is at most N.
Decomposition: RANGE($[X_1, \dots, X_n], \{1, \dots, n\}, T$) $\wedge$ $|T| \leq N$.

BALANCE:

Semantics: BALANCE($[X_1, \dots, X_n], op \in [\leq, \geq, <, >, \neq, =], N$) holds iff the difference d between the number of occurrences of the value that occurs the most and the value that occurs the least satisfies d *op* N.
Decomposition: GCC($[X_1, \dots, X_n], [d_1, \dots, d_{|D|}], [O_1, \dots, O_{|D|}]$) $\wedge$ RANGECTR($[O_1, \dots, O_{|D|}], op, N$) where $\{d_1, \dots, d_{|D|}\} = D$.

BALANCEINTERVAL:

Semantics: BALANCEINTERVAL($[X_1, \dots, X_n], N, k$) holds iff N is the difference between the minimum and the maximum number of variables in $X_1, \dots, X_n$ that take values from the same interval. The intervals are defined by $i * k..i * k + k - 1$ where i is an integer.
Decomposition: ($\forall i \in \{0, \dots, max(D)/k\}$. AMONGINTERVAL($[X_1, \dots, X_n], i * k, i * k + k - 1, N_i$)) $\wedge$ RANGECTR($[N_0, \dots, N_{max(D)/k}], =, N$).

BALANCEMODULO:

Semantics: BALANCEMODULO($[X_1, \dots, X_n], N, m$) holds iff N is the difference between the minimum and the maximum number of variables in $X_1, \dots, X_n$ that have the same rest when divided by m.
Decomposition:
($\forall i \in \{0, \dots, m-1\}$. AMONGMODULO($[X_1, \dots, X_n], i, m, N_i$)) $\wedge$ RANGECTR($[N_0, \dots, N_{m-1}], =, N$).

BalancePartition:

Semantics: BalancePartition($[X_1, \ldots, X_n], N, [p_1, \ldots, p_m]$) holds iff N is the difference between the m imum and the maximum number of variables in $X_1, \ldots, X_n$ that take values from the same partition wh each p_i defines a partition.
Decomposition: $(\forall i \in \{1, \ldots, m\}.$ Among$([X_1, \ldots, X_n], p_i, N_i)) \wedge$ RangeCtr$([N_1, \ldots, N_m], =, N)$.

BinPacking:

Semantics: BinPacking($[X_1, \ldots, X_n], c, weights$) holds iff by the assignment of a bin to each of n ite described by a weight (by the *weights* function), the total capacity of each bin does not exceed c.
Decomposition: $\forall i \in D.$ Roots$([X_1, \ldots, X_n], S_i, \{i\}) \wedge$ Apply$(weights, S_i, T_i) \wedge$ Sum$(T_i) \leq c$.

CardAtLeast:

Semantics: CardAtLeast($[X_1, \ldots, X_n], [d_1, \ldots, d_m], N$) holds iff N is the minimum number of times th a value of $d_1, \ldots, d_m$ is taken by the variables $X_1, \ldots, X_n$.
Decomposition: GCC$([X_1, \ldots, X_n], [d_1, \ldots, d_m], [O_1, \ldots, O_m]) \wedge$ Min$(N, [O_1, \ldots, O_m])$.
Note: This constraint is very similar to MinNValue in which the values are not specified.

CardAtmost:

Semantics: CardAtMost($[X_1, \ldots, X_n], [d_1, \ldots, d_m], N$) holds iff N is the maximum number of times th a value of $d_1, \ldots, d_m$ is taken by the variables $X_1, \ldots, X_n$.
Decomposition: GCC$([X_1, \ldots, X_n], [d_1, \ldots, d_m], [O_1, \ldots, O_m]) \wedge$ Max$(N, [O_1, \ldots, O_m])$.
Note: This constraint is very similar to MaxNValue in which the values are not specified.

CardAtmostPartition:

Semantics: CardAtMostPartition($[X_1, \ldots, X_n], [p_1, \ldots, p_m], N$) holds iff N is the maximum number times that values of a same partition of $p_1, \ldots, p_m$ are taken by the variables $X_1, \ldots, X_n$.
Decomposition: $(\forall i \in \{1, \ldots, m\}.$ Among$([X_1, \ldots, X_n], p_i, N_i) \wedge$ Max $(N, [N_1, \ldots, N_m])$.

ChangePartition:

Semantics: ChangePartition $(N, [X_1, \ldots, X_n], [p_1, \ldots, p_m])$ holds iff there are N consecutive pairs variables in $X_1, \ldots, X_n$ which take values from different partitions where each p_i defines a partition.
Decomposition: $(\forall i \in \{1, \ldots, n-1\}.$ AlldifferentPartition$([X_i, X_{i+1}], [p_1, \ldots, p_m]) \leftrightarrow N_i = 1) \wedge$ $\sum_{i \in \{1, \ldots, n-1\}} N_i = N$.

Common:

Semantics: Common($[X_1, \ldots, X_n], [Y_1, \ldots, Y_m], N, M$) holds iff N (resp. M) variables in $X_1, \ldots, X_n$ (re in $Y_1, \ldots, Y_m$) take their values in $Y_1, \ldots, Y_m$ (resp. in $X_1, \ldots, X_n$).
Decomposition: Among$([X_1, \ldots, X_n], [Y_1, \ldots, Y_m], N) \wedge$ Among$([Y_1, \ldots, Y_m], [X_1, \ldots, X_n], M)$.

COMMONMODULO:

Semantics: COMMONMODULO$([X_1,\ldots,X_n],[Y_1,\ldots,Y_m],N,M,k)$ holds iff N (resp. M) variables in $X_1,\ldots,X_n$ (resp. in $Y_1,\ldots,Y_m$) take their values in one of the equivalence classes derived from the values of $Y_1,\ldots,Y_m$ (resp. $X_1,\ldots,X_n$) modulo k.
Decomposition: RANGE$([Y_1,\ldots,Y_m],\{1,\ldots,m\},S_1)$ $\wedge$ APPLY$(\mathit{mod}\,k,S_1,T_1)$ $\wedge$ AMONG$([X_1,\ldots,X_n],T_1,N)$ $\wedge$ RANGE$([X_1,\ldots,X_n],\{1,\ldots,n\},S_2)$ $\wedge$ APPLY$(\mathit{mod}\ k,S_2,T_2)$ $\wedge$ AMONG$([Y_1,\ldots,Y_n],T_2,M)$.

COUNT:

Semantics: COUNT$([X_1,\ldots,X_n],v,op\in[\leq,\geq,<,>,\neq,=],N)$ holds iff the operation op is satisfied between the number of variables assigned to v and N.
Decomposition: ROOTS$([X_1,\ldots,X_n],S,\{v\})$ $\wedge$ $|S|\ op\ N$.

DISJOINT:

Semantics: DISJOINT$([X_1,\ldots,X_n],[Y_1,\ldots,Y_m])$ holds iff there is no overlap between the set of values assigned to $X_1,\ldots,X_n$ and those assigned to $Y_1,\ldots,Y_m$.
Decomposition: RANGE$([X_1,\ldots,X_n],\{1,\ldots,n\},S)$ $\wedge$ RANGE$([Y_1,\ldots,Y_m],\{1,\ldots,m\},T)$ $\wedge$ $S\cap T=\{\}$.
A special case of COMMON: COMMON$([X,\ldots,X_n],[Y_1,\ldots,Y_m],0,0)$.

ELEMENT:

Semantics: ELEMENT$(I,[X_1,\ldots,X_n],V)$ holds iff $X_I=V$.
Decomposition: $V\in T$ $\wedge$ $|T|=1$ $\wedge$ ROOTS$([X_1,\ldots,X_n],S,T)$ $\wedge$ $I\in S$.

GCC:

Semantics: GCC$([X_1,\ldots,X_n],[d_1,\ldots,d_m],[O_1,\ldots,O_m])$ holds iff the value d_i is used O_i times in $X_1,\ldots,X_n$.
Decomposition: $\forall i\in\{1\ldots,m\}$ ROOTS$([X_1,\ldots,X_n],S_i,\{d_i\})$ $\wedge$ $|S_i|=O_i$.

GLOBALCONTINUITY:

Semantics: GLOBALCONTINUITY$([X_1,\ldots,X_n])$ holds iff all 1's are consecutive.
Decomposition: ROOTS$([X_1,\ldots,X_n],S,\{1\})$ $\wedge$ MIN(S,Low) $\wedge$ MAX(S,Hi) $\wedge$ $|S|=\mathit{Hi}-\mathit{Low}+1$.

INSAMEPARTITION:

Semantics: INSAMEPARTITION(V_1,V_2,class) holds iff the values assigned to V_1 and V_2 are in the same class (with respect to class).
Decomposition: APPLY(class,V_1,S) $\wedge$ APPLY(class,V_2,T) $\wedge$ $S=T$.

INTERVALANDCOUNT:

Semantics: INTERVALANDCOUNT$(\mathit{colours},[X_1,\ldots,X_n],[Y_1,\ldots,Y_n],k,N)$ holds iff by the assignment of origin and colour to each of n tasks, the number of items d_i of colour colour in each interval satisfies $d_i\leq N$.
Decomposition:
$(\forall i\in\{0,\ldots,max(D)/k\}$. ROOTS$([X_1,\ldots,X_n],S_i,\{i*k..i*k+k-1\})$ $\wedge$ RANGE$([Y_1,\ldots,Y_n],S_i,T_i)$ $\wedge$ $|T_i\cap \mathit{colours}|\ \leq\ N$.

LinkSetToBooleans:

Semantics: LinkSetToBooleans $(S, [B_1, \ldots, B_n])$ holds iff the 0/1 variables $B_1, \ldots, B_n$ which are as ciated to a value belonging to the set variable S are 1, while the remaining 0/1 variables are all equal 0.
Decomposition: Roots$([B_1, \ldots, B_n], S, \{1\})$.

Max:

Semantics: Max$(Max, [X_1, \ldots, X_n])$ holds iff Max is the maximum value of the variables $X_1, \ldots, X_n$.
Decomposition: Range$([X_1, \ldots, X_n], \{1, \ldots, n\}, S) \;\wedge\;$ Max(S, Max).

MaxIndex:

Semantics: MaxIndex$(MaxI, [X_1, \ldots, X_n])$ holds iff $MaxI$ is the indices of the variables $X_1, \ldots, X_n$ c responding to the maximum value of the variables.
Decomposition: Max $(Max, [X_1, \ldots, X_n]) \;\wedge\;$ Roots$([X_1, \ldots, X_n], MaxI, \{Max\})$.

MaxModulo:

Semantics: Max$(Max, M, [X_1, \ldots, X_n])$ holds iff Max is the maximum value of the variables $X_1, \ldots,$ according to the partial order $(X \; mod \; M) < (Y \; mod \; M)$.
Decomposition: Range$([X_1, \ldots, X_n], \{1, \ldots, n\}, S) \;\wedge\;$ Apply$(mod \; M, S, T) \;\wedge\;$ Max(T, Max).

MaxN:

Semantics: MaxN$([X_1, \ldots, X_n], N, k)$ holds iff N is the maximum value of rank k (i.e. the k^{th} largest distin value) in $X_1, \ldots, X_n$.
Decomposition: Range$([X_1, \ldots, X_n], \{1, \ldots, n\}, S) \;\wedge\;$ KthMax(S, N).

MaxNValue:

Semantics: MaxNValue$([X_1, \ldots, X_n], N)$ holds iff N is the maximum number of times that the same val is taken by the variables $X_1, \ldots, X_n$.
Decomposition: GCC$([X_1, \ldots, X_n], [d_1, \ldots, d_{|D|}], [O_1, \ldots, O_{|D|}]) \wedge$ Max$(N, [O_1, \ldots, O_{|D|}])$ where $\{d_1, \ldots$ D.

Min:

Semantics: Min$(Min, [X_1, \ldots, X_n])$ holds iff Min is the minimum value of the variables $X_1, \ldots, X_n$.
Decomposition: Range$([X_1, \ldots, X_n], \{1, \ldots, n\}, S) \;\wedge\;$ Min(S, Min).

MinExcept0:

Semantics: MinExcept0$(Min, [X_1, \ldots, X_n])$ holds iff Min is the minimum value of the variables $X_1, \ldots,$ ignoring all variables that take 0 as value.
Decomposition: Range$([X_1, \ldots, X_n], \{1, \ldots, n\}, S) \;\wedge\; T = S - \{0\} \;\wedge\;$ Min(T, Min).

MinGreaterThan:

Semantics: MinGreaterThan($Var_1, Var_2, [X_1, \ldots, X_n]$) holds iff Var_1 is the smallest value strictly greater than Var_2 in $X_1, \ldots, X_n$.
Decomposition: MinN($[X_1, \ldots, X_n], Var_1, k+1$) $\wedge$ MinN($[X_1, \ldots, X_n], Var_2, k$).

MinIndex:

Semantics: MinIndex($MinI, [X_1, \ldots, X_n]$) holds iff $MinI$ is the indices of the variables $X_1, \ldots, X_n$ corresponding to the minimum value of the variables.
Decomposition: Min ($Min, [X_1, \ldots, X_n]$) $\wedge$ Roots($[X_1, \ldots, X_n], MinI, \{Min\}$).

MinModulo:

Semantics: Min($Min, M, [X_1, \ldots, X_n]$) holds iff Min is the minimum value of the variables $X_1, \ldots, X_n$ according to the partial order $(X\ mod\ M) < (Y\ mod\ M)$.
Decomposition: Range($[X_1, \ldots, X_n], \{1, \ldots, n\}, S$) $\wedge$ Apply($mod\ M, S, T$) $\wedge$ Min(T, Min).

MinN:

Semantics: MinN($[X_1, \ldots, X_n], N, k$) holds iff N is the minimum value of rank k (i.e. the k^{th} smallest distinct value) in $X_1, \ldots, X_n$.
Decomposition: Range($[X_1, \ldots, X_n], \{1, \ldots, n\}, S$) $\wedge$ KthMin(S, N).

MinNValue:

Semantics: MinNValue($[X_1, \ldots, X_n], N$) holds iff N is the minimum number of times that the same value is taken by the variables $X_1, \ldots, X_n$.
Decomposition: GCC($[X_1, \ldots, X_n], [d_1, \ldots, d_{|D|}], [O_1, \ldots, O_{|D|}]$) $\wedge$ MinExcept0($N, [O_1, \ldots, O_{|D|}]$) where $\{d_1, \ldots, d_{|D|}\} = D$.

NClass:

Semantics: NClass($[X_1, \ldots, X_n], N, class$) holds iff the number of distinct class (with respect to $class$) of values assigned to $X_1, \ldots, X_n$ is N.
Decomposition: Range($[X_1, \ldots, X_n], \{1, \ldots, n\}, S$) $\wedge$ Apply($class, S, T$) $\wedge |T| = N$.

NEquivalence:

Semantics: NEquivalence($[X_1, \ldots, X_n], N, m$) holds iff the number of distinct values modulo m assigned to $X_1, \ldots, X_n$ is N.
Decomposition: NClass($[X_1, \ldots, X_n], N, mod\ m$).

NInterval:

Semantics: NInterval($[X_1, \ldots, X_n], N, k$) holds iff N is the number of intervals for which at least one value is taken by at least one variable of $X_1, \ldots, X_n$. The intervals are defined by $i * k..i * k + k - 1$ where i is an integer.
Decomposition:
($\forall i \in \{0, \ldots, max(D)/k\}$. AmongInterval($[X_1, \ldots, X_n], i{*}k, i{*}k{+}k{-}1, N_i$) $\wedge$ Min($D_i, [1, N_i]$)) $\wedge$ $N = \sum_{i \in \{0, \ldots, max(D)/k\}} D_i$.

NotAllEqual:

Semantics: NotAllEqual($[X_1, \ldots, X_n]$) holds iff the variables $X_1, \ldots, X_n$ take more than one single valu
Decomposition: NValue($[X_1, \ldots, X_n], N$) $\wedge$ $N > 1$.

NPair:

Semantics: NPair($[X_1, \ldots, X_n], [Y_1, \ldots, Y_n], N$) holds iff N is the number of distinct pairs of values assign to $\langle X_1, Y_1 \rangle \ldots, \langle X_n, Y_n \rangle$. We assume the maximum domain size is d.
Decomposition: $\forall i \in \{1, \ldots, n\}$. $Z_i = X_i * d + Y_i$ $\wedge$ $NValue([Z_1, \ldots, Z_n], N)$.

NValue:

Semantics: NValue($[X_1, \ldots, X_n], N$) holds iff the number of distinct values assigned to $X_1, \ldots, X_n$ is N
Decomposition: Range($[X_1, \ldots, X_n], \{1, \ldots, n\}, T$) $\wedge$ $|T| = N$.

RangeCtr:

Semantics: RangeCtr($[X_1, \ldots, X_n], op \in [\leq, \geq, <, >, \neq, =], N$) holds iff the difference d between the ma imum and the minimum values assigned to the variables $X_1, \ldots, X_n$ satisfies d op N.
Decomposition: Max($Max, [X_1, \ldots, X_n]$) $\wedge$ Min($Min, [X_1, \ldots, X_n]$) $\wedge$ $Max - Min$ op N.

Same:

Semantics: Same1($[X_1, \ldots, X_n], [Y_1, \ldots, Y_m]$) holds iff the same set of values is shared by $X_1, \ldots, X_n$ a $Y_1, \ldots, Y_m$. Same2($[X_1, \ldots, X_n], [Y_1, \ldots, Y_n]$) holds iff the values assigned to $X_1, \ldots, X_n$ correspond to t values assigned to $Y_1, \ldots, Y_n$ according to a permutation.
Decomposition: Same1: Range($[X_1, \ldots, X_n], \{1, \ldots, n\}, S$) $\wedge$ Range($[Y_1, \ldots, Y_m], \{1, \ldots, m\}, T$) $\wedge$ S T. By defining Range($[X_1, \ldots, X_n], S, T$) as $T = \biguplus_{i \in S} \{X_i\}$, we can decompose Same2 similarly to Sam

SameInterval:

Semantics: SameInterval($[X_1, \ldots, X_n], [Y_1, \ldots, Y_n], k$) holds iff N_i (resp. M_i) is the number of variab in $X_1, \ldots, X_n$ (resp. $Y_1, \ldots, Y_n$) which take their values in the interval $k * i..k * i + k - 1$ and $N_i = M_i$ for i.
Decomposition: $\forall i \in \{0, \ldots, max(D)/k\}$. AmongInterval($[X_1, \ldots, X_n], i * k, i * k + k - 1, N_i$) $\wedge$ AmongInterval($[Y_1, \ldots, Y_n], i * k, i * k + k - 1, M_i$) $\wedge$ $N_i = M_i$.

SameModulo:

Semantics: SameModulo($[X_1, \ldots, X_n], [Y_1, \ldots, Y_n], m$) holds iff N_i (resp. M_i) is the number of variab in $X_1, \ldots, X_n$ (resp. $Y_1, \ldots, Y_n$) which have i as a rest when divided by m and $N_i = M_i$ for all i in $0..m -$
Decomposition:
$\forall i \in \{0, \ldots, m - 1\}$. AmongModulo($[X_1, \ldots, X_n], i, m, N_i$) $\wedge$ AmongModulo ($[Y_1, \ldots, Y_n], i, m, M$ $\wedge$ $N_i = M_i$.

SAMEPARTITION:

Semantics: SAMEPARTITION($[X_1, \ldots, X_n], [Y_1, \ldots, Y_n], [p_1, \ldots, p_m]$) holds iff N_i (resp. M_i) is the number of variables in $X_1, \ldots, X_n$ (resp. $Y_1, \ldots, Y_n$) which take their value in p_i and $N_i = M_i$ for all i in $1..m$.
Decomposition: $\forall i \in \{1, \ldots, m\}$. AMONG($[X_1, \ldots, X_n], p_i, N_i$) $\wedge$ AMONG ($[Y_1, \ldots, Y_n], p_i, M_i$) $\wedge$ $N_i = M_i$.

SOFTALLDIFFERENTCTR:

Semantics: SOFTALLDIFFERENTCTR($[X_1, \ldots, X_n], N$) holds iff N is the number of equality constraints ($=$) which hold between the variables of $X_1, \ldots, X_n$.
Decomposition: GCC($[X_1, \ldots, X_n], [d_1, \ldots, d_{|D|}], [O_1, \ldots, O_{|D|}]$) $\wedge$ $N = \sum_{i \in \{1, \ldots, |D|\}} O_i * (O_i - 1)/2$ where $\{d_1, \ldots, d_{|D|}\} = D$.

SOFTALLDIFFERENTVAR:

Semantics: SOFTALLDIFFERENTVAR($[X_1, \ldots, X_n], N$) holds iff N is the minimum number of variables in $X_1, \ldots, X_n$ for which the value needs to be changed in order that all the variables take a distinct value.
Decomposition: NVALUE($[X_1, \ldots, X_n], K$) $\wedge$ $N = n - K$.

SUMOFWEIGHTSOFDISTINCTVALUES:

Semantics: SUMOFWEIGHTSOFDISTINCTVALUES($[X_1, \ldots, X_n], [\langle d_1, c_1 \rangle, \ldots, \langle d_m, c_m \rangle], C$) holds iff the variables $X_1, \ldots, X_n$ take their values from $d_1, \ldots, d_m$ and C is the sum of the cost values (described by c_i) of the distinct values assigned.
Decomposition:
$D = \{d_1, \ldots, d_m\} \wedge$ GCC($[X_1, \ldots, X_n], [d_1, \ldots, d_m], [O_1, \ldots, O_m]$) $\wedge$ ($\forall i \in \{1, \ldots, m\}$. MIN($NO_i, [1, O_i]$))$\wedge$ $\sum_{i \in \{1, \ldots, m\}} c_i * NO_i = C$.

SURJECTION:

Semantics: SURJECTION($[X_1, \ldots, X_n], S$) holds iff for each $j \in S$ there exists $X_i = j$.
Decomposition: RANGE($[X_1, \ldots, X_n], \{1, \ldots, n\}, T$) $\wedge$ $S \subseteq T$.

SYMETRICALLDIFFERENT:

Semantics: SYMETRICALLDIFFERENT($[X_1, \ldots, X_n]$) holds iff $X_i = j \leftrightarrow X_j = i$ (it is therefore implied that ALLDIFFERENT($[X_1, \ldots, X_n]$))
Decomposition:
RANGE($[X_1, \ldots, X_n], \{1, \ldots, n\}, \{1, \ldots, n\}$) $\wedge$ ($\forall i \in \{1, \ldots, n\}$. ROOTS($[X_1, \ldots, X_n], S_i, \{i\}$) $\wedge$ $X_i \in S_i$)

SYMETRICGCC:

Semantics: SYMETRICGCC($[X_1, \ldots, X_n], [Y_1, \ldots, Y_n], [OX_1, \ldots, OX_n], [OY_1, \ldots, OY_n]$) holds iff $i \in X_j \leftrightarrow j \in Y_i$ and OX_i (resp. OY_i) is the number of occurrences of i in the sets assigned to $Y_1, \ldots, Y_n$ (resp. $X_1, \ldots, X_n$).
Decomposition:
$\forall i \in \{1, \ldots, n\}$. LINKSETTOBOOLEANS($X_i, [B_{i1}, B_{i2}, \ldots, B_{in}]$) $\wedge$ LINKSETTOBOOLEANS($Y_i, [B_{1i}, B_{2i}, \ldots, B_{ni}]$) $\wedge$ $OX_i = |X_i|$ $\wedge$ $OY_i = |Y_i|$.

USEDBY:

Semantics: USEDBY$([X_1, \dots, X_n], [Y_1, \dots, Y_m])$ holds iff the multiset of values assigned to $Y_1, \dots, Y_m$ is
subset of the one assigned to $X_1, \dots, X_n$.
Decomposition: By defining RANGE$([X_1, \dots, X_n], S, T)$ as $T = \biguplus_{i \in S} \{X_i\}$, we can decompose USEDB
similarly to USES.

USES:

Semantics: USES$([X_1, \dots, X_n], [Y_1, \dots, Y_m])$ holds iff the set of values assigned to $Y_1, \dots, Y_m$ is a subset
the one assigned to $X_1, \dots, X_n$.
Decomposition: RANGE$([X_1, \dots, X_n], \{1, \dots, n\}, S) \wedge$ RANGE$([Y_1, \dots, Y_m], \{1, \dots, m\}, T) \ \wedge \ T \subseteq S$.