A Grand Challenge for Computing Research: a mathematical assistant

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The mathematical assistant

Scientists, engineers and students would all benefit greatly from the help of a mathematical assistant. Such an assistant should be rigorous and indefatigable, and have vast amounts of mathematical knowledge at her fingertips. Since these are precisely the qualities we appreciate most in computers, computers ought to make excellent mathematical assistants. Indeed, in specialized domains, computers already are useful mathematical assistants. For example:

- Computer algebra systems can compute complex indefinite integrals and solve difficult algebraic equations;
- Matrix packages can perform large and tedious matrix computations.

However, we lack systems that have the breadth as well as the depth of knowledge of a working mathematician. Systems typically do not reason at the meta-level about how they solve problems. They are unable therefore to explain their answers, to apply their expertise to new domains, or to reason about the quality of their answers. In addition, systems are neither pro-active nor adaptive. They do not leap in and offer the user help. They require the user to know when and how to call them.

The challenge then is to develop an automated mathematical assistant with both breadth and depth of mathematical expertise. The assistant should cooperatively help users solve their mathematical problems, adapting and learning over time. Such an assistant would be able to:

- Prove that a complicated series diverges;
- Identify parameters for which an indefinite integration is "dangerous";
- Construct a counter-example to the security of the user's cryptographic scheme, and suggest how to modify it;
- Explain an integral over the real line by identifying a suitable contour in the complex plane and locating all the poles;
- Find a large combinatorial object like a projective plane of order 10;
- Prove the uniqueness of a solution to Laplace's equation by appealing to a general purpose uniqueness proof method.

A mathematical assistant will have skills across a wide range of topics, from the very formal and axiomatic (e.g. constructing theories, identifying inconsistencies, proving meta-theoretic results) to the very applied (e.g. numerically solving a set of partial differential equations).

Is this research?

Such an assistant will require research in a wide number of areas. These include:

Knowledge representation: a mathematical assistant will need a large ontology of mathematical information at both the object and the meta level;
Automated reasoning: a mathematical assistant will need rich and complex inference mechanisms;
Learning: a mathematical assistant will need to learn new mathematics;
User modelling: a mathematical assistant will need to infer the user's goals and intentions from their actions;
Databases: a mathematical assistant will need to access vast mathematical databases in complex ways (e.g. search a database for a balanced incomplete block designs with some given properties)
Distributed computation: a mathematical assistant will need to know how to break large computations down to tap into the GRID;

Is it a grand challenge?

It is certainly a challenge since we could fail. AI has had success in narrow domains (witness expert systems) but broad expertise, like that proposed here, is a much more challenging and uncertain goal. What about the other criteria identified in the call for submissions to the workshop? This challenge arises from curiosity about the limits of how much mathematics we can automate. It aims to build something never seen before. It ought to be obvious when the challenge is met since we will stop asking our mathematical colleagues for help. It will be useful to the whole scientific community so should gain their support. It is of a scale that will require international participation. It will be comprehensible to the general public. It was formulated long ago (at least as far back as Leibnitz's desire to reduce all mathematics to calculation). It will take us way beyond the domain specific mathematical tools available today. It will require planned co-operation between many different research projects. Even partial success will improve the mathematical tools available. Finally, given the scale and ambition of the challenge, it is unlikely to happen through the evolution of existing commercial products.