$$\alpha.E \xrightarrow{\alpha} E \qquad\qquad \frac{E_j \xrightarrow{\alpha} E_j'}{\sum_{i\in I} E_i \xrightarrow{\alpha} E_j'}\ (j\in I)$$

$$\frac{E \xrightarrow{\alpha} E'}{E|F \xrightarrow{\alpha} E'|F} \qquad \frac{E \xrightarrow{a} E',\ F \xrightarrow{\bar{a}} F'}{E|F \xrightarrow{\tau} E'|F'} \qquad \frac{F \xrightarrow{\alpha} F'}{E|F \xrightarrow{\alpha} E|F'}$$

$$\frac{E \xrightarrow{\alpha} E',\ \alpha \notin L \cup \overline{L}}{E\backslash L \xrightarrow{\alpha} E'\backslash L} \qquad \frac{E \xrightarrow{\alpha} E'}{E[f] \xrightarrow{f(\alpha)} E'[f]} \qquad \frac{S(X)[\mathbf{fix}_Y S/Y]_{Y\in dom(S)} \xrightarrow{\alpha} E}{\mathbf{fix}_X S \xrightarrow{\alpha} E}$$

Table 1: Structural operational semantics of CCS

1 CCS

CCS [4] is parametrised with a set $\mathscr{A}$ of *names*. The set $\bar{\mathscr{A}}$ of *co-names* is $\bar{\mathscr{A}} := \{\bar{a} \mid a \in \mathscr{A}\}$, and $\mathscr{L} := \mathscr{A} \cup \bar{\mathscr{A}}$ is the set of *labels*. The function $\bar{\cdot}$ is extended to $\mathscr{L}$ by declaring $\bar{\bar{a}} = a$. Finally, $Act := \mathscr{L} \dot\cup \{\tau\}$ is the set of *actions*. Below, a, b, c, … range over $\mathscr{L}$ and α, β over Act. A *relabelling function* is a function $f : \mathscr{L} \to \mathscr{L}$ satisfying $f(\bar{a}) = \overline{f(a)}$; it extends to Act by $f(\tau) := \tau$. Let $\mathscr{X}$ be a set $X, Y, \dots$ of *process variables*. The set $\mathscr{E}$ of CCS terms or *process expressions* is the smallest set including:

$\alpha.E$	for $\alpha \in Act$ and $E \in \mathscr{E}$	*prefixing*
$\sum_{i\in I} E_i$	for I an index set and $E_i \in \mathscr{E}$	*choice*
$E\|F$	for $E, F \in \mathscr{E}$	*parallel composition*
$E\backslash L$	for $L \subseteq \mathscr{L}$ and $E \in \mathscr{E}$	*restriction*
$E[f]$	for f a relabelling function and $E \in \mathscr{E}$	*relabelling*
X	for $X \in \mathscr{X}$	a *process variable*
$\mathbf{fix}_X S$	for $S : \mathscr{X} \rightharpoonup \mathscr{E}$ and $X \in dom(S)$	*recursion.*

One writes $E_1 + E_2$ for $\sum_{i\in I} E_i$ with $I = \{1,2\}$, and 0 for $\sum_{i\in\emptyset} E_i$. A partial function $S : \mathscr{X} \rightharpoonup \mathscr{E}$ is called a *recursive specification*. The variables in its domain $dom(S)$ are called *recursion variables* and the equations $Y = S(Y)$ for $Y \in dom(S)$ *recursion equations*. A recursive specification $S : \mathscr{X} \rightharpoonup \mathscr{E}$ is traditionally written as $\{Y = S(Y) \mid Y \in dom(S)\}$.

The operational semantics of CCS is given by the labelled transition relation $\rightarrow \subseteq \mathrm{T}_{\mathrm{CCS}} \times Act \times \mathrm{T}_{\mathrm{CCS}}$ between closed CCS expressions. The transitions $p \xrightarrow{\alpha} q$ with $p, q \in \mathrm{T}_{\mathrm{CCS}}$ and $\alpha \in Act$ are derived from the rules of Table 1. Formally a transition $p \xrightarrow{\alpha} q$ is part of the transition relation of CCS if there exists a well-founded, upwards branching tree (a *proof* of the transition) of which the nodes are labelled by transitions, such that

- the root is labelled by $p \xrightarrow{\alpha} q$, and
- if φ is the label of a node n and K is the set of labels of the nodes directly above n, then $\frac{K}{\varphi}$ is a rule from Table 1, with closed CCS expressions substituted for the variables $E, F, \dots$.

2 CSP

CSP [1, 5, 2, 3] is parametrised with a set $\mathscr{A}$ of *communications*; $Act := \mathscr{A} \dot\cup \{\tau\}$ is the set of *actions*. Below, a, b range over $\mathscr{A}$ and α, β over Act. The set $\mathscr{E}$ of CSP terms is the smallest set including:

STOP		*inaction*
DIV		*divergence*
$(a \rightarrow E)$	for $a \in \mathscr{A}$ and $E \in \mathscr{E}$	*prefixing*
$E \,\Box\, F$	for $E, F \in \mathscr{E}$	*external choice*
$E \sqcap F$	for $E, F \in \mathscr{E}$	*internal choice*
$E \Vert_A F$	for $E, F \in \mathscr{E}$ and $A \subseteq \mathscr{A}$	*parallel composition*
E/b	for $b \in \mathscr{A}$ and $E \in \mathscr{E}$	*concealment*
$f(E)$	for $E \in \mathscr{E}$ and $f : Act \rightarrow Act$ with $f(\tau) = \tau$ and $f^{-1}(a)$ finite	*renaming*
X	for $X \in \mathscr{X}$	a *process variable*
$\mu X \cdot E$	for $E \in \mathscr{E}$ and $X \in \mathscr{X}$	*recursion*.

As in [5], I here leave out the guarded choice $(x : B \rightarrow P(x))$ and the constant RUN of [1], and the inverse image and sequential composition operator, with constant SKIP, of [1, 2]. The semantics of CSP was originally given in quite a different way [1, 2], but [5] provided an operational semantics of CSP in the same style as the one of CCS, and showed its consistency with the original semantics. It is this operational semantics I will use here; it is given by the rules in Table 2. Let $\mathscr{L} := \mathscr{A}$.

$$\text{DIV} \xrightarrow{\tau} \text{DIV} \qquad (a \rightarrow E) \xrightarrow{a} E \qquad E \sqcap F \xrightarrow{\tau} E \qquad E \sqcap F \xrightarrow{\tau} F$$

$$\frac{E \xrightarrow{a} E'}{E \,\Box\, F \xrightarrow{a} E'} \qquad \frac{F \xrightarrow{a} F'}{E \,\Box\, F \xrightarrow{a} F'} \qquad \frac{E \xrightarrow{\tau} E'}{E \,\Box\, F \xrightarrow{\tau} E' \,\Box\, F} \qquad \frac{F \xrightarrow{\tau} F'}{E \,\Box\, F \xrightarrow{\tau} E \,\Box\, F'}$$

$$\frac{E \xrightarrow{\alpha} E' \quad (\alpha \notin A)}{E \Vert_A F \xrightarrow{\alpha} E' \Vert_A F} \qquad \frac{E \xrightarrow{a} E' \quad F \xrightarrow{a} F' \quad (a \in A)}{E \Vert_A F \xrightarrow{a} E' \Vert_A F'} \qquad \frac{F \xrightarrow{\alpha} F' \quad (\alpha \notin A)}{E \Vert_A F \xrightarrow{\alpha} E \Vert_A F'}$$

$$\frac{E \xrightarrow{b} E'}{E/b \xrightarrow{\tau} E'/b} \qquad \frac{E \xrightarrow{\alpha} E' \quad (\alpha \neq b)}{E/b \xrightarrow{\alpha} E'/b} \qquad \frac{E \xrightarrow{\alpha} E'}{f(E) \xrightarrow{f(\alpha)} f(E')} \qquad \mu X \cdot E \xrightarrow{\tau} E[\mu X \cdot E / X]$$

Table 2: Structural operational semantics of CSP

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