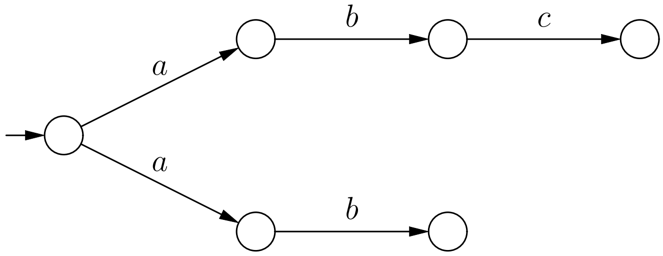
Module Title: Modelling Concurrent Systems
Exam Diet: April 2025

1.

$$\begin{aligned}
X_{\downarrow\downarrow} &= 1^{\uparrow}.X_{\uparrow\downarrow} + 2^{\uparrow}.X_{\downarrow\uparrow}\\
X_{\uparrow\downarrow} &= 1^{\downarrow}.X_{\downarrow\downarrow} + 2^{\uparrow}.X_{\uparrow\uparrow} + \mathit{light}.X_{\uparrow\downarrow}\\
X_{\downarrow\uparrow} &= 1^{\uparrow}.X_{\uparrow\uparrow} + 2^{\downarrow}.X_{\downarrow\downarrow} + \mathit{light}.X_{\downarrow\uparrow}\\
X_{\uparrow\uparrow} &= 1^{\downarrow}.X_{\downarrow\uparrow} + 2^{\downarrow}.X_{\uparrow\downarrow}
\end{aligned}$$

2.



3. (a)

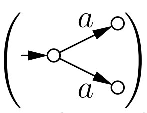
$$\frac{P \xrightarrow{b} P' \quad P \xrightarrow{c} P''}{f(P) \xrightarrow{d} f(P')} \qquad \frac{P \xrightarrow{b} P' \quad P \not\xrightarrow{c}}{f(P) \xrightarrow{b} f(P')} \qquad \frac{P \xrightarrow{\alpha} P'}{f(P) \xrightarrow{\alpha} f(P')}\ (\alpha \in \{c,d\})$$

(b) No, because $a.b + a.c$ is strong completed trace equivalent with $a.(b+c)$ but $f(a.b+a.c) = a.b+a.c$ is not strong completed trace equivalent with $f(a.(b+c)) = a.(d+c)$.

(c) Yes, because the above operational rules are in GSOS format.

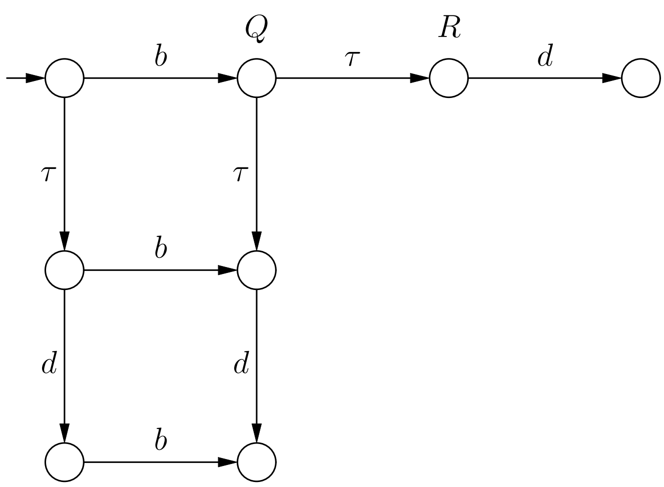
(d) $f(P) \models \mathbf{AG}(\neg(\mathbf{EX}b \wedge \mathbf{EX}c))$.

(e) Expressing the existence of a choice between two futures requires branching time semantics, so we need to use CTL.

4. Consider the CCS processes $P = a.0 + a.(0|0)$ () and $Q = a.0$. Then

$P \models \mathbf{E}_2\mathbf{X}a$, whereas $Q \not\models \mathbf{E}_2\mathbf{X}a$. Yet, P and Q are bisimilar, so there is no CTL formula that can tell them apart. It follows that $\mathbf{E}_2\mathbf{X}\phi$ cannot be expressed in CTL.

5. (a)

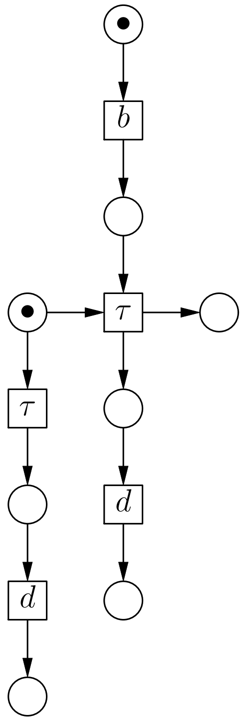


(b) $Q = ((c.d)|(\bar{c} + \tau.d))\backslash c$.

(c)

$$\dfrac{\dfrac{\dfrac{}{c.d \xrightarrow{\bar{c}} d}\ \textit{action prefixing} \qquad \dfrac{\dfrac{}{\bar{c} \xrightarrow{\bar{c}} 0}\ \textit{action prefixing}}{\bar{c} + \tau.d \xrightarrow{\bar{c}} 0}\ \textit{L-choice}}{(c.d)|(\bar{c} + \tau.d) \xrightarrow{\tau} d|0}\ \textit{synchronisation}}{((c.d)|(\bar{c} + \tau.d))\backslash c \xrightarrow{\tau} (d|0)\backslash c}\ \textit{restriction}$$

(d)



(e) bd and db.

(f) $b \rightarrow \tau \rightarrow d$ and $\tau \xrightarrow{b} d$.

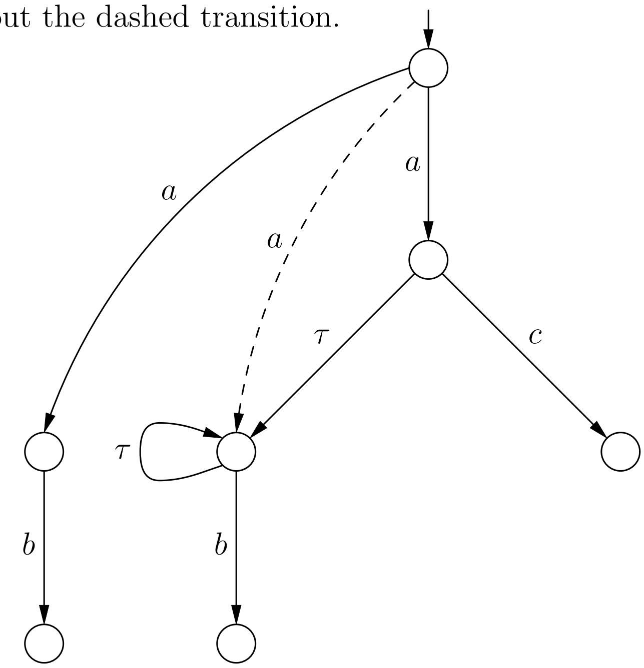
(g) $\tau_{\{c\}}(\partial_{\{a,e\}}((b \cdot a \cdot d)|(e + \tau.d)))$ with $\gamma(a, e) = c$.

(h) $((b.c.d)\|_{\{c\}}(c \,\Box\, e.d))/c/e$.

(i) 6 states. $b|(\tau.d)$

(j) 4 states. $b|d$

6. Two such processes are as drawn here, with and without the dashed transition.



$\mathbf{EF}(a \wedge (\neg\mathbf{EF}c) \wedge \mathbf{EG}\neg b)$.