Module Title: MCS
Exam Diet (Dec/April/Aug): April/May 2024

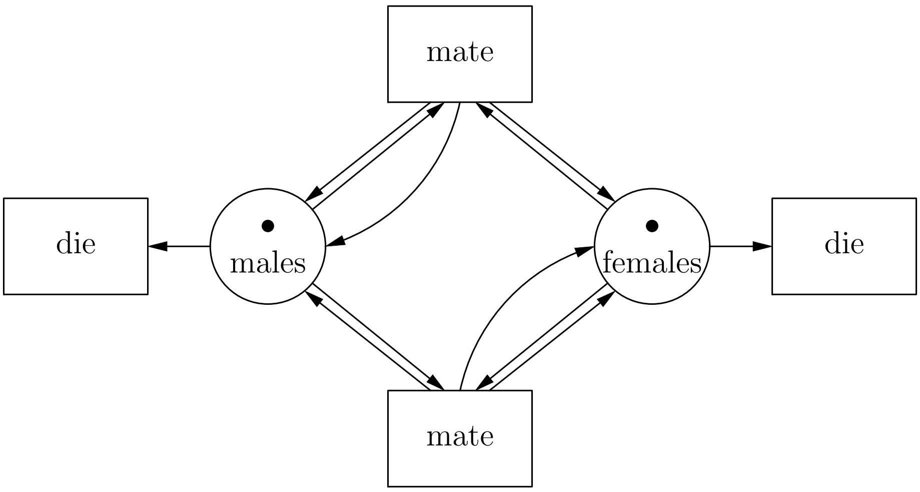
1. (a) $(M|F)\backslash m$ where $M \stackrel{\text{def}}{=} d.\mathbf{0} + \bar{m}.M$ and $F \stackrel{\text{def}}{=} d.\mathbf{0} + m.(F|M) + m.(F|F)$.
   This is a CCS expression where actions d and m stand for *die* and *mate*, resp.

   (b) $(M^k|F^\ell)\backslash m$ with $k, \ell \in \mathbb{N}$. Here M^k stands for k parallel copies $M|\dots|M$ of the process M. This expression is simplified modulo the laws $P|Q = Q|P$, $P|(Q|R) = (P|Q)|R$ and $P|\mathbf{0} = P$.

   (c)

$$
\dfrac{\dfrac{
\textit{recursion}\ \dfrac{\textit{R-choice}\ \dfrac{\textit{action prefixing}\ \dfrac{}{\bar{m}.M \xrightarrow{\bar{m}} M}}{d.\mathbf{0} + \bar{m}.M \xrightarrow{\bar{m}} M}}{M \xrightarrow{\bar{m}} M}
\qquad
\dfrac{\dfrac{\dfrac{\dfrac{}{m.(F|M) \xrightarrow{m} F|M}\ \textit{action prefixing}}{m.(F|M) + m.(F|F) \xrightarrow{m} F|M}\ \textit{L-choice}}{d.\mathbf{0} + (m.(F|M) + m.(F|F)) \xrightarrow{m} F|M}\ \textit{R-choice}}{F \xrightarrow{m} F|M}\ \textit{recursion}
}{M|F \xrightarrow{\tau} M|(F|M)}\ \textit{synchronisation}}{(M|F)\backslash m \xrightarrow{\tau} (M|(F|M))\backslash m}\ \textit{restriction}
$$

   (d)



   (e) A finite safe Petri net has only finitely many different states. This system has infinitely many different states, for if there are n fruit flies, it is possible to perform n *die* actions without any *mate*s.

2. $(b\|c) + b$.

3. (a) $\dfrac{P \stackrel{c}{\not\longrightarrow}}{f(P) \xrightarrow{c} f(P)} \qquad \dfrac{P \xrightarrow{\alpha} Q}{f(P) \xrightarrow{\alpha} f(Q)}$ (for $\alpha := a, b, c$)

   (b) No, for $ab + ac =_{PT} a(b+c)$, yet only $f(ab+ac)$ has a partial trace $accb$.

   (c) No, for $ab + ac =_{CT} a(b+c)$, yet only $f(ab+ac)$ has a partial trace $accb$.
   Here we use that completed trace equivalent processes also have the same partial traces.

   (d) Yes, for the structural operational rules above are in GSOS format.

4. (a) $Q_\varepsilon = (B_\varepsilon[c/s] \mid B_\varepsilon[c/r])\backslash c$.
Here $[c/s]$ is the CCS relabelling operator that renames s into c.

(b) Write $P_{\sigma,\rho}$ for $(B_\sigma[c/s] \mid B_\rho[c/r])\backslash c$. The expansion law for $|$, together with the laws for the restriction operator $\backslash m$ gives

$$P_{\varepsilon,\varepsilon} = \sum_{d\in\mathcal{D}} r(d).P_{d,\varepsilon}$$

$$P_{d,\varepsilon} = \tau.P_{\varepsilon,d}$$

$$P_{\varepsilon,e} = \bar{s}(e).P_{\varepsilon,\varepsilon} + \sum_{d\in\mathcal{D}} r(d).P_{d,e}$$

$$P_{d,e} = \bar{s}(e).P_{d,\varepsilon}$$

Now fill in $\tau.P_{\varepsilon,d}$ for $P_{d,\varepsilon}$ in the first and last equation, and apply the axiom $a.\tau.P = a.P$, which holds for branching bisimilarity. This yields

$$P_{\varepsilon,\varepsilon} = \sum_{d\in\mathcal{D}} r(d).P_{\varepsilon,d}$$

$$P_{\varepsilon,e} = \bar{s}(e).P_{\varepsilon,\varepsilon} + \sum_{d\in\mathcal{D}} r(d).P_{d,e}$$

$$P_{d,e} = \bar{s}(e).P_{\varepsilon,d}$$

Now $P_{\varepsilon,\varepsilon}$, $P_{\varepsilon,e}$ and $P_{d,e}$ satisfy the defining equations of Q_ε, Q_e and Q_{de}, respectively. Hence RDP yields $P_{\varepsilon,\varepsilon} = Q_\varepsilon$.

5. (a) $\mathbf{F}(ec_B)$ holds for A_{33}, but not for A_{32}.

(b) $\mathbf{G}(ec_A \Rightarrow (\neg ec_B)\mathbf{U}(lc_A)) \wedge \mathbf{G}(ec_B \Rightarrow (\neg ec_A)\mathbf{U}(lc_B))$.

(c) Yes. There are no reachable states where Process A is between ec_A and lc_A while Process B is between ec_B and lc_B.

(d) $\mathbf{G}(ln_A \Rightarrow \mathbf{F}\,ec_A)$.

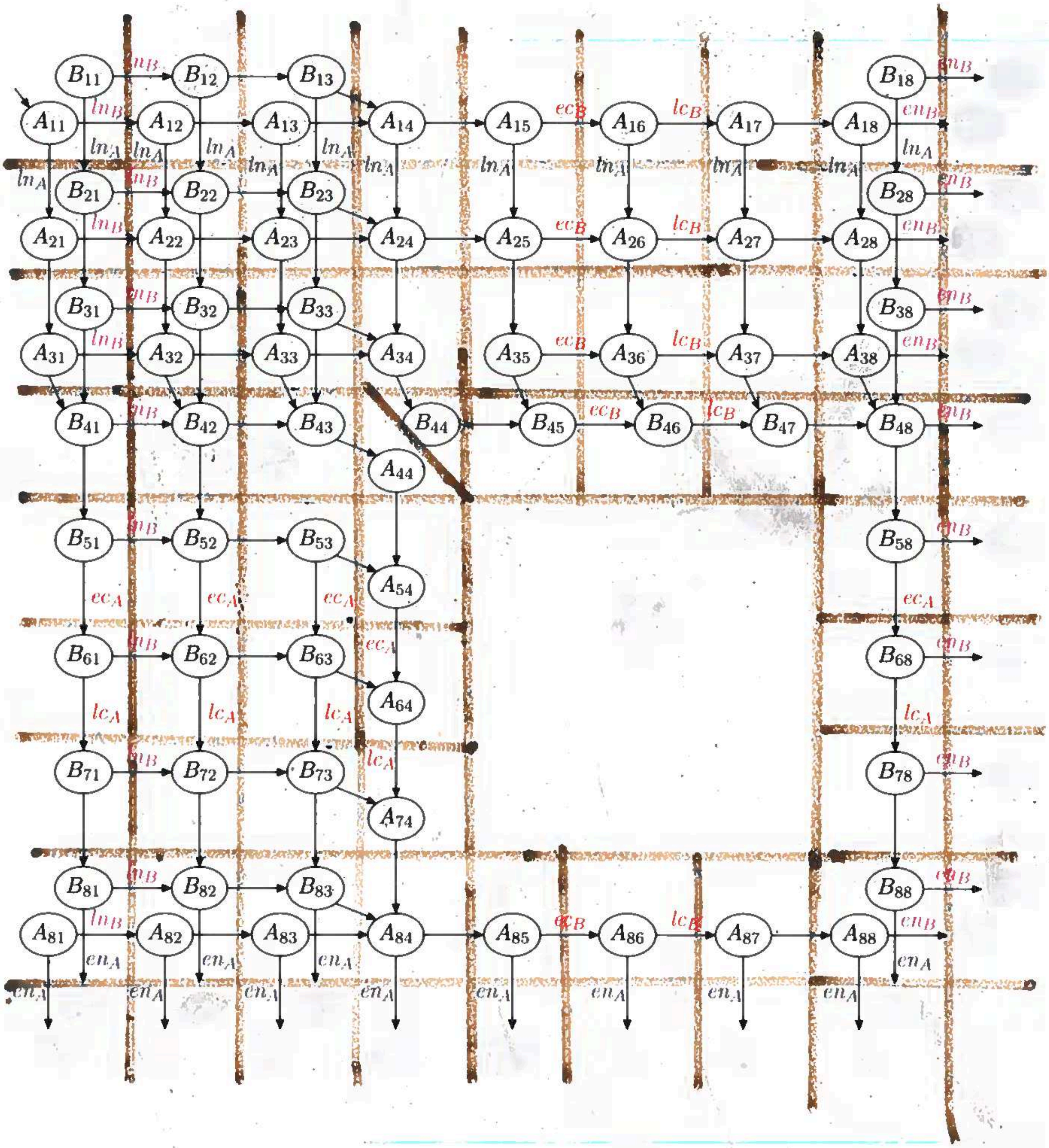
(e) No. One could go forever to the right from state A_{21}.

(f) 64-9+1=56 states. See Figure 1 on the next page.

(g) 42. (This question has been anticipated for a long time.) See Figure 2.

(h) The dark blue separator lines in Figure 2 can be deleted for weak bisimilarity.
Let formula ψ say that ec_B will occur after the first, but prior to the second occurrence of ec_A. That formula holds exclusively in stated B_{43}, A_{44} and A_{54}. It does not hold in B_{53}. Let $\varphi = \mathbf{EG}\neg ec_B$ be the formula that says that ec_B might never occur. It holds in B_{42} and B_{52}, but not in B_{53}. Now B_{42} and B_{52} for instance can be separated by the formula $\mathbf{E}(\varphi\mathbf{U}\psi)$.
The formula ψ can be given as

$$\mathbf{A}(\neg(ec_B)\mathbf{U}(ec_A \wedge \mathbf{A}(ec_A\mathbf{U}((\neg ec_A) \wedge \mathbf{A}(\neg(ec_A)\mathbf{U}\,ec_B))))).$$


B_{11} B_{12} B_{13} B_{18}
A_{11} A_{12} A_{13} A_{14} A_{15} A_{16} A_{17} A_{18}
B_{21} B_{22} B_{23} B_{28}
A_{21} A_{22} A_{23} A_{24} A_{25} A_{26} A_{27} A_{28}
B_{31} B_{32} B_{33} B_{38}
A_{31} A_{32} A_{33} A_{34} A_{35} A_{36} A_{37} A_{38}
B_{41} B_{42} B_{43} B_{44} B_{45} B_{46} B_{47} B_{48}
A_{44}
B_{51} B_{52} B_{53} A_{54} B_{58}
B_{61} B_{62} B_{63} A_{64} B_{68}
B_{71} B_{72} B_{73} A_{74} B_{78}
B_{81} B_{82} B_{83} B_{88}
A_{81} A_{82} A_{83} A_{84} A_{85} A_{86} A_{87} A_{88}
ln_B ln_A ec_B lc_B en_B ec_A lc_A en_A