SCS&E Report 9403
March, 1994

Motion planning in Prototypical Corridors

N. Ahmed and A. Sowmya

SCHOOL OF COMPUTER SCIENCE AND ENGINEERING
THE UNIVERSITY OF NEW SOUTH WALES

Abstract

We discuss the motion planning of a rectangular moving object in certain prototypical situations arising in a 2-D isothetic workspace. We have suggested three possible motion strategies involving rotation and translation of the moving object negotiating an L-shaped corridor. We have also given simulation results to compare the three cases of the proposed motion.

E-mail:
sowmya@cse.unsw.edu.au

First author's affiliation: Department of Computer Science James Cook University of North Queensland Townsville QLD 4811, Australia

1 Introduction

Motion planning is required for moving a robot from a source placement (s) to a destination placement (d) within a workspace occupied by obstacles. Motion planning for a mobile robot can be viewed as a two-level process. At level one, the primary interest is in solving the FINDPATH problem, called path planning under global constraints. The result of this level is a path description of the desired motion from s to d. Once a path is proposed by level one, the level two performs *trajectory planning* wherein local motion is decided based on local geometric properties of the workspace, the moving object and other constraints along the specified path. The output of trajectory planning is a sequence of placements of the moving object. Normally, trajectory planning based on local constraints does not change a path computed by FINDPATH, since the latter would have considered all such constraints before computing the path.

Freespace decomposition is one of the geometric methods employed to solve the path planning problem [HWA92]. The freespace method works in two stages. In the first stage, an appropriate representation scheme that captures the essential properties of the problem environment is devised; the second stage utilizes this representation to solve the problem at hand. In other words, this method imposes an abstract geometric structure upon the physical structure of obstacles and the moving object. Generally, such a representation uses a primitive geometric shape to describe the freespace, so that the freespace can be viewed as a collection of connected corridors of primitive shapes. Due to the geometric simplicity of the primitive shape, corridors and their connections may be categorized into a finite number of prototypical situations [MAD86]. In order to plan collision free robot motion both at FINDPATH and at trajectory planning levels, it is essential that we understand the mechanism of moving the robot through prototypical situations.

The problem of moving an object from one point to another inside a complex 2-D and 3-D workspace has been well studied [LAT91]. However moving an object in prototypical situations has not been given much attention. Often complex general solutions which address global aspects of the motion planning problem are not suitable for a simplified instance of the problem. Also certain techniques for solving the general motion planning problem, like freespace decomposition, naturally lead to subproblems involving prototypical situations. We briefly mention some efforts in this direction below.

The SOFA PROBLEM, determining the largest region (or sofa) which can be moved through a 2-D corridor was originally proposed by Moser [MOS66]. In one of the early attempts, Goldberg [GOL69] and Sebastian [SEB70] considered an analytical solution to find the largest *sized* object that can be maneuvered through a variety of corridors. Maruyama [MAR73] gave an approximation method for solving the SOFA PROBLEM for shapes which were angularly simple polygons. He used a sequence of transformations to move the sofa through a given corridor. Howden [HOW68] used chain representation to move the largest rectangle of a given width through an L-shaped corridor. His approach was applicable only when the given objective function specifies the generic shape, e.g., the largest square or the widest rectangle, but problems with Moser's objective function, like the largest-area sofa, cannot be tackled. Strang [STR82] and Yap [YAP87] have dealt with complexity theoretic analysis of moving a rigid object through a door and its generalization. Strang studied the motion of convex objects through a door in 2-D and 3-D and tried to compute the minimum width of the door which allows the passage of the object. Yap then extended the work to the motion of nonconvex polygonal objects and showed that all passages of the moving objects through the door can be reduced to a sequence of certain elementary motions.

This paper addresses the issue of motion planning in certain prototypical situations arising in a 2-D *isothetic* workspace for a moving robot modelled as a rectangle. The term isothetic means that all object boundaries are parallel to the principal axes. An isothetic workspace is a realistic model of a controlled indoor robot world while a rectangle is a good first approximation of the footprint of a mobile robot. A rectangle is also the natural choice of a primitive shape to represent the freespace in an isothetic workspace as it is the simplest of isothetic shapes. Ahmed and Biswas [AHM90a] have devised a representation for describing such a workspace. In this representation, rectangle-based primitive corridors arise naturally and include the I-, L-, T- and X-shaped prototypical segments. In this research, we propose a solution to the motion planning problem in such prototypical corridors, based on geometric constraint analysis of the motion.

the rest of the paper is organized as follows. Section 2 describes the prototypical situations arising in the isothetic workspace. In section 3, we identify three strategies for motion through an L-shaped path and deal them in detail. Thereafter section 4 discusses the relative merits and demerits of these models based on experimental results and the final section gives concluding remarks.

2 Prototypical Corridor Segments

We consider a moving object within a planar workspace containing obstacles which are modelled as isothetic polygons. The workspace itself is bounded by an isothetic polygon. The moving object is assumed to be a rectangle. The motion of the robot consists of translations and rotations. The translation is restricted to the directions of the two principal axes and the rotation is about any point on the edge of the moving object [AHM90b] [SOW91].

In this setting, the freespace is modelled as overlapping rectangular regions- [AHM90a]. Motion of an object through these regions is viewed as crossingovers from one region to the neighbouring region. We can think of a path from one point to another as a corridor through which the moving object manoeuvers itself. See Figure (1), where two points s and d are connected by a corridor.

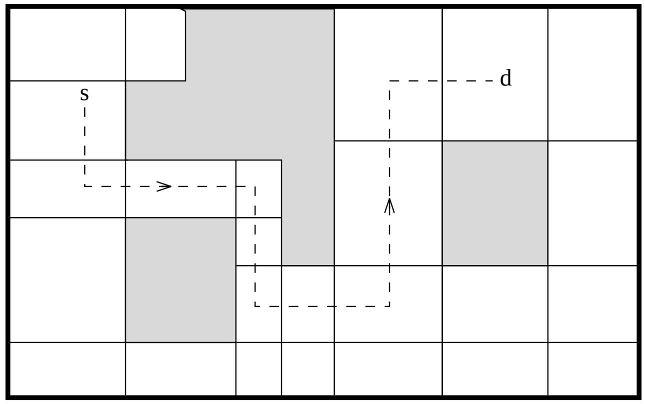


Figure 1: An Isothetic Workspace

A long corridor connecting two points can be considered as composed of corridor segments. In the isothetic workspace with rectangular corridors, four prototypical corridor segments may be recognized:

- Straight segment I which is horizontal or vertical
- L-segment, which has two I-segments intersecting at right angles
- T-segment, where three I-segments join in a T shape
- X-segment, consisting of four I-segments intersecting in an X shape

See Figure (2) for illustration. .

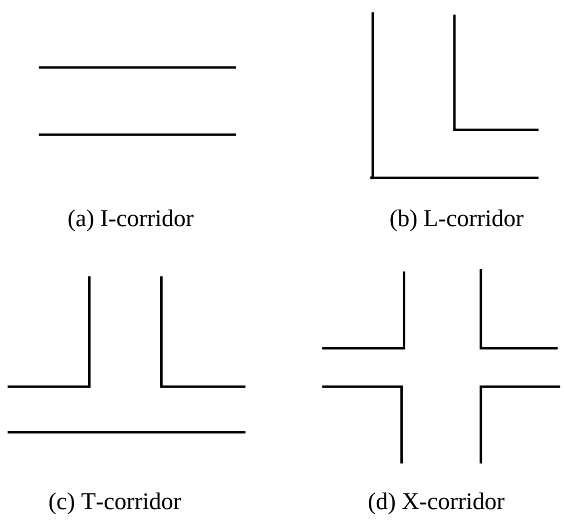


Figure 2: Prototypical Corridors

Motion planning along the I-segment is straightforward. Once the moving object is in the corridor, it can be moved using translation alone. The largest object which can pass through is determined by the length and width of the corridor. Moving along the L-segment involves translation and if necessary rotation. Rotation is confined to the region where two I-segments meet. The remaining corridor segment can be maneouvered through by translating the moving object.

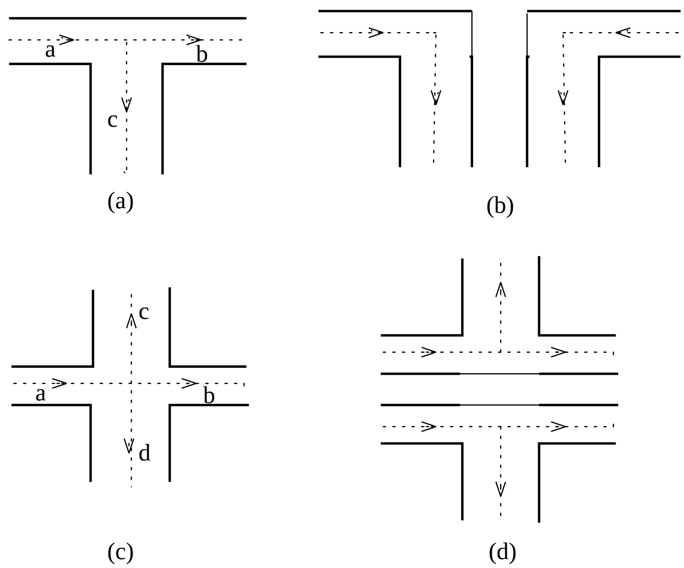


Figure 3: Motion in T- and X-corridor

The case of moving through T- and X-segments has two possibilities. First, the motion is along the sleeve, i.e., the moving object enters from one end of the sleeve and exits from the other end. See path marked by arrows a and b in Figure (3). It is important to note that in this representation, the T- and X-segments can never have sleeves with two varying entry and exit widths. thus, the motion in these cases simplifies to moving along an I-segment. In the second situation, the moving object enters through one end of the sleeve and exits from the vest end (path marked by arrows a and c). It can be shown that for a convex moving object like a rectangle, this motion can be simplified to moving through an L-segment (Figure (3.b) and (3.d)). Therefore we concentrate on motion within an L-segment which can be applied to T- and X-segments. For notational simplicity, we will use the term corridor for corridor segment now onwards.

3 Motion through L-corridor

When motion through an L-corridor comprises a sequence of translations and rotations, three basic cases of such motion can be identified. In CASE 1, the motion is effected in several steps of translation and a single step of rotation. CASE 2 involves incremental translation and incremental rotation simultaneously about a pivot. Lastly, in CASE 3, the object moves using incremental translation and incremental rotation such that the moving object slides against the corridor walls, touching the walls at two points, unlike earlier cases where it touches the walls only at the pivot point [AHM90b] [SOW91]. We now consider these cases of moving through an L-corridor in detail. In each case, we first consider the motion of a rigid rod through an L-corridor and then extend it to a rectangle. We compute the maximum length of the moving rectangle for a given width. For simplicity we first assume that the corridor is symmetric, i.e., its width is constant, and extend the result to an asymmetric corridor.

It is convenient to have the following definitions. Referring to Figure (4), let the four walls forming the L-corridor be defined as

$$\begin{array}{lcll} X &=& \{(x,0) &:\ x \geq 0\} \\ Y &=& \{(0,y) &:\ y \geq 0\} \\ X' &=& \{(x,W_2) &:\ x \geq W_1\} \\ Y' &=& \{(W_1,y) &:\ y \geq W_2\} \end{array}$$

Let o be the origin and a the bendpoint of the L-corridor. Let the rectangular moving object of length l and width w enter the corridor at the end whose width is marked W_1 and exit through the end of width W_2 . When the corridor is symmetric, $W_1 = W_2 = W$. If $W_1 \neq W_2$, i.e., for an asymmetric corridor, it is assumed that $W_1 > W_2$. When the moving object degenerates to a rod then $w = 0$. For a given w, let l_{max} be defined as the maximum length which can pass through the corridor. Let edges pq and mn of the moving object move close to walls Y and Y' respectively.

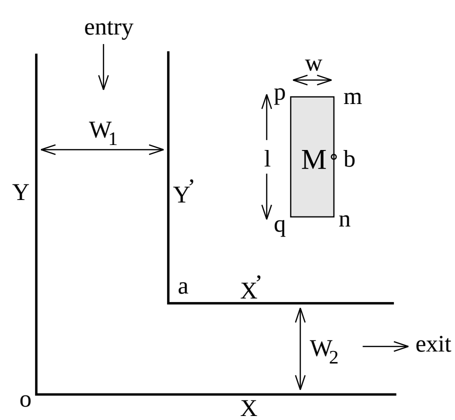


Figure 4: Moving Object in an L-corridor

In general, the length of the moving object which can pass through the corridor can be expressed by a functional relationship given by Eqn (1).

$$l_{max} = f(W_1, W_2, w, r(a,b)) \tag{1}$$

In other words, maximum length is a function of the widths of the entry and exit ends of the corridor, W_1 and W_2, width w of the moving object and a relationship $r(a,b)$ between bendpoint a and an arbitrarily chosen, but fixed, point b on the edge

mn of the moving object. We choose corner m of edge mn on the moving object as the point b. In fact $r(a, b)$ determines the physical motion of the moving object while transiting from the segment YY' to XX'. The transition involving only translation is simple and the corresponding relationship can be defined easily. But the transitions which combine both translation and rotation, and ultimately result in change of orientation of the moving object, are more complicated. Of several ways for such a transition to take place, we present three basic ones in the following subsections. Later we discuss their relative importance and utility.

3.1 CASE 1: Single rotation

In CASE 1, the moving object is translated along wall Y', rotated by 90^o about the pivot point a and then translated along X' to the exit. See Figure (5). Various situations corresponding to corridor widths and dimensions of the moving object are discussed below.

1. If $W_1 = W_2 = W$, and $w = 0$, that is, the moving object is a rod, then the curve traced by the endpoints of the rod is a portion of a circle due to the pivot point being fixed at the corner a of the corridor and the midpoint of the moving object rod. Radius of the circle is equal to $l/2$. See Figure (5.a). The equation of the circle is given by Eqn (2).

$$(x - W)^2 + (y - W)^2 = l^2/4 \tag{2}$$

   The condition for a successful rotation is that no portion of the circle may lie outside the corridor extents. Using simple geometry, it is easy to compute the value of the maximum length of rod, l_{max} which is given by Eqn (3).

$$l_{max} = 2W \tag{3}$$

   If $W_1 \neq W_2$, that is, when the corridor is asymmetric, the end points of the rod trace a quarter each of two circles whose radii are proportional to the entry and exit widths of the corridor. The centres of these two quarter circles are located at the same point a. Thus l_{max} is the sum of two radii of the largest such circles enclosed by the corridor and is given by Eqn (4).

$$l_{max} = W_1 + W_2 \tag{4}$$

   See Figure (5.b) for the illustration of such a motion.

2. If $w > 0$ then the four corners p, q, m and n of the rectangle trace four quarter-circles with the same centre at point a but varying radii. In order to rotate successfully, the quarter circles traced by points p and q must lie within the

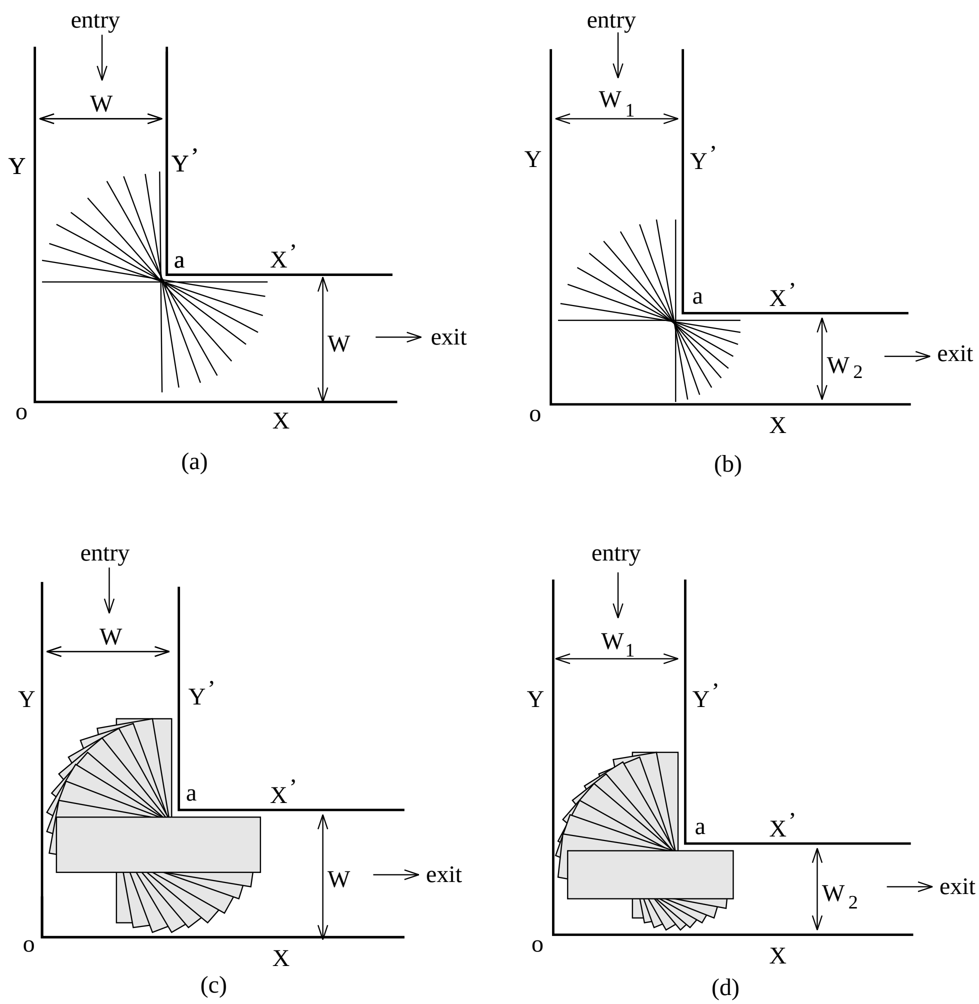


Figure 5: CASE 1: Single Rotation

corridor. Hence l_{max} is less than the quantity given in Eqn (4) as more space is be required to rotate due to the width of the moving object, which will cut down on length (Figures (5.c and 5.d)).

If $W_1 = W_2 = W$ then the radii of circles traced by points p and q are equal to $\sqrt{(W^2 - w^2)}$ and therefore l_{max} corresponding to the largest circles within the corridor extents can be computed as in Eqn (5).

$$l_{max} = 2\sqrt{(W^2 - w^2)} \tag{5}$$

and in the general case when $W_1 \neq W_2$, l_{max} is given as Eqn (6).

$$l_{max} = \sqrt{(W_1^2 - w^2)} + \sqrt{(W_2^2 - w^2)} \tag{6}$$

3.2 CASE 2: Incremental translation and rotation

CASE 2 also deals with rotation of the moving object at a single pivot point but it is combined with translation. The moving object is translated along Y' till one end reaches bend point a of the corridor; then it is rotated by making a as the pivot point and translated simultaneously such that after 90^o rotation, it is aligned along wall X', whence it is translated to the exit point. For simplicity, it is assumed that the amount of translation is linearly proportional to the angle of rotation; however a nonlinear relationship between the translation and rotation may also be used to produce certain desired effects in the motion. For a successful motion through the corridor, the curves traced by the endpoints of the moving object should lie within the corridor extents. See Figure (6) for the motion in CASE 2. The details of computing l_{max} are given below.

1. If $W_1 = W_2 = W$ and $w = 0$, that is, if moving object is a rod, then the curves traced by the locii of endpoints m and n (or p and q, since both coincide) determine the maximum length of rod which can pass through the corridor. To ensure a collision-free rotation, the two curves must lie within the corridor extents. In general, for a rod of length l, the equation of the curve traced by point m is given by Eqns (7) and (8).

$$x = W - (l - \frac{2l\phi}{\pi}) sin\phi \tag{7}$$

$$y = W + (l - \frac{2l\phi}{\pi}) cos\phi \tag{8}$$

   where ϕ is the angle of rotation measured from the wall Y'. During rotation it varies from 0^o to 90^o while moving object is also being translated. The amount of translation is linearly proportional to rotation. The quantity $2l/\pi$ in the equations above is the constant of proportionality. The equation corresponding

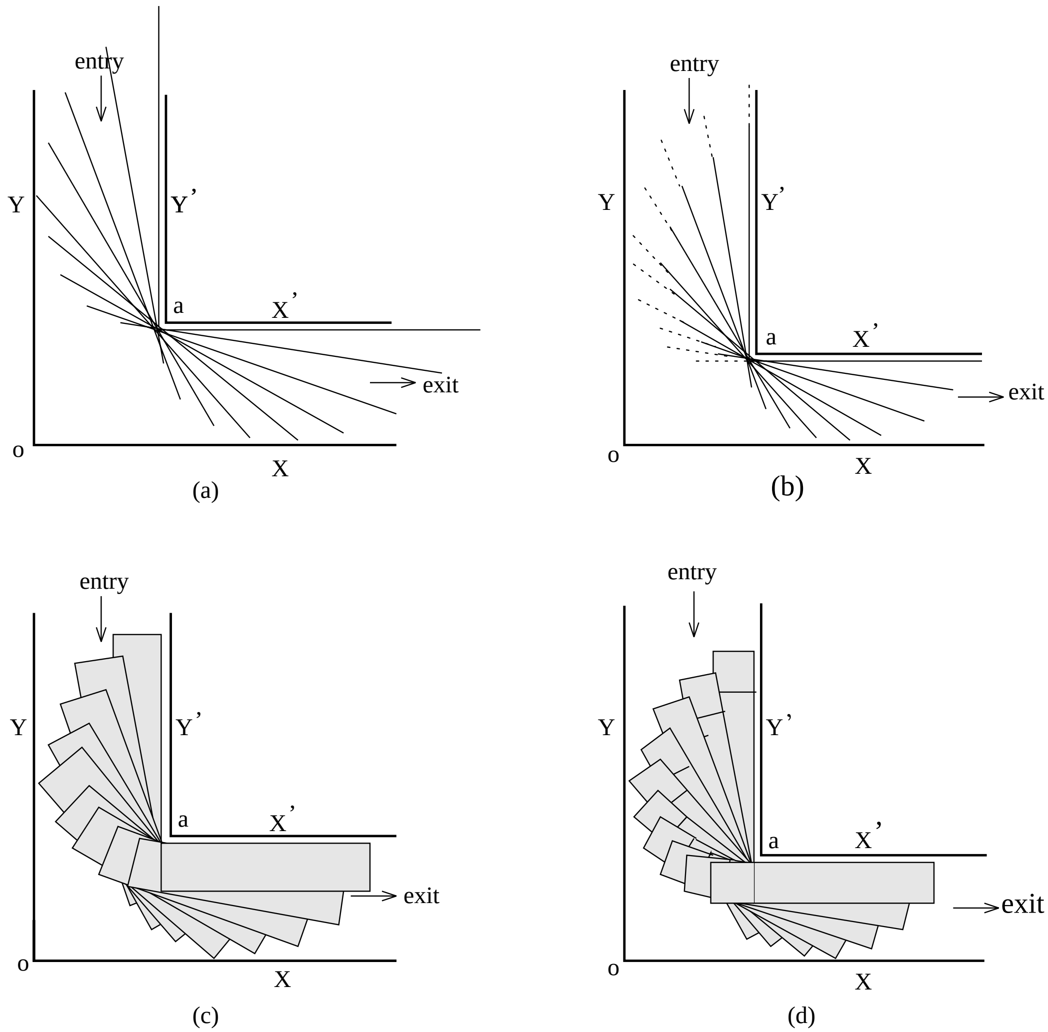


Figure 6: CASE 2: Incremental Translation and Rotation

to endpoint n (or q) is similar since the curve traced is mirror symmetric. See Figure (6.a). Therefore, it is sufficient in this case to deal with the curve traced by point m. The kind of rotation envisaged ensures that the minimum value of x in Eqn (7) is greater than 0. We need to find the angle ϕ at which it happens. This angle can be obtained by equating $\frac{dx}{d\phi} = 0$ in the general case, i.e.

$$\pi cos\phi - 2\phi cos\phi - 2sin\phi = 0 \tag{9}$$

Eqn (9) can be solved numerically for ϕ. For the curve which has the largest X-axis extent confined within the corridor, let this angle be denoted by ϕ_{max}. In other words, ϕ_{max} is the angle at which placement of the rod is critical for a collision-free motion. It is interesting to note that ϕ_{max} is independent of object and corridor dimensions in this case. Now the corridor should be wide enough to accommodate this curve which will ensure the passage of the moving object. Hence the peak distance must fit within the corridor, i.e.,

$$W = (l - \frac{2l}{\pi}\phi_{max})sin\phi_{max} \tag{10}$$

or

$$l_{max} = \frac{W}{sin\phi_{max}(1 - \frac{2}{\pi}\phi_{max})} \tag{11}$$

If $W_1 \neq W_2$, that is, for an asymmetric corridor, a length l corresponding to W_2 (assuming $W_1 > W_2$) and computed by Eqn (11) can easily pass; however an additional length Δl can also pass through due to more space provided by W_1. The locus of the point m is therefore an identically shaped curve but shifted by Δl from the previous curve. Hence the Eqns (7) and (8) must be modified as

$$x = W_1 - (l + \Delta l - \frac{2l\phi}{\pi})sin\phi \tag{12}$$

$$y = W_1 + (l + \Delta l - \frac{2l\phi}{\pi})cos\phi \tag{13}$$

Let ϕ'_{max} be the angle of rotation at which maximum X-axis extent of the curve traced by the endpoint occurs; then in order to restrict the motion within the corridor, Eqn (12) must be maximized with respect to angle of rotation ϕ. As a result, Eqn (14) must be satisfied, i.e.,

$$W_1 = (l - \frac{2l}{\pi\phi'_{max}})sin\phi'_{max} + \Delta l sin\phi'_{max} \tag{14}$$

and therefore, Eqn (14) may be used to calculate $l_{max} = l + \Delta l$. See Figure (6.b) for the illustration. The dotted lines show the extra length Δl which can pass through in an asymmetric case. Note that now ϕ'_{max} is dependent on corridor width and size of the moving object unlike the previous case of symmetric corridor.

2. If $w > 0$ and $W_1 = W_2 = W$, i.e., for a rectangular moving object, the length that can pass through the corridor is less than that in the case of rod. The locus of outer vertices p and q are now important to determine the maximum length of the moving object that can pass through without colliding with the walls of the corridor. For vertex p, for example, the x-coordinate is now a quantity $wcos\phi$ less than the x-coordinate of vertex m and similarly, the y-coordinate is a quantity $wsin\phi$ less than the y-coordinate of vertex m. Using the derivation above, the equation of the locus of the point p can be written as (see Figure (6.c))

$$x = W - wcos\phi - (l - 2l\phi/\pi)sin\phi \tag{15}$$

$$y = W - wsin\phi + (l - 2l\phi/\pi)cos\phi \tag{16}$$

Using Eqns (15) and (16) the l_{max} can be obtained by Eqn (17) as follows.

$$l_{max} = \frac{(W - wcos\phi_{max})}{(1 - 2/\pi\phi_{max})sin\phi_{max}} \tag{17}$$

where ϕ_{max} is the angle of rotation of the moving object at which the peak of the curve given by Eqns (15) and (16) is attained.

Similarly when $W_1 \neq W_2$, a length based on W_2 can pass as per Eqn (17); however an additional length Δl can also pass through due to extra space provided by $max(W_1, W_2)$. This extended l_{max} can be computed as done in the case of rod. See Figure (6.d) for moving a rectangle in an asymmetric corridor.

3.3 CASE 3: Sliding along two walls

In this method of moving through an L-corridor, the moving object is translated and rotated simultaneously by sliding against the two walls of the corridor. When the transition is completed, the moving object is aligned in such a way that it is ready to be translated to the exit point. See Figure (7). Details of computing l_{max} for the case of rod and rectangle in symmetric and asymmetric corridors are discussed below.

In this sliding motion, the moving object starts sliding initially with its two ends touching two walls of the corridor. During sliding, the moving object reaches a critical position when it touches the bendpoint a, as well as the two walls. Therefore three points as shown in Figure (8) constrain the size of the moving object in the corridor. Let ϕ be the angle that the moving object makes with the X wall of the corridor. From simple geometry, length mn can be written as follows in Eqn (18).

$$mn = \frac{mu}{cos\phi} + \frac{vn}{sin\phi} \tag{18}$$

Eqn (18) can be rewritten in terms of l, w, W_1 and W_2 as follows.

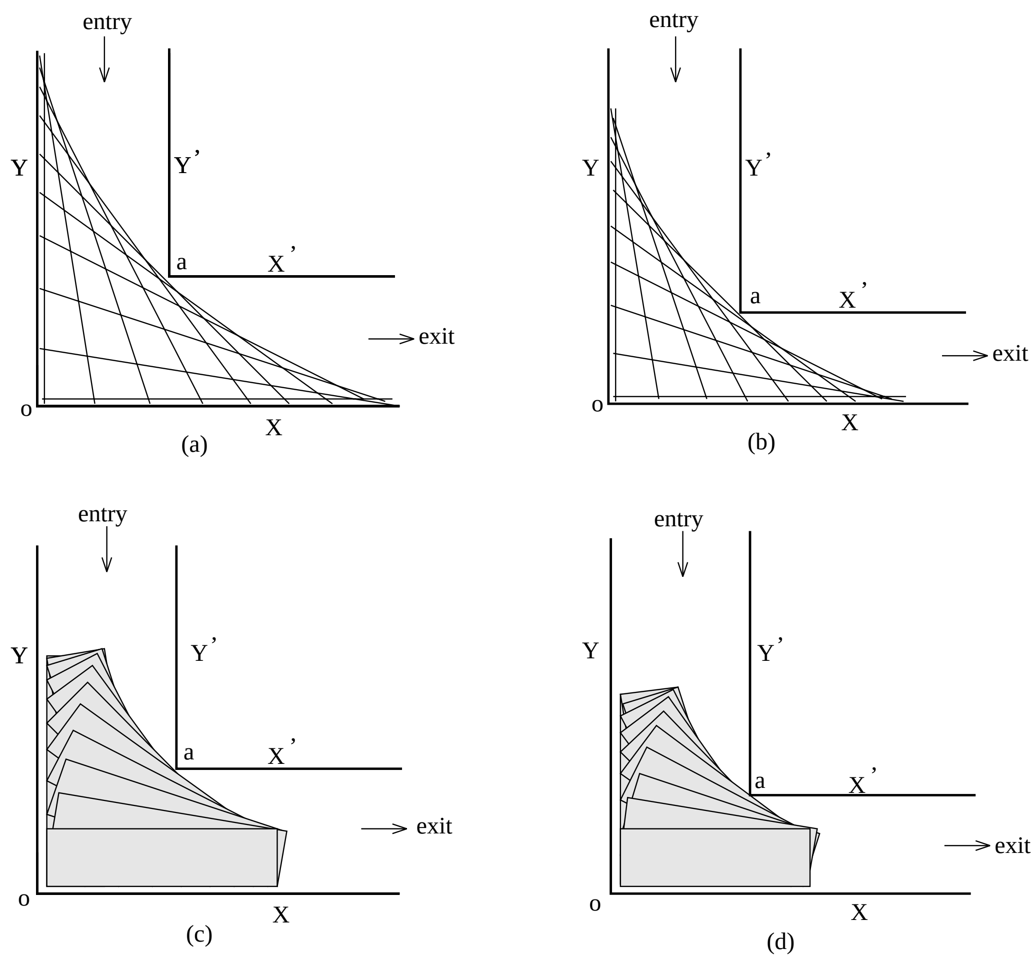


Figure 7: CASE 3: Sliding along Two Walls

$$l = \frac{(W_1 - w sin\phi)}{cos\phi} + \frac{(W_2 - w cos\phi)}{sin\phi} \tag{19}$$

The nature of the curve defined by Eqn (19) is shown in Figure (9) as a set of graphs. The X-axis shows the angle ϕ and Y-axis shows the corresponding value of l. We have chosen arbitrary values of $W_1 = 2, W_2 = 1$ and a range of w values varying from $0.0 \ldots 1.2$. Plot for $w = 1.2$ is incomplete because no moving object whose width is greater than 1.0 can pass through.

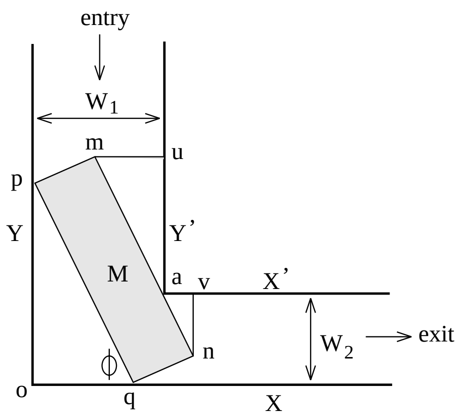


Figure 8: Computing l_{max} for CASE 3

When entering ($\phi = 90^o$) or when exiting the corridor ($\phi = 0^o$), the moving object may be of infinite length, provided the width fits into the corridor. As the moving object slides through the corridor, ϕ decreases from (90^o) to (0^o). Consequently, the permissible length first decreases to a minimum value then it starts to increase. As is obvious from the Figure (9), the minimum l for each w is the l_{max} that will successfully slide through the corridor. To find this value analytically, Eqn (19) must be minimized with respect to ϕ by equating $\frac{dl}{d\phi} = 0$ to obtain l_{max}, i.e.,

$$(W_1 - w sin\phi) sin^3\phi - (W_2 - w cos\phi) cos^3\phi = 0 \tag{20}$$

Figure (10) plots $\frac{dl}{d\phi}$ as a curve against ϕ for a range of W_1/W_2 varying from $1 \ldots 3$ and w values varying from $0.0 \ldots 1.2$. The angle at which the curve crosses the line $\frac{dl}{d\phi} = 0$ (the X-axis of Figure (10)) is the orientation of the moving object at which l_{max} is achieved. As seen from the Figure (10.a), for a symmetric corridor, l_{max} is achieved at the same angle (45^o), irrespective of the corridor-width and the length and the width of the moving object. In the case of an asymmetric corridor, l_{max} depends on corridor dimensions and the width of the moving object (Figure (10.b,c)). The nature of the plot in Figure (10) appears interesting, as the slope $\frac{dl}{d\phi}$ is constant for all widths of the moving object when $\phi = 45^o$. It is evident from Eqn (20) that $\frac{dl}{d\phi}$ depends on corridor widths and not on dimensions of the moving object when $\phi = 45^o$.

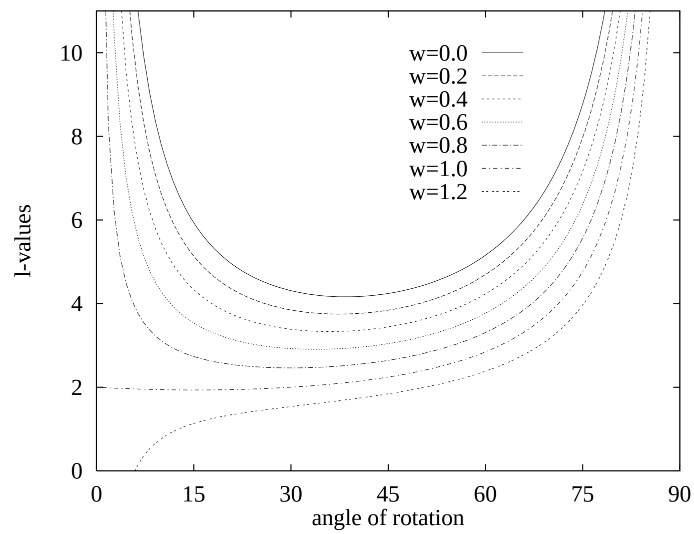


Figure 9: l vs Angle of Rotation for CASE 3 ($W_1 = 2, W_2 = 1$)

Eqns(19) and (20) can be used to obtain l_{max} which can pass through a specific corridor. A closed form solution for l_{max} is, in general, not obtainable; so, a solution must be obtained numerically. However, we will use Eqns(19) and (20) to obtain l_{max} in special cases of this motion.

1. First consider the case of moving a rod in a symmetric corridor, i.e., $W_1 = W_2 = W$ and $w = 0$. Corner points p and q (or m and n) constantly touch walls Y and X respectively. To begin with the moving object is aligned parallel to wall Y and as the rotation progresses the endpoints keep sliding against the respective walls such that in the end the moving object lies parallel to the wall X. See Figure (7.a). To ensure that the entire the moving object remains within the corridor, the curve traced such that the rod is always tangent to the curve is important. The curve traced in this manner is called an *astroid* [LOC71].

   Under these conditions, Eqns(19) and (20) can be rewritten as follows:

$$W sin\phi + W cos\phi - l sin\phi cos\phi = 0 \tag{21}$$

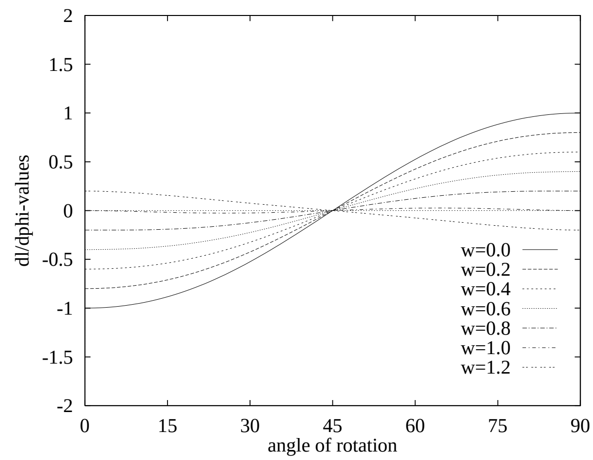
$$W(sin^3\phi - cos^3\phi) = 0 \tag{22}$$

   Substitute the value of ϕ $\quad (\phi = 45^0)$ obtained from Eqn(21) in Eqn(22) to obtain l_{max} as:

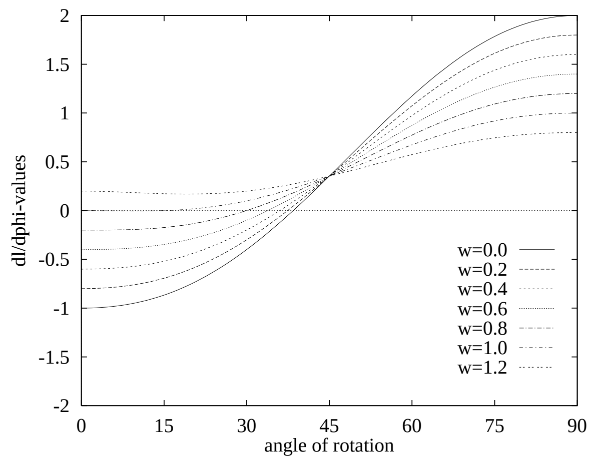
$$l_{max} = 2\sqrt{2}W \tag{23}$$

2. When $W_1 \neq W_2$ and $w = 0$ (Figure (7.b)), the curve passes through coordinates (W_1, W_2). Now the general Eqns (19) and (20) can be modified as follows:

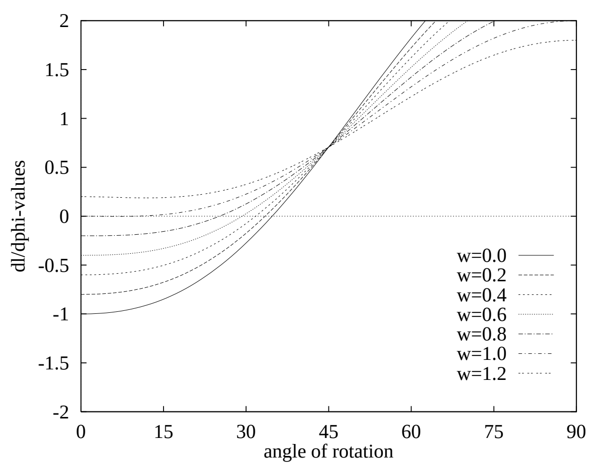
$$W_1 sin\phi + W_2 cos\phi - l sin\phi cos\phi = 0 \tag{24}$$



(a) $W_1/W_2 = 1.0$



(b) $W_1/W_2 = 2.0$



(c) $W_1/W_2 = 3.0$

Figure 10: $\frac{dl}{d\phi}$ vs ϕ (angle of rotation) for CASE 3

$$W_1 sin^3\phi - W_2 cos^3\phi = 0 \tag{25}$$

Eqns(24) and (25) can be solved to obtain l_{max} as:

$$l_{max} = (W_1^{2/3} + W_2^{2/3})^{3/2} \tag{26}$$

3. If $W_1 = W_2 = W$ and $w \neq 0$, that is, for moving a rectangular object in a symmetric corridor (Figure (7.c)), Eqns (19) and (20) are simplified to:

$$W(sin\phi + cos\phi) - w - l sin\phi cos\phi = 0 \tag{27}$$

$$W(sin^3\phi - cos^3\phi) + w(sin^2\phi - cos^2\phi) = 0 \tag{28}$$

Substitute the value of ϕ $(\phi = 45^0)$ obtained from Eqn(28) in Eqn(27) to obtain l_{max} as given by Eqn (29)

$$l_{max} = 2(\sqrt{2}W - w) \tag{29}$$

4 Results

A number of interesting results may be inferred from this study. The first is a characterization of the rotation process that may be derived from the analysis. Consider $r(a,b)$, defined already as the distance between the bend point a and the corner m of the moving object during the rotation. Then $r(a,b)$ may be derived for each case. In CASE 1,

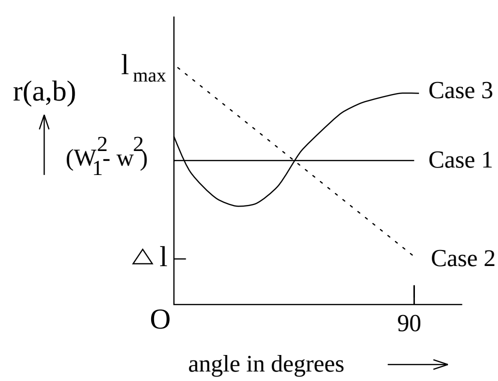


Figure 11: Comparing r(a,b)

$$r(a,b) = \sqrt{(W_1^2 - w^2)} \tag{30}$$

and thus the value of $r(a,b)$ is constant throughout the rotation from 0^o to 90^o. See Figure (11).

In CASE 2, the function $r(a, b)$ is linearly decreasing as shown in Figure (11). At the beginning of rotation when $\phi = 0$, the function value is l_{max} corresponding to the corridor size (W_1, W_2) and width of the moving object w; after the completion of the rotation when $\phi = 90$, it linearly decreases to Δl [SOW91]. This linear decrease rests solely on the assumption that the rate of rotation is linearly proportional to the rate of translation.

In CASE 3, $r(a, b)$ is given by

$$r(a, b) = \sqrt{(wcos\phi + lsin\phi - W_2)^2 + (W_1 - wsin\phi)^2} \tag{31}$$

and is derivable from simple geometry. The nature of the curve is shown in Figure (11).

The second set of observations relates to a comparison of the equations of motion derived for the three cases of motion strategies, in order to gain insights into the motion planning problem. We looked at two of many possible relations, which we think are useful both for motion planning and mobile robot design. The first is the relationship of l_{max} to the corridor widths, assuming that the width of the moving object is fixed. The aim is to answer the question "what is the maximum length object, of fixed width, that can pass through any corridor?", which is analogous to the SOFA problem and its derivatives. The second is the relationship of l_{max} to the width w of the moving object, assuming that the corridors are fixed but arbitrary. The goal is an answer to the question, "given a corridor, what are the possible object dimensions that would permit the robot to pass through the corridor?". Answers to these questions would be of great use both in motion planning and in the design of mobile robots which are to operate in restricted but well-known domains.

Both relationships are highlighted in the graphs of Figures (12 -15). In plotting these graphs, the analytical forms of the motion equations that were derived in the earlier sections were used. Note that CASEs 1 and 3 have convenient analytical forms, while CASE 2 equations are complicated by the fact that they involve the solutions of trigonometric equations, which themselves have no analytical solutions but must be solved numerically. To draw the graphs, we first computed sets of numerical values using the corresponding equations. These values were then plotted as graphs, as displayed in the figures. While plotting l_{max} as a function of corridor widths W_1 and W_2, for convenience we plot l_{max} against W_1, keeping W_2 fixed when appropriate, and varying the ratio W_1/W_2.

Figure (12) contains four graphs which show l_{max} for a rod $w = 0$ as a function of corridor width. Figure (12.a) illustrates motion of the rod in a symmetric L-corridor ($W_1 = W_2 = W$) by plotting l_{max} against corridor width W. It turns out that CASE 1 motion always gives the worst maximum length for any corridor width W. In Figures (12.b,c and d), we consider motion of a rod in an L-shaped corridor, with $W_1 \neq W_2$. We plot l_{max} against W_1, with W_2 fixed at arbitrary values of 30,40 and 50, with varying W_1/W_2. These graphs exhibit similar behaviour. CASE 1 is always the worst; it degrades more and more when the corridor width increases. Of the other cases, CASE 3 is slightly better than CASE 2, though they start nearly equal when the ratio of the corridor widths is close to 1. To summarize, the maximum length of a rod moving through an L-shaped corridor increases monotonically with corridor width; of the three motions, CASE 1 is the worst though the simplest to execute. CASEs 2 and 3 give nearly the same maximum length when corridor widths are nearly equal. As W_1/W_2 increases, CASE 3 overtakes CASE 2.

Figure (13) illustrates motion of a rectangle of fixed width w in an asymmetric corridor, for various fixed but arbitrary values of w. l_{max} is plotted against W_1, for a fixed value of $W_2 = 40$, and varying the ratio W_1/W_2 from 1.0 to 3.0. As obvious in Figure (13.a), CASE 1 seems worst, while CASE 2 is better than CASE 3 as the ratio of the corridor widths increases. In Figures (13.b,c and d), CASE 2 has a *non-linear* curve, and CASE 1 gives good performance for low values of W_1/W_2 in Figure (13.c) and most of the time in Figure (13.d). In summary, we can say that as w increases and tends towards a corridor dimension, CASE 1 gives good performance.

Figure (14) illustrates the motion of a rectangle by comparing l_{max} with w, for varying corridor width W of a symmetric corridor. It is obvious that as width w of the moving object increases, the maximum length decreases. Also, even though CASE 1 starts off worst for small w, it does well as width w increases.

Figure (15) repeats the experiment of Figure (14) for an asymmetric corridor, with similar results.

The analysis of motion in CASE 3 gives some important insights. As already discussed, for the angle value of (45^o), $\frac{dl}{d\phi}$ is independent of moving object width and depends only on corridor widths. Since we have assumed that $W_1 > W_2$, this slope is always positive. Further, the slope is negative at $\phi = 0$, so that the equation $\frac{dl}{d\phi} = 0$ always has a solution lying between (0^o) and (45^o), which is a very useful observation.

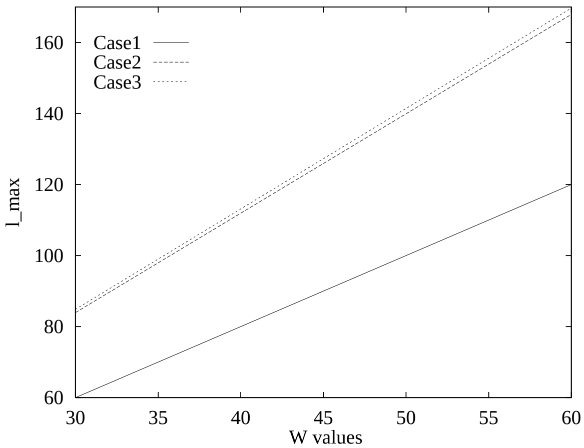
5 Conclusions

This paper deals with motion planning for a rectangular moving object in certain prototypical situations. These prototypical situations arise due to the freespace representation in a 2-D isothethetic workspace. The solution for the L-corridor, optimized with respect to geometric constraints on the moving object and the corridor, has been given. All other cases may be dealt with using this solution; see [AHM90b] [SOW91].

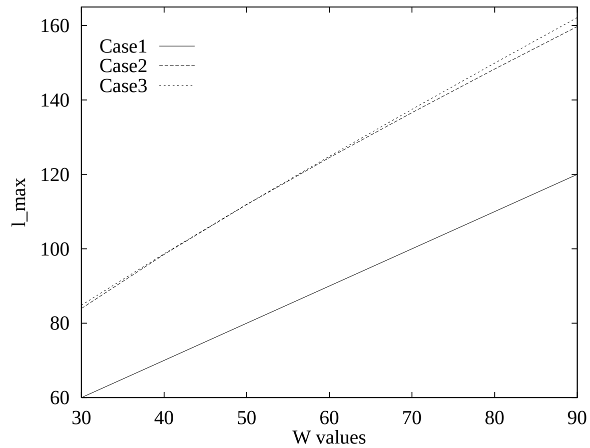
In the motion strategies suggested in this paper, CASE 3 in general is the best for a given width, as far as motion of the largest sized object is concerned. CASE 2 is close to CASE 3 when the width of the moving object is small and deteriorates rapidly for larger widths. CASE 1 is the worst when the width is small but shows improvement over CASE 2 and CASE 3 when moving object width becomes close to the corridor width.

CASE 1 is obviously very simple and requires not-so-hard kinematics to achieve the motion. But the drawback of this method is that comparatively smaller length (when width is small) of the moving object can pass through the corridor. CASE 2 and CASE 3 allow larger lengths to pass through, but the mechanism of motion is complicated and the corresponding kinematics has demanding requirements.

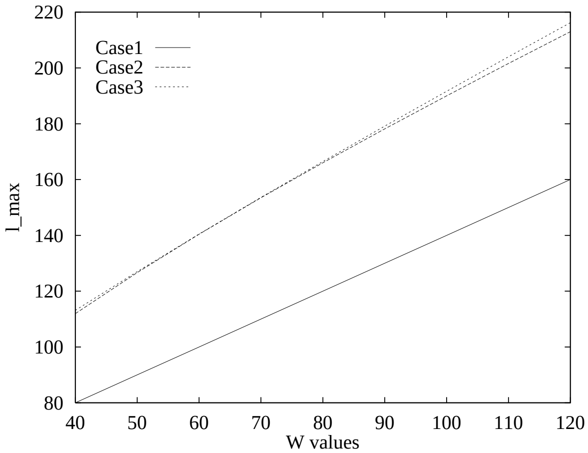
Finally, the significant contribution of the paper is the recognition that the theoretical analysis for computing the largest objects which can pass through certain prototypical situations has to be modulated by motion type as well. Also certain mechanisms of motion may be attractive in terms physical dimensions of the moving object but may not be achievable due to limitations on robot kinematics.



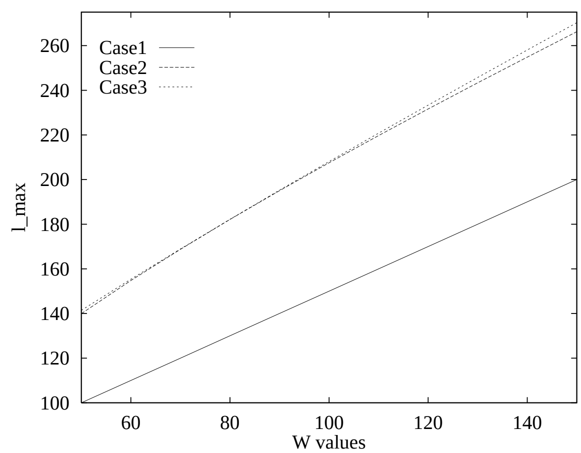
(a) $W_1 = W_2 = W$



(b) $W_2 = 30, W_1/W_2 = 1.0 \ldots 3.0$

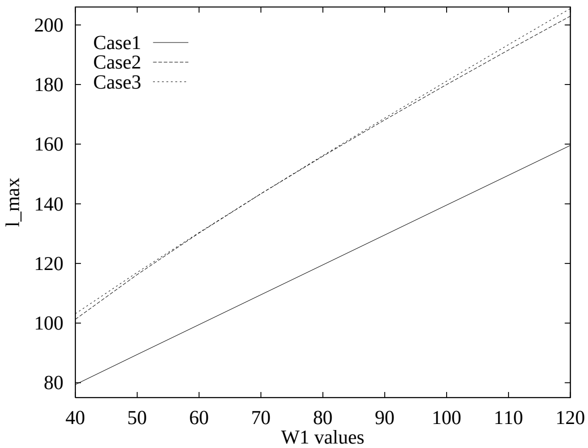


(c) $W_2 = 40, W_1/W_2 = 1.0 \ldots 3.0$

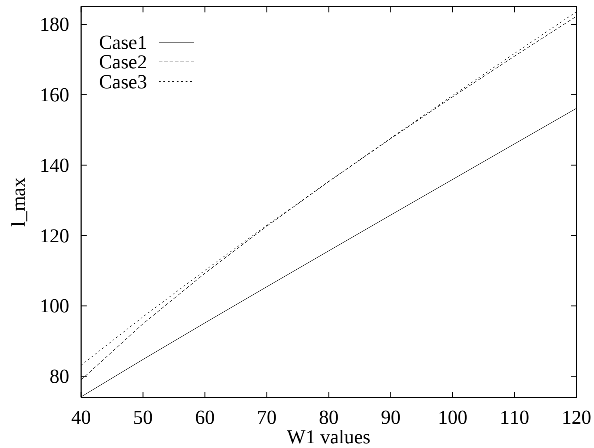


(d) $W_2 = 50, W_1/W_2 = 1.0 \ldots 3.0$

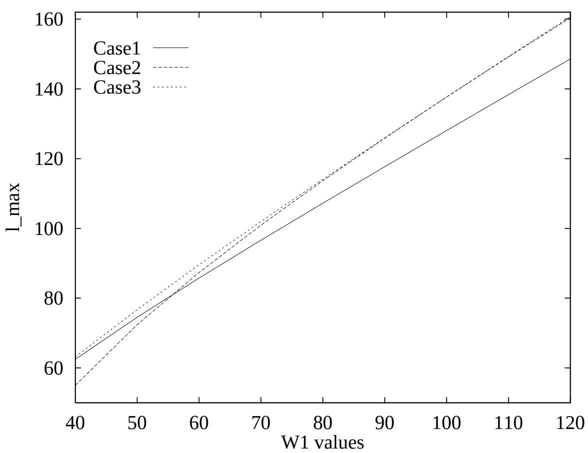
Figure 12: l_{max} for Moving a Rod



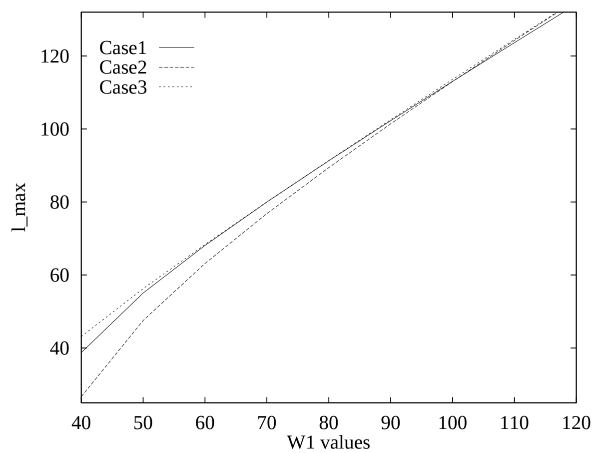
(a) $w = 5, W_2 = 40, W_1/W_2 = 1.0 \ldots 3.0$



(b) $w = 15, W_2 = 40, W_1/W_2 = 1.0 \ldots 3.0$

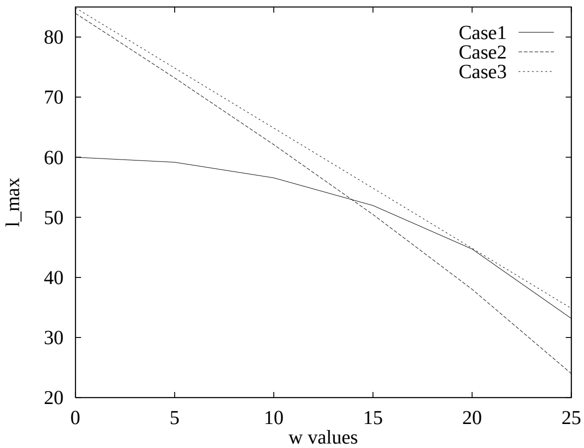


(c) $w = 25, W_2 = 40, W_1/W_2 = 1.0 \ldots 3.0$

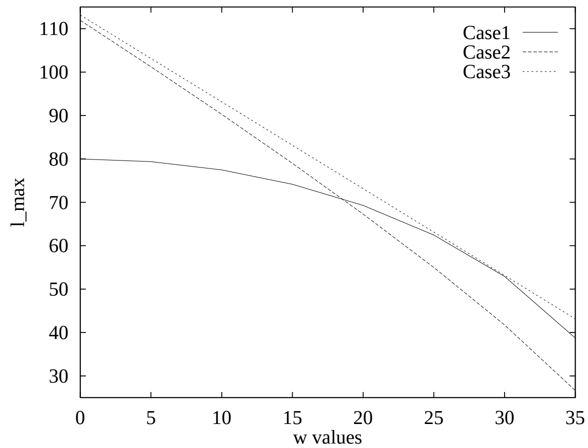


(d) $w = 35, W_2 = 40, W_1/W_2 = 1.0 \ldots 3.0$

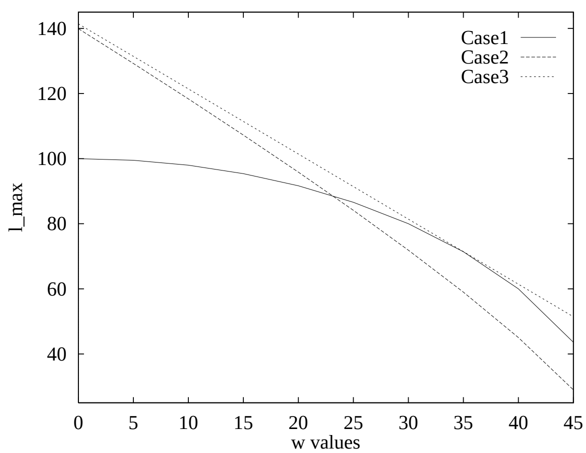
Figure 13: l_{max} for Moving a Rectangular Object



(a) $W_1 = W_2 = 30$

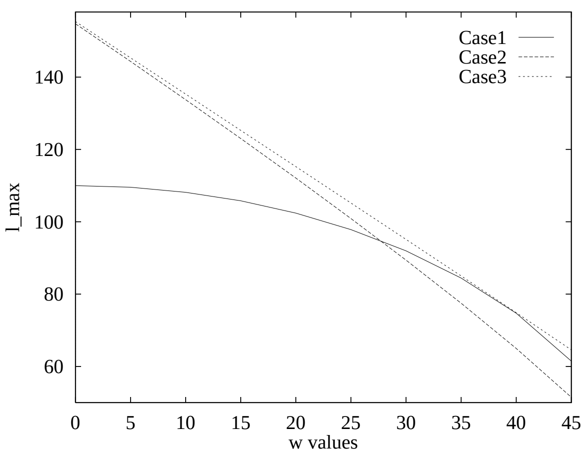


(b) $W_1 = W_2 = 40$

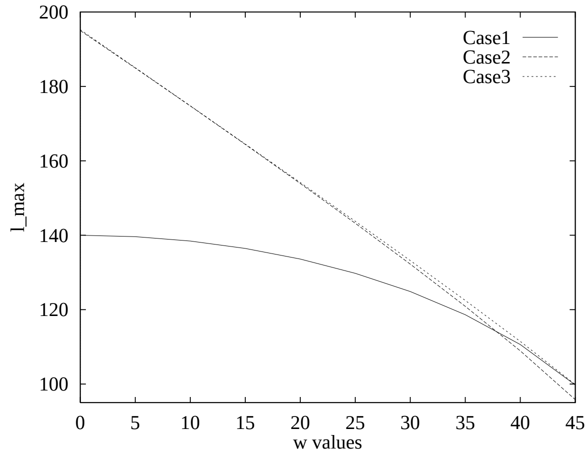


(c) $W_1 = W_2 = 50$

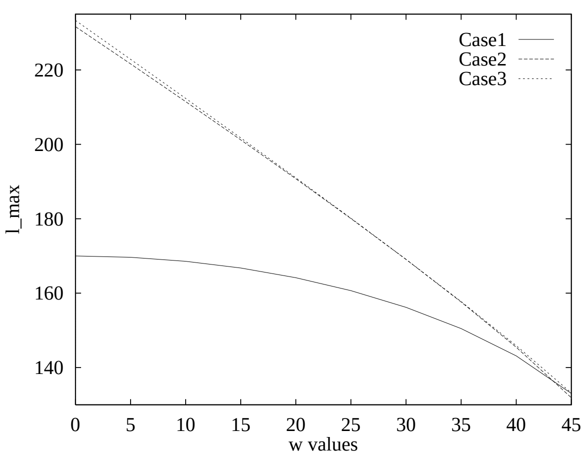
Figure 14: Limiting Motion of a Rectangular Object in a Symmetric Corridor



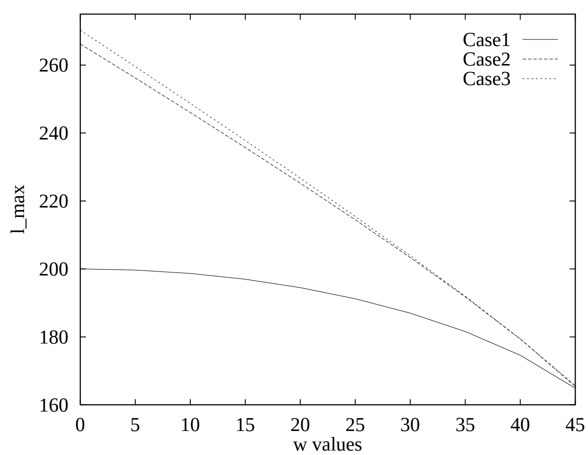
(a) $W_1 = 60, W_2 = 50$



(b) $W_1 = 80, W_2 = 50$



(c) $W_1 = 120, W_2 = 50$



(d) $W_1 = 150, W_2 = 50$

Figure 15: Limiting Motion of a Rectangular Object in a Symmetric Corridor

References

[AHM90a] Ahmed, N. and Biswas, S., 'Path planning: A new look at freespace characterization', Proc. International Conference on Automation, Robotics and Computer Vision, pp. 539-543, McGraw-Hill Book Co. Singapore, 1990.

[AHM90b] Ahmed, N., Sowmya, A., 'How to move in a corridor', Proc. International Conference on Automation, Robotics and Computer Vision, Mc Graw-Hill Book Co. Singapore, pp. 579-583, 1990.

[GOL69] Goldberg, M., 'A solution of problem 66-11: Moving furniture through a hallway', SIAM Review, 11(1), pp.75-78, 1969.

[HOW68] Howden, W.E., 'The Sofa Problem', Computer Journal, 11(3), pp. 299-301, 1968.

[HWA92] Hwang, Y.K., N. Ahuja, 'Gross motion planning- A survey', ACM Computing Reviews, 24(3), pp. 219-291, 1992.

[LAT91] Latombe, J. C., Robot Motion Planning, Kluwer Academic Publishers, Boston, 1991.

[LOC71] Lockwood, E. H., A Book of Curves, Cambridge Univ. Press, London, 1971.

[MAD86] Maddila, S.R., 'Decomposition algorithm for moving a ladder among rectangular obstacles', Proc. IEEE International Conference on Robotics and Automation (San Francisco, Apr 7-10), IEEE New York, pp. 1413-1418, 1986.

[MAR73] Maruyama, K., 'An approximation method for solving the sofa problem', International Journal of Computer and Information Sciences, 2(1), pp. 29-48, 1973.

[MOS66] Moser, L., 'Problem 66-11', SIAM Review, 8(3), pp. 381-381, 1966.

[SEB70] Sebastian, J., 'A solution to problem 66-11: Moving furniture through a hallway', SIAM Review, 12, pp. 582-586, 1970.

[SOW91] Sowmya, A., Ahmed, N., 'How to move in a corridor revisited', SPIE Symposium on Advances in Intelligent Robotics Systems:Mobile Robot V, Boston, Nov., pp. 127-138, 1991.

[STR82] Strang, G., 'The width of a chair', American Math. Monthly, 89, pp. 529-535, 1982.

[YAP87] Yap, C.K., 'How to move a chair through a door', IEEE Journal of Robotics and Automation, 3(3), pp. 172-181, June 1987.