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S2747975.

Modelling Concurrent Systems

Homework 9

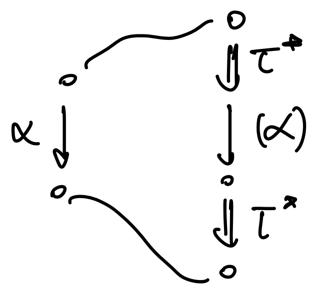
1. $P_1 = a.0 + b.0$, $P_2 = \tau.(a.0 + b.0)$

$P_3 = \tau.a.0 + b.0$, $P_4 = a.\tau.0 + b.0$

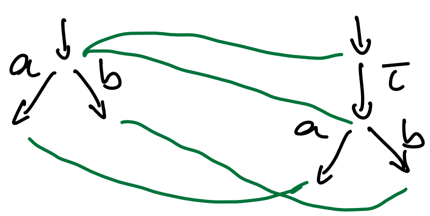
$P_5 = \tau.(a.0 + b.0) + a.0$

a) Since we don't have an axiomatization of WB (because WB is not a congruence), I decided to approach this problem by analysing the LTS.

$\Rightarrow$ Recalling the "definition" of WB :

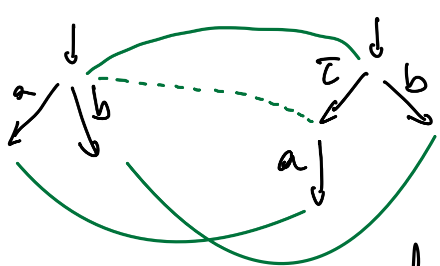


<u>P_1 vs. P_2</u>



$P_1 =_{WB} P_2$

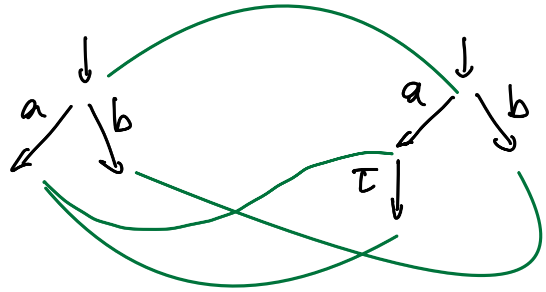
<u>P_1 vs. P_3</u>



The dashed line relation would be mandatory, but P_3 doesn't have an outgoing action b.

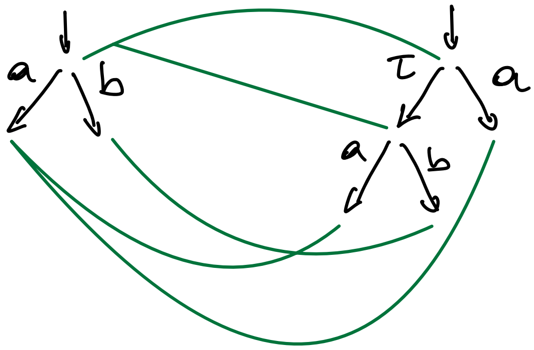
$\Rightarrow P_1 \neq_{WB} P_3$.

P_1 vs. P_4



$P_1 =_{WB} P_4$.

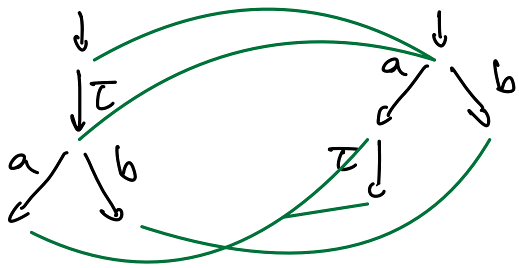
P_1 vs. P_5



$P_1 =_{WB} P_5$.

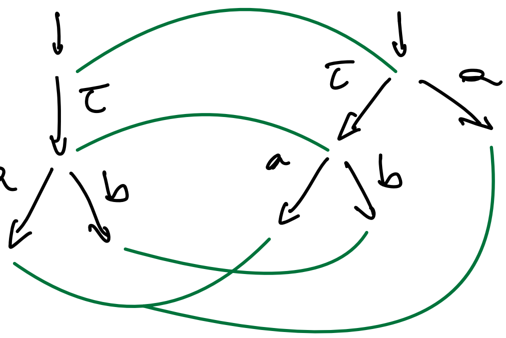
P_2 vs. P_3 $\Rightarrow$ Since $P_1 =_{WB} P_2$ and $P_2 \neq_{WB} P_3 \Rightarrow P_2 \neq_{WB} P_3$.

P_2 vs. P_4



$P_2 =_{WB} P_4$.
(also because
$P_2 =_{WB} P_1 =_{WB} P_4$)

P_2 vs. P_5



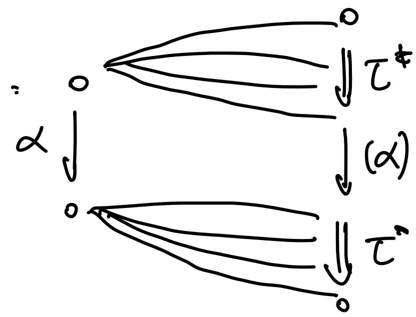
$P_2 =_{WB} P_5$
(also because
$P_2 =_{WB} P_1 =_{WB} P_5$)

P_3 vs. P_4 $\Rightarrow$ Since $P_2 \neq_{WB} P_3$ and $P_2 =_{WB} P_4 \Rightarrow P_3 \neq_{WB} P_4$.

P_3 vs. P_5 $\Rightarrow$ Since $P_2 \neq_{WB} P_3$ and $P_2 =_{WB} P_5 \Rightarrow P_3 \neq_{WB} P_5$

P_4 vs. P_5 $\Rightarrow$ Since $P_2 =_{WB} P_4$ and $P_2 =_{WB} P_5 \Rightarrow P_4 =_{WB} P_5$.

b) Recalling the "definition" of BB :

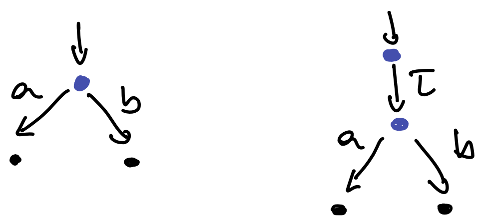


$\Rightarrow$ We also note that BB is finer than WB.

Since $P_1 \neq_{WB} P_3$, $P_2 \neq_{WB} P_3$, $P_3 \neq_{WB} P_4$, $P_3 \neq_{WB} P_5$ $\Rightarrow$ $P_1 \neq_{BB} P_3$, $P_2 \neq_{BB} P_3$, $P_3 \neq_{BB} P_4$, $P_3 \neq_{BB} P_5$.

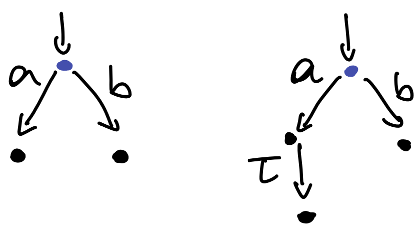
$$\begin{array}{l|l} P_1 \neq_{WB} P_3 & P_1 \neq_{BB} P_3 \\ P_2 \neq_{WB} P_3 & P_2 \neq_{BB} P_3 \\ P_3 \neq_{WB} P_4 & P_3 \neq_{BB} P_4 \\ P_3 \neq_{WB} P_5 & P_3 \neq_{BB} P_5 . \end{array}$$

<u>P_1 vs. P_2</u>



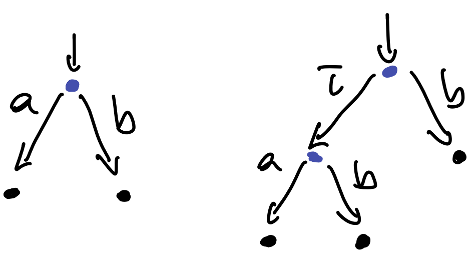
$\{\}$
$\{(a,\bullet),(b,\bullet)\}$
$P_1 =_{BB} P_2$

<u>P_1 vs. P_4</u>



$\{\}$
$\{(a,\bullet),(b,\bullet)\}$
$P_1 =_{BB} P_4$

<u>P_1 vs. P_5</u>



$\{\}$
$\{(a,\bullet),(b,\bullet)\}$
$P_1 =_{BB} P_5$

<u>P_2 vs. P_4</u> $\Rightarrow$ Since $P_1 =_{BB} P_2$ and $P_1 =_{BB} P_4$ $\Rightarrow$ $P_2 =_{BB} P_4$

<u>P_2 vs. P_5</u> $\Rightarrow$ Since $P_1 =_{BB} P_2$ and $P_1 =_{BB} P_5$ $\Rightarrow$ $P_2 =_{BB} P_5$.

<u>P_4 vs. P_5</u> $\Rightarrow$ $P_4 =_{BB} P_1 =_{BB} P_5$ $\Rightarrow$ $P_4 =_{BB} P_5$.

c) $P =_{RWB} Q \;:\Longleftrightarrow\; \begin{cases} \forall P.\ P \xrightarrow{\alpha} P'.\ \exists Q'.\ Q \Rightarrow\xrightarrow{\alpha}\Rightarrow Q'.\ P' =_{WB} Q' \\ \forall Q.\ Q \xrightarrow{\alpha} Q'.\ \exists P'.\ P \Rightarrow\xrightarrow{\alpha}\Rightarrow P'.\ P' =_{WB} Q' \end{cases}$

Since

$$\left.\begin{array}{l} P_1 \neq_{WB} P_3 \\ P_2 \neq_{WB} P_3 \\ P_3 \neq_{WB} P_4 \\ P_3 \neq_{WB} P_5 \end{array}\right| \Rightarrow \begin{array}{l} P_1 \neq_{RWB} P_3 \\ P_2 \neq_{RWB} P_3 \\ P_3 \neq_{RWB} P_4 \\ P_3 \neq_{RWB} P_5. \end{array}$$

Axiomatisation of RWB (WB^c)

$$\left.\begin{array}{rcl} (P+Q)+R &=& P+(Q+R) \\ P+Q &=& Q+P \\ P+P &=& P \\ P+0 &=& P \end{array}\right\} \text{From strong bisimulation equivalence on recursion-free CCS}$$

$$\left.\begin{array}{lrcl} (T_1) & \alpha.\tau.P &=& \alpha.P \\ (T_2) & \tau.P &=& \tau.P+P \\ (T_3) & \alpha.(\tau.P+Q) &=& \alpha.(\tau.P+Q)+\alpha.P \end{array}\right\} \text{Extra for axiomatisation of RWB.}$$

P_1 vs. P_2

$P_1 = a.0 + b.0$

$P_2 = \tau.(a.0+b.0)$

$P_1 \neq_{RWB} P_2$ because you can't perform P_2's τ transition from root in P_1.

P_1 vs. P_4

$P_1 = a.0 + b.0$

$P_4 = a.\tau.0 + b.0 \underset{(T_1)}{=} a.0 + b.0 \;\Big|\!\Rightarrow P_1 =_{RWB} P_4$

$\underline{P_1 \text{ vs. } P_5}$ $\quad P_1 = a.0+b.0$

$P_5 = \tau.(a.0+b.0)+a.0$ $\Rightarrow$ $P_1 \neq_{RWB} P_5$ - same τ-action argument as $P_1 \neq_{RWB} P_2$.

$\underline{P_2 \text{ vs. } P_4}$ $\Rightarrow$ Since $P_1 \neq_{RWB} P_2$ and $P_1 =_{RWB} P_4 \Rightarrow P_2 \neq_{RWB} P_4$

$\underline{P_2 \text{ vs. } P_5}$

$$\left.\begin{aligned} P_2 &= \tau.(a.0+b.0) \overset{(T_2)}{=} \tau.(a.0+b.0)+a.0+b.0 \\ P_5 &= \tau.(a.0+b.0)+a.0 \underset{(T_2)}{=} \tau.(a.0+b.0)+a.0+a.0+b.0 = \\ &= \tau.(a.0+b.0)+a.0+b.0 \end{aligned}\right\} \Rightarrow P_2 =_{RWB} P_5$$

$\underline{P_4 \text{ vs. } P_5}$ $\Rightarrow$ Since $P_2 \neq_{RWB} P_4$ and $P_2 =_{RWB} P_5 \Rightarrow P_4 \neq_{RWB} P_5$.

d) $$P =_{RBB} Q :\Leftrightarrow \begin{cases} \forall P.\ P \xrightarrow{\alpha} P' \Rightarrow \exists Q'.\ Q \xrightarrow{\alpha} Q'.\ P' =_{BB} Q' \\ \forall Q.\ Q \xrightarrow{\alpha} Q' \Rightarrow \exists P'.\ P \xrightarrow{\alpha} P'.\ P' =_{BB} Q' \end{cases}$$

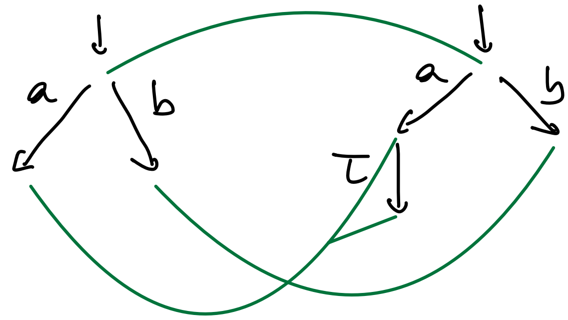
Axiomatisation of BB^c (RBB)

$\Rightarrow$ The axioms for B $\oplus$

(CB) $\alpha.(\tau.(P+Q)+Q) = \alpha.(P+Q)$.

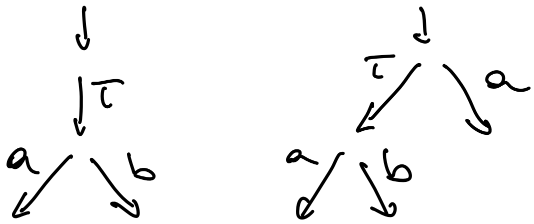
We only have to check P_1 vs. P_4 and P_2 vs. P_5. All the others are not RBB because they're not RWB, and RWB is coarser.

P_1 vs. P_4



$P_1 =_{RBB} P_4$

P_2 vs. P_5



We clearly see that $P_2 \neq_{RBB} P_5$ because we can't make the first action behave in a B manner, because P_5's a- action from root has no equivalent in P_2.

2. RTP: (B) : $\alpha.(\tau.(P+Q)+Q) = \alpha.(P+Q)$.

$$\alpha.(\tau.(P+Q)+Q) \overset{(T3)}{=} \alpha.(\underline{\tau.(P+Q)}+Q) + \alpha(P+Q) =$$

$$= \alpha.(\tau.(P+Q) + \underline{(P+Q)+Q}) + \alpha.(P+Q) =$$

$$= \alpha.(\tau.(P+Q) + P + \underline{(Q+Q)}) + \alpha.(P+Q) =$$

$$= \alpha.(\underline{\tau.(P+Q) + (P+Q)}) + \alpha.(P+Q) =$$

$$= \underline{\alpha.\tau.(P+Q)} + \alpha.(P+Q) =$$

$$= \underline{\alpha.(P+Q) + \alpha.(P+Q)} =$$

$$= \alpha.(P+Q)$$

I have underlined where appropriate axioms were used.