1 Deadlock, Livelock and Divergence

Definition 2 A process p *is deadlocked* if $p \stackrel{a}{\longrightarrow}$ for no $a \in A_\tau$.
p *has a deadlock trace* σ if $p \stackrel{\sigma}{\Longrightarrow} q$ for some deadlocked process q.
p *has an initial deadlock* if $p \stackrel{\varepsilon}{\Longrightarrow} q$ for some deadlocked process q.
p *has a deadlock* if a process reachable from p is deadlocked.
p *is deadlock-free* if no process reachable from p is deadlocked.

An equivalence relation $\sim$ on processes
respects deadlock traces if $p \sim q$ implies that any deadlock trace of p is also a deadlock trace of q,
respects initial deadlock if $p \sim q$ and p has an initial deadlock implies that q has an initial deadlock
and *respects deadlock-freedom* if $p \sim q$ and p is deadlock-free implies that q is deadlock-free.

A process has an initial deadlock iff ε is one of its deadlock traces. It is deadlock-free iff it has no deadlocks iff the set of its deadlock traces is empty. Thus, an equivalence that respects deadlock traces surely respects initial deadlock, as well as deadlock-freedom.

Definition 3 A process p *is livelocked* if any q with $p \stackrel{\varepsilon}{\Longrightarrow} q$ satisfies $q \stackrel{\tau}{\longrightarrow}$ but $q \stackrel{a}{\longrightarrow}$ for no $a \in A$.
p *has a livelock trace* σ if $p \stackrel{\sigma}{\Longrightarrow} q$ for some livelocked process q.
p *has an initial livelock* if $p \stackrel{\varepsilon}{\Longrightarrow} q$ for some livelocked process q.
p *has a livelock* if a process reachable from p is livelocked.
p *is livelock-free* if no process reachable from p is livelocked.

An equivalence relation $\sim$ on processes
respects livelock traces if $p \sim q$ implies that any livelock trace of p is also a livelock trace of q,
respects initial livelock if $p \sim q$ and p has an initial livelock implies that q has an initial livelock
and *respects livelock-freedom* if $p \sim q$ and p is livelock-free implies that q is livelock-free.

A process has an initial livelock iff ε is one of its livelock traces. It is livelock-free iff it has no livelocks iff the set of its livelock traces is empty. Thus, an equivalence that respects livelock traces surely respects initial livelock, as well as livelock-freedom.

Definition 4 A process p *is locked* if $p \stackrel{a}{\Longrightarrow}$ for no $a \in A$.
p *has a deadlock/livelock trace* σ if $p \stackrel{\sigma}{\Longrightarrow} q$ for some locked process q.
p *has an initial deadlock/livelock* if $p \stackrel{\varepsilon}{\Longrightarrow} q$ for some locked process q.
p *has a deadlock/livelock* if a process reachable from p is locked.
p *is lock-free* if no process reachable from p is locked.

An equivalence relation $\sim$ on processes *respects deadlock/livelock traces* if $p \sim q$ implies that any deadlock/livelock trace of p is also a deadlock/livelock trace of q. It *respects initial deadlock/livelock* if $p \sim q$ and p has an initial deadlock/livelock implies that q has an initial deadlock/livelock. It *respects lock-freedom* if $p \sim q$ and p is lock-free implies that q is lock-free.

A process has an initial deadlock/livelock iff ε is one of its deadlock/livelock traces. It is lock-free iff the set of its deadlock/livelock traces is empty. Thus, an equivalence that respects deadlock/livelock traces surely respects initial deadlock/livelock, as well as lock-freedom.

If a process is deadlocked or livelocked, it surely is locked. However, a locked process need not be deadlocked or livelocked. Still, a locked process is either livelocked or has a initial deadlock. Thus, the set of deadlock/livelock traces of a process is the union of the sets of its deadlock traces and its livelock traces. A process has an initial deadlock/livelock iff it has an initial deadlock or an initial livelock. And a process is lock-free iff it is deadlock-free as well as livelock-free.

Definition 5 A process p *is divergent* if there are p_i for $i \in \mathbb{N}^+$ with $p \xrightarrow{\tau} p_1 \xrightarrow{\tau} p_2 \xrightarrow{\tau} \cdots$.
p *has a divergence trace* σ if $p \stackrel{\sigma}{\Longrightarrow} q$ for some divergent process q.
p *has an initial divergence* if $p \stackrel{\varepsilon}{\Longrightarrow} q$ for some divergent process q.
p *has a divergence* if a process reachable from p is divergent.
p *is divergence-free* if no process reachable from p is divergent.

An equivalence relation $\sim$ on processes *respects divergence traces* if $p \sim q$ implies that any divergence trace of p is also a divergence trace of q. It *respects (initial) divergence* if $p \sim q$ and p is divergent implies that q is divergent. It *respects divergence-freedom* if $p \sim q$ and p is divergence-free implies that q is divergence-free.

A process has an initial divergence iff it is divergent iff ε is one of its divergence traces. It is divergence-free iff it has no divergences iff the set of its divergence traces is empty. Thus, an equivalence that respects divergence traces surely respects divergence, as well as divergence-freedom.

Definition 6 A process p *has a deadlock/divergence trace* σ if $p \stackrel{\sigma}{\Longrightarrow} q$ for a deadlocked or divergent process q. It *is deadlock/divergence-free* if no process reachable from p is deadlocked or divergent.

An equivalence relation $\sim$ on processes *respects deadlock/divergence traces* if $p \sim q$ implies that any deadlock/divergence trace of p is also a deadlock/divergence trace of q. It *respects initial deadlock/divergence* if $p \sim q$ and p has an initial deadlock or initial divergence implies that also q has an initial deadlock or initial divergence. It *respects deadlock/divergence-freedom* if $p \sim q$ and p is deadlock/divergence-free implies that q is deadlock/divergence-free.

A process is deadlock/divergence-free iff the set of its deadlock/divergence traces is empty. Thus, an equivalence that respects deadlock/divergence traces surely respects deadlock/divergence-freedom, as well as initial deadlock/divergence.