The Range and Roots Constraints: some applications

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Let $X_1, \ldots, X_n$ be a set of integer variables each with domain $dom(X_i)$, and let $D = \bigcup_i dom(X_i)$. Let $X : \{1, \ldots n\} \rightarrow D$ be the assignment function from the set of variable indices to their domain set D.

The Range constraint constrains T to be the range of function X restricted to the variables whose index belong to S. When S is not used it is implicitly equal to $\{1, ..., n\}$.

$$\textsc{Range}([X_1, \ldots, X_n], S, T) \text{ iff } T = \bigcup_{i \in S} \{X_i\}$$

The Roots constraint constrains S to be the set of indices which are inverses of an element of T.

$$\textsc{Roots}([X_1, \ldots, X_n], S, T) \text{ iff } S = \bigcup_{j \in T} X^{-1}(j)$$

where $X^{-1}(j)$ is the set of indices whose image is j.

We now give an extensive catalog of global constraints that can be expressed in terms of Roots and Range.

AllDifferent:

Semantics: $\textsc{AllDifferent}([X_1, \ldots, X_n])$ holds iff all X_i take different values.
Decomposition: $\textsc{Range}([X_1, \ldots, X_n], \{1, \ldots, n\}, T) \wedge |T| = n$.

AllDifferentExcept0:

Semantics: $\textsc{AllDifferentExcept0}([X_1, \ldots, X_n])$ holds iff all X_i take different values except those variables which are assigned to 0.
Decomposition: $\textsc{Range}([X_1, \ldots, X_n], \{1, \ldots, n\}, T) \wedge \textsc{Count}([X_1, \ldots, X_n], 0, =, N) \wedge |T - \{0\}| = n - N$.

AlldifferentInterval:

Semantics: $\textsc{AlldifferentInterval}([X_1, \ldots, X_n], k)$ holds iff all X_i take values from different intervals. The intervals are defined by $i * k..i * k + k - 1$ where i is an integer.
Decomposition: $\forall i \in \{0, \ldots, max(D)/k\}$. $\textsc{AmongInterval}([X_1, \ldots, X_n], i*k, i*k+k-1, N) \wedge 0 \leq N \leq 1$.

AlldifferentModulo:

Semantics: $\textsc{AlldifferentModulo}([X_1, \ldots, X_n], m)$ holds iff all X_i have distinct rest when divided by m.
Decomposition: $\forall i \in \{0, \ldots, m-1\}$. $\textsc{AmongModulo}([X_1, \ldots, X_n], i, m, N) \wedge 0 \leq N \leq 1$.

ALLDIFFERENTPARTITION:

Semantics: ALLDIFFERENTPARTITION $([X_1, \ldots, X_n], [p_1, \ldots, p_m])$ holds iff all X_i take values from different partitions where each p_i defines a partition.
Decomposition: $\forall i \in \{1, \ldots, m\}$. AMONGLOWUP$(0, 1, [X_1, \ldots, X_n], p_i)$.

AMONG:

Semantics: AMONG$([X_1, \ldots, X_n], [d_1, \ldots, d_m], N)$ holds iff the number of variables in $X_1, \ldots, X_n$ which take their value in $d_1, \ldots, d_m$ is N.
Decomposition: ROOTS$([X_1, \ldots, X_n], S, \{d_1, \ldots, d_m\}) \ \wedge \ |S| = N$.

AMONGINTERVAL:

Semantics: AMONGINTERVAL$([X_1, \ldots, X_n], low, up, N)$ holds iff the number of variables in $X_1, \ldots, X_n$ which take their value in the interval $low..up$ is N.
Decomposition:AMONG$([X_1, \ldots, X_n], [low, low + 1 \ldots, up - 1, up], N)$.

AMONGLOWUP:

Semantics: AMONGLOWUP$(low, up, [X_1, \ldots, X_n], [d_1, \ldots, d_m])$ holds iff the variables in $X_1, \ldots, X_n$ take at least low and at most up values in $d_1, \ldots, d_m$.
Decomposition: AMONG$([X_1, \ldots, X_n], [d_1, \ldots, d_m], N) \ \wedge \ low \leq N \leq up$.

AMONGMODULO:

Semantics: AMONGMODULO$([X_1, \ldots, X_n], rem, quot, N)$ holds iff the number of variables in $X_1, \ldots, X_n$ which take their value as rem by modulo $quot$ is N.
Decomposition: AMONG$([X_1, \ldots, X_n], [rem, rem + quot \ldots, max], N)$ where max is the maximum value in D which is rem by modulo $quot$.

AMONGSEQ:

Semantics: AMONGSEQ$(low, up, seq, [X_1, \ldots, X_n], [d_1, \ldots, d_m])$ holds iff all sequences of seq variables in $X_1, \ldots, X_n$ take at least low and at most up values in $d_1, \ldots, d_m$.
Decomposition: $\forall i \in \{1, \ldots, n - seq\}$, AMONGLOWUP$(low, up, [X_i, \ldots, X_{i+seq-1}], [d_1, \ldots, d_m])$.

APPLY:

Semantics: APPLY(f, S, T) holds iff T is the set of values constructed by applying the function f to the set S.
Decomposition: RANGE$([X_1, \ldots, X_n], S, T)$ where $X_i = j$ iff $f(i) = j$.

ASSIGN&COUNTS:

Semantics: ASSIGN&COUNTS$(colours, [X_1, \ldots, X_n], [Y_1, \ldots, Y_n], op \in [\leq, \geq, <, >, \neq, =], N)$ holds iff by the assignment of bin and colour to each of n items, the number of items d_i of colour $colour$ in each bin satisfies $d_i \ op \ N$.
Decomposition: $\forall i \in D$. ROOTS$([X_1, \ldots, X_n], S_i, \{i\}) \ \wedge$ RANGE$([Y_1, \ldots, Y_n], S_i, T_i) \ \wedge \ |T_i \cap colours| \ op \ N$.

ASSIGN&NVALUES:

Semantics: ASSIGN&NVALUES($[X_1, \ldots, X_n], [Y_1, \ldots, Y_n], op \in [\leq, \geq, <, >, \neq, =], N$) holds iff by the assignment of bin and value to each of n items, the number of distinct values d_i in each bin satisfies d_i *op* N.
Decomposition: $\forall i \in D$. ROOTS($[X_1, \ldots, X_n], S_i, \{i\}$) $\wedge$ RANGE($[Y_1, \ldots, Y_n], S_i, T_i$) $\wedge$ $|T_i|$ *op* N.

ATLEAST:

Semantics: ATLEAST($[X_1, \ldots, X_n], v, N$) holds iff the number of variables in $X_1, \ldots, X_n$ assigned to v is at least N.
Decomposition: ROOTS($[X_1, \ldots, X_n], S, \{v\}$) $\wedge$ $|S| \geq N$.

ATLEASTNVALUE:

Semantics: ATLEASTNVALUE($[X_1, \ldots, X_n], N$) holds iff the number of distinct values assigned to $X_1, \ldots, X_n$ is at least N.
Decomposition: RANGE($[X_1, \ldots, X_n], \{1, \ldots, n\}, T$) $\wedge$ $|T| \geq N$.

ATMOST:

Semantics: ATMOST($[X_1, \ldots, X_n], v, N$) holds iff the number of variables in $X_1, \ldots, X_n$ assigned to v is at most N.
Decomposition: ROOTS($[X_1, \ldots, X_n], S, \{v\}$) $\wedge$ $|S| \leq N$.

ATMOSTNVALUE:

Semantics: ATMOSTNVALUE($[X_1, \ldots, X_n], N$) holds iff the number of distinct values assigned to $X_1, \ldots, X_n$ is at most N.
Decomposition: RANGE($[X_1, \ldots, X_n], \{1, \ldots, n\}, T$) $\wedge$ $|T| \leq N$.

BALANCE:

Semantics: BALANCE($[X_1, \ldots, X_n], op \in [\leq, \geq, <, >, \neq, =], N$) holds iff the difference d between the number of occurrences of the value that occurs the most and the value that occurs the least satisfies d *op* N.
Decomposition: GCC($[X_1, \ldots, X_n], [d_1, \ldots, d_{|D|}], [O_1, \ldots, O_{|D|}]$) $\wedge$ RANGECTR($[O_1, \ldots, O_{|D|}], op, N$) where $\{d_1, \ldots, d_{|D|}\} = D$.

BALANCEINTERVAL:

Semantics: BALANCEINTERVAL($[X_1, \ldots, X_n], N, k$) holds iff N is the difference between the minimum and the maximum number of variables in $X_1, \ldots, X_n$ that take values from the same interval. The intervals are defined by $i * k..i * k + k - 1$ where i is an integer.
Decomposition: ($\forall i \in \{0, \ldots, max(D)/k\}$. AMONGINTERVAL($[X_1, \ldots, X_n], i * k, i * k + k - 1, N_i$)) $\wedge$ RANGECTR($[N_0, \ldots, N_{max(D)/k}], =, N$).

BALANCEMODULO:

Semantics: BALANCEMODULO($[X_1, \ldots, X_n], N, m$) holds iff N is the difference between the minimum and the maximum number of variables in $X_1, \ldots, X_n$ that have the same rest when divided by m.
Decomposition:
($\forall i \in \{0, \ldots, m-1\}$. AMONGMODULO($[X_1, \ldots, X_n], i, m, N_i$)) $\wedge$ RANGECTR($[N_0, \ldots, N_{m-1}], =, N$).

BALANCEPARTITION:

Semantics: $\text{BALANCEPARTITION}([X_1,\ldots,X_n],N,[p_1,\ldots,p_m])$ holds iff N is the difference between the minimum and the maximum number of variables in $X_1,\ldots,X_n$ that take values from the same partition where each p_i defines a partition.
Decomposition: $(\forall i \in \{1,\ldots,m\}.\ \text{AMONG}([X_1,\ldots,X_n],p_i,N_i)) \ \wedge \ \text{RANGECTR}([N_1,\ldots,N_m],=,N)$.

BINPACKING:

Semantics: $\text{BINPACKING}([X_1,\ldots,X_n],c,weights)$ holds iff by the assignment of a bin to each of n items described by a weight (by the *weights* function), the total capacity of each bin does not exceed c.
Decomposition: $\forall i \in D.\ \text{ROOTS}([X_1,\ldots,X_n],S_i,\{i\}) \ \wedge \ \text{APPLY}(weights,S_i,T_i) \ \wedge \ \text{SUM}(T_i) \leq c$.

CARDATLEAST:

Semantics: $\text{CARDATLEAST}([X_1,\ldots,X_n],[d_1,\ldots,d_m],N)$ holds iff N is the minimum number of times that a value of $d_1,\ldots,d_m$ is taken by the variables $X_1,\ldots,X_n$.
Decomposition: $\text{GCC}([X_1,\ldots,X_n],[d_1,\ldots,d_m],[O_1,\ldots,O_m]) \ \wedge \ \text{MIN}(N,[O_1,\ldots,O_m])$.
Note: This constraint is very similar to MINNVALUE in which the values are not specified.

CARDATMOST:

Semantics: $\text{CARDATMOST}([X_1,\ldots,X_n],[d_1,\ldots,d_m],N)$ holds iff N is the maximum number of times that a value of $d_1,\ldots,d_m$ is taken by the variables $X_1,\ldots,X_n$.
Decomposition: $\text{GCC}([X_1,\ldots,X_n],[d_1,\ldots,d_m],[O_1,\ldots,O_m]) \ \wedge \ \text{MAX}(N,[O_1,\ldots,O_m])$.
Note: This constraint is very similar to MAXNVALUE in which the values are not specified.

CARDATMOSTPARTITION:

Semantics: $\text{CARDATMOSTPARTITION}([X_1,\ldots,X_n],[p_1,\ldots,p_m],N)$ holds iff N is the maximum number of times that values of a same partition of $p_1,\ldots,p_m$ are taken by the variables $X_1,\ldots,X_n$.
Decomposition: $(\forall i \in \{1,\ldots,m\}.\ \text{AMONG}([X_1,\ldots,X_n],p_i,N_i) \ \wedge \ \text{MAX}\ (N,[N_1,\ldots,N_m])$.

CHANGEPARTITION:

Semantics: $\text{CHANGEPARTITION}\ (N,[X_1,\ldots,X_n],[p_1,\ldots,p_m])$ holds iff there are N consecutive pairs of variables in $X_1,\ldots,X_n$ which take values from different partitions where each p_i defines a partition.
Decomposition: $(\forall i \in \{1,\ldots,n-1\}.\ \text{ALLDIFFERENTPARTITION}([X_i,X_{i+1}],[p_1,\ldots,p_m]) \ \leftrightarrow \ N_i = 1) \ \wedge \ \sum_{i\in\{1,\ldots,n-1\}} N_i = N$.

COMMON:

Semantics: $\text{COMMON}([X_1,\ldots,X_n],[Y_1,\ldots,Y_m],N,M)$ holds iff N (resp. M) variables in $X_1,\ldots,X_n$ (resp. in $Y_1,\ldots,Y_m$) take their values in $Y_1,\ldots,Y_m$ (resp. in $X_1,\ldots,X_n$).
Decomposition: $\text{AMONG}([X_1,\ldots,X_n],[Y_1,\ldots,Y_m],N) \wedge \text{AMONG}([Y_1,\ldots,Y_m],[X_1,\ldots,X_n],M)$.

Commonmodulo:

Semantics: $\textsc{CommonModulo}([X_1,\ldots,X_n],[Y_1,\ldots,Y_m],N,M,k)$ holds iff N (resp. M) variables in $X_1,\ldots,X_n$ (resp. in $Y_1,\ldots,Y_m$) take their values in one of the equivalence classes derived from the values of $Y_1,\ldots,Y_m$ (resp. $X_1,\ldots,X_n$) modulo k.
Decomposition: $\textsc{Range}([Y_1,\ldots,Y_m],\{1,\ldots,m\},S_1) \wedge \textsc{Apply}(mod\,k,S_1,T_1) \wedge \textsc{Among}([X_1,\ldots,X_n],T_1,N) \wedge \textsc{Range}([X_1,\ldots,X_n],\{1,\ldots,n\},S_2) \wedge \textsc{Apply}(mod\ k,S_2,T_2) \wedge \textsc{Among}([Y_1,\ldots,Y_n],T_2,M)$.

Count:

Semantics: $\textsc{Count}([X_1,\ldots,X_n],v,op \in [\leq,\geq,<,>,\neq,=],N)$ holds iff the operation op is satisfied between the number of variables assigned to v and N.
Decomposition: $\textsc{Roots}([X_1,\ldots,X_n],S,\{v\}) \wedge |S|\ op\ N$.

Disjoint:

Semantics: $\textsc{Disjoint}([X_1,\ldots,X_n],[Y_1,\ldots,Y_m])$ holds iff there is no overlap between the set of values assigned to $X_1,\ldots,X_n$ and those assigned to $Y_1,\ldots,Y_m$.
Decomposition: $\textsc{Range}([X_1,\ldots,X_n],\{1,\ldots,n\},S) \wedge \textsc{Range}([Y_1,\ldots,Y_m],\{1,\ldots,m\},T) \wedge S \cap T = \{\}$.
A special case of Common: $\textsc{Common}([X,\ldots,X_n],[Y_1,\ldots,Y_m],0,0)$.

Element:

Semantics: $\textsc{Element}(I,[X_1,\ldots,X_n],V)$ holds iff $X_I = V$.
Decomposition: $V \in T \wedge |T| = 1 \wedge \textsc{Roots}([X_1,\ldots,X_n],S,T) \wedge I \in S$.

GCC:

Semantics: $\text{GCC}([X_1,\ldots,X_n],[d_1,\ldots,d_m],[O_1,\ldots,O_m])$ holds iff the value d_i is used O_i times in $X_1,\ldots,X_n$.
Decomposition: $\forall i \in \{1\ldots,m\}\ \textsc{Roots}([X_1,\ldots,X_n],S_i,\{d_i\}) \wedge |S_i| = O_i$.

GlobalContinuity:

Semantics: $\textsc{GlobalContinuity}([X_1,\ldots,X_n])$ holds iff all 1's are consecutive.
Decomposition: $\textsc{Roots}([X_1,\ldots,X_n],S,\{1\}) \wedge \textsc{Min}(S,Low) \wedge \textsc{Max}(S,Hi) \wedge |S| = Hi - Low + 1$.

InSamePartition:

Semantics: $\textsc{InSamePartition}(V_1,V_2,class)$ holds iff the values assigned to V_1 and V_2 are in the same class (with respect to $class$).
Decomposition: $\textsc{Apply}(class,V_1,S) \wedge \textsc{Apply}(class,V_2,T) \wedge S = T$.

IntervalAndCount:

Semantics: $\textsc{IntervalAndCount}(colours,[X_1,\ldots,X_n],[Y_1,\ldots,Y_n],k,N)$ holds iff by the assignment of origin and colour to each of n tasks, the number of items d_i of colour *colour* in each interval satisfies $d_i \leq N$.
Decomposition:
$(\forall i \in \{0,\ldots,max(D)/k\}.\ \textsc{Roots}([X_1,\ldots,X_n],S_i,\{i*k..i*k+k-1\}) \wedge \textsc{Range}([Y_1,\ldots,Y_n],S_i,T_i) \wedge |T_i \cap colours| \leq N$.

LinkSetToBooleans:

Semantics: LinkSetToBooleans $(S, [B_1, \ldots, B_n])$ holds iff the 0/1 variables $B_1, \ldots, B_n$ which are associated to a value belonging to the set variable S are 1, while the remaining 0/1 variables are all equal to 0.
Decomposition: Roots$([B_1, \ldots, B_n], S, \{1\})$.

Max:

Semantics: Max$(Max, [X_1, \ldots, X_n])$ holds iff Max is the maximum value of the variables $X_1, \ldots, X_n$.
Decomposition: Range$([X_1, \ldots, X_n], \{1, \ldots, n\}, S)$ $\wedge$ Max(S, Max).

MaxIndex:

Semantics: MaxIndex$(MaxI, [X_1, \ldots, X_n])$ holds iff $MaxI$ is the indices of the variables $X_1, \ldots, X_n$ corresponding to the maximum value of the variables.
Decomposition: Max $(Max, [X_1, \ldots, X_n])$ $\wedge$ Roots$([X_1, \ldots, X_n], MaxI, \{Max\})$.

MaxModulo:

Semantics: Max$(Max, M, [X_1, \ldots, X_n])$ holds iff Max is the maximum value of the variables $X_1, \ldots, X_n$ according to the partial order $(X \ mod \ M) < (Y \ mod \ M)$.
Decomposition: Range$([X_1, \ldots, X_n], \{1, \ldots, n\}, S)$ $\wedge$ Apply$(mod \ M, S, T)$ $\wedge$ Max(T, Max).

MaxN:

Semantics: MaxN$([X_1, \ldots, X_n], N, k)$ holds iff N is the maximum value of rank k (i.e. the k^{th} largest distinct value) in $X_1, \ldots, X_n$.
Decomposition: Range$([X_1, \ldots, X_n], \{1, \ldots, n\}, S)$ $\wedge$ KthMax(S, N).

MaxNValue:

Semantics: MaxNValue$([X_1, \ldots, X_n], N)$ holds iff N is the maximum number of times that the same value is taken by the variables $X_1, \ldots, X_n$.
Decomposition: GCC$([X_1, \ldots, X_n], [d_1, \ldots, d_{|D|}], [O_1, \ldots, O_{|D|}]) \wedge$ Max$(N, [O_1, \ldots, O_{|D|}])$ where $\{d_1, \ldots, d_{|}$
D.

Min:

Semantics: Min$(Min, [X_1, \ldots, X_n])$ holds iff Min is the minimum value of the variables $X_1, \ldots, X_n$.
Decomposition: Range$([X_1, \ldots, X_n], \{1, \ldots, n\}, S)$ $\wedge$ Min(S, Min).

MinExcept0:

Semantics: MinExcept0$(Min, [X_1, \ldots, X_n])$ holds iff Min is the minimum value of the variables $X_1, \ldots, X_n$ ignoring all variables that take 0 as value.
Decomposition: Range$([X_1, \ldots, X_n], \{1, \ldots, n\}, S)$ $\wedge$ $T = S - \{0\}$ $\wedge$ Min(T, Min).

MinGreaterThan:

Semantics: MinGreaterThan$(Var_1, Var_2, [X_1, \ldots, X_n])$ holds iff Var_1 is the smallest value strictly greater than Var_2 in $X_1, \ldots, X_n$.
Decomposition: MinN$([X_1, \ldots, X_n], Var_1, k+1) \ \wedge$ MinN$([X_1, \ldots, X_n], Var_2, k)$.

MinIndex:

Semantics: MinIndex$(MinI, [X_1, \ldots, X_n])$ holds iff $MinI$ is the indices of the variables $X_1, \ldots, X_n$ corresponding to the minimum value of the variables.
Decomposition: Min $(Min, [X_1, \ldots, X_n]) \ \wedge$ Roots$([X_1, \ldots, X_n], MinI, \{Min\})$.

MinModulo:

Semantics: Min$(Min, M, [X_1, \ldots, X_n])$ holds iff Min is the minimum value of the variables $X_1, \ldots, X_n$ according to the partial order $(X\ mod\ M) < (Y\ mod\ M)$.
Decomposition: Range$([X_1, \ldots, X_n], \{1, \ldots, n\}, S) \ \wedge$ Apply$(mod\ M, S, T) \ \wedge$ Min(T, Min).

MinN:

Semantics: MinN$([X_1, \ldots, X_n], N, k)$ holds iff N is the minimum value of rank k (i.e. the k^{th} smallest distinct value) in $X_1, \ldots, X_n$.
Decomposition: Range$([X_1, \ldots, X_n], \{1, \ldots, n\}, S) \ \wedge$ KthMin(S, N).

MinNValue:

Semantics: MinNValue$([X_1, \ldots, X_n], N)$ holds iff N is the minimum number of times that the same value is taken by the variables $X_1, \ldots, X_n$.
Decomposition: GCC$([X_1, \ldots, X_n], [d_1, \ldots, d_{|D|}], [O_1, \ldots, O_{|D|}]) \wedge$ MinExcept0$(N, [O_1, \ldots, O_{|D|}])$ where $\{d_1, \ldots, d_{|D|}\} = D$.

NClass:

Semantics: NClass$([X_1, \ldots, X_n], N, class)$ holds iff the number of distinct class (with respect to $class$) of values assigned to $X_1, \ldots, X_n$ is N.
Decomposition: Range$([X_1, \ldots, X_n], \{1, \ldots, n\}, S) \ \wedge$ Apply$(class, S, T) \ \wedge |T| = N$.

NEquivalence:

Semantics: NEquivalence$([X_1, \ldots, X_n], N, m)$ holds iff the number of distinct values modulo m assigned to $X_1, \ldots, X_n$ is N.
Decomposition: NClass$([X_1, \ldots, X_n], N, mod\ m)$.

NInterval:

Semantics: NInterval$([X_1, \ldots, X_n], N, k)$ holds iff N is the number of intervals for which at least one value is taken by at least one variable of $X_1, \ldots, X_n$. The intervals are defined by $i*k..i*k+k-1$ where i is an integer.
Decomposition:
$(\forall i \in \{0, \ldots, max(D)/k\}.$ AmongInterval$([X_1, \ldots, X_n], i*k, i*k+k-1, N_i) \wedge$ Min$(D_i, [1, N_i])) \wedge N = \sum_{i \in \{0, \ldots, max(D)/k\}} D_i$.

NotAllEqual:

Semantics: NotAllEqual($[X_1, \ldots, X_n]$) holds iff the variables $X_1, \ldots, X_n$ take more than one single value.
Decomposition: NValue($[X_1, \ldots, X_n], N$) $\wedge$ $N > 1$.

NPair:

Semantics: NPair($[X_1, \ldots, X_n], [Y_1, \ldots, Y_n], N$) holds iff N is the number of distinct pairs of values assigned to $\langle X_1, Y_1 \rangle \ldots, \langle X_n, Y_n \rangle$. We assume the maximum domain size is d.
Decomposition: $\forall i \in \{1, \ldots, n\}.\ Z_i = X_i * d + Y_i \ \wedge\ NValue([Z_1, \ldots, Z_n], N)$.

NValue:

Semantics: NValue($[X_1, \ldots, X_n], N$) holds iff the number of distinct values assigned to $X_1, \ldots, X_n$ is N.
Decomposition: Range($[X_1, \ldots, X_n], \{1, \ldots, n\}, T$) $\wedge$ $|T| = N$.

RangeCtr:

Semantics: RangeCtr($[X_1, \ldots, X_n], op \in [\leq, \geq, <, >, \neq, =], N$) holds iff the difference d between the maximum and the minimum values assigned to the variables $X_1, \ldots, X_n$ satisfies d *op* N.
Decomposition: Max($Max, [X_1, \ldots, X_n]$) $\wedge$ Min($Min, [X_1, \ldots, X_n]$) $\wedge$ $Max - Min$ *op* N.

Same:

Semantics: Same1($[X_1, \ldots, X_n], [Y_1, \ldots, Y_m]$) holds iff the same set of values is shared by $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_m$. Same2($[X_1, \ldots, X_n], [Y_1, \ldots, Y_n]$) holds iff the values assigned to $X_1, \ldots, X_n$ correspond to the values assigned to $Y_1, \ldots, Y_n$ according to a permutation.
Decomposition: Same1: Range($[X_1, \ldots, X_n], \{1, \ldots, n\}, S$) $\wedge$ Range($[Y_1, \ldots, Y_m], \{1, \ldots, m\}, T$) $\wedge$ $S = T$. By defining Range($[X_1, \ldots, X_n], S, T$) as $T = \biguplus_{i \in S} \{X_i\}$, we can decompose Same2 similarly to Same1.

SameInterval:

Semantics: SameInterval($[X_1, \ldots, X_n], [Y_1, \ldots, Y_n], k$) holds iff N_i (resp. M_i) is the number of variables in $X_1, \ldots, X_n$ (resp. $Y_1, \ldots, Y_n$) which take their values in the interval $k * i..k * i + k - 1$ and $N_i = M_i$ for all i.
Decomposition: $\forall i \in \{0, \ldots, max(D)/k\}$. AmongInterval($[X_1, \ldots, X_n], i * k, i * k + k - 1, N_i$) $\wedge$ AmongInterval($[Y_1, \ldots, Y_n], i * k, i * k + k - 1, M_i$) $\wedge$ $N_i = M_i$.

SameModulo:

Semantics: SameModulo($[X_1, \ldots, X_n], [Y_1, \ldots, Y_n], m$) holds iff N_i (resp. M_i) is the number of variables in $X_1, \ldots, X_n$ (resp. $Y_1, \ldots, Y_n$) which have i as a rest when divided by m and $N_i = M_i$ for all i in $0..m - 1$.
Decomposition:
$\forall i \in \{0, \ldots, m - 1\}$. AmongModulo($[X_1, \ldots, X_n], i, m, N_i$) $\wedge$ AmongModulo ($[Y_1, \ldots, Y_n], i, m, M_i$) $\wedge$ $N_i = M_i$.

SamePartition:

Semantics: SamePartition$([X_1,\ldots,X_n],[Y_1,\ldots,Y_n],[p_1,\ldots,p_m])$ holds iff N_i (resp. M_i) is the number of variables in $X_1,\ldots,X_n$ (resp. $Y_1,\ldots,Y_n$) which take their value in p_i and $N_i = M_i$ for all i in $1..m$.
Decomposition: $\forall i \in \{1,\ldots,m\}$. Among$([X_1,\ldots,X_n],p_i,N_i) \wedge$ Among $([Y_1,\ldots,Y_n],p_i,M_i) \wedge N_i = M_i$.

SoftAllDifferentCtr:

Semantics: SoftAllDifferentCtr$([X_1,\ldots,X_n],N)$ holds iff N is the number of equality constraints (=) which hold between the variables of $X_1,\ldots,X_n$.
Decomposition: GCC$([X_1,\ldots,X_n],[d_1,\ldots,d_{|D|}],[O_1,\ldots,O_{|D|}]) \wedge N = \sum_{i\in\{1,\ldots,|D|\}} O_i * (O_i - 1)/2$ where $\{d_1,\ldots,d_{|D|}\} = D$.

SoftAllDifferentVar:

Semantics: SoftAllDifferentVar$([X_1,\ldots,X_n],N)$ holds iff N is the minimum number of variables in $X_1,\ldots,X_n$ for which the value needs to be changed in order that all the variables take a distinct value.
Decomposition: NValue$([X_1,\ldots,X_n],K) \wedge N = n - K$.

SumOfWeightsOfDistinctValues:

Semantics: SumOfWeightsOfDistinctValues$([X_1,\ldots,X_n],[\langle d_1,c_1\rangle,\ldots,\langle d_m,c_m\rangle],C)$ holds iff the variables $X_1,\ldots,X_n$ take their values from $d_1,\ldots,d_m$ and C is the sum of the cost values (described by c_i) of the distinct values assigned.
Decomposition:
$D = \{d_1,\ldots,d_m\} \wedge$ GCC$([X_1,\ldots,X_n],[d_1,\ldots,d_m],[O_1,\ldots,O_m]) \wedge (\forall i \in \{1,\ldots,m\}.$ Min$(NO_i,[1,O_i])) \wedge \sum_{i\in\{1,\ldots,m\}} c_i * NO_i = C$.

Surjection:

Semantics: Surjection$([X_1,\ldots,X_n],S)$ holds iff for each $j \in S$ there exists $X_i = j$.
Decomposition: Range$([X_1,\ldots,X_n],\{1,\ldots,n\},T) \wedge S \subseteq T$.

SymetricAllDifferent:

Semantics: SymetricAllDifferent$([X_1,\ldots,X_n])$ holds iff $X_i = j \leftrightarrow X_j = i$ (it is therefore implied that AllDifferent$([X_1,\ldots,X_n])$)
Decomposition:
Range$([X_1,\ldots,X_n],\{1,\ldots,n\},\{1,\ldots,n\}) \wedge (\forall i \in \{1,\ldots,n\}.$ Roots$([X_1,\ldots,X_n],S_i,\{i\}) \wedge X_i \in S_i)$

SymetricGCC:

Semantics: SymetricGCC$([X_1,\ldots,X_n],[Y_1,\ldots,Y_n],[OX_1,\ldots,OX_n],[OY_1,\ldots,OY_n])$ holds iff $i \in X_j \leftrightarrow j \in Y_i$ and OX_i (resp. OY_i) is the number of occurrences of i in the sets assigned to $Y_1,\ldots,Y_n$ (resp. $X_1,\ldots,X_n$).
Decomposition:
$\forall i \in \{1,\ldots,n\}$. LinkSetToBooleans$(X_i,[B_{i1},B_{i2},\ldots,B_{in}]) \wedge$ LinkSetToBooleans$(Y_i,[B_{1i},B_{2i},\ldots,B_{ni}]) \wedge OX_i = |X_i| \wedge OY_i = |Y_i|$.

UsedBy:

Semantics: UsedBy($[X_1, \ldots, X_n], [Y_1, \ldots, Y_m]$) holds iff the multiset of values assigned to $Y_1, \ldots, Y_m$ is a subset of the one assigned to $X_1, \ldots, X_n$.
Decomposition: By defining Range($[X_1, \ldots, X_n], S, T$) as $T = \biguplus_{i \in S}\{X_i\}$, we can decompose UsedBy similarly to Uses.

Uses:

Semantics: Uses($[X_1, \ldots, X_n], [Y_1, \ldots, Y_m]$) holds iff the set of values assigned to $Y_1, \ldots, Y_m$ is a subset of the one assigned to $X_1, \ldots, X_n$.
Decomposition: Range($[X_1, \ldots, X_n], \{1, \ldots, n\}, S$) $\wedge$ Range($[Y_1, \ldots, Y_m], \{1, \ldots, m\}, T$) $\wedge\ T \subseteq S$.