Efficient Resource Allocation for Delay Sensitive Multicast in Future WiMAX Systems

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Abstract

This paper studies efficient resource (radio spectrum and transmission power) allocation in orthogonal frequency-division multiple-access (OFDMA) based multicast wireless system under guaranteed QoS to users. Since most multicast applications are delay sensitive (e.g. Voice over IP, video gaming, online conference etc.), this paper takes minimizing average transmission delay with individual delay requirement as the objective of resource allocation. We first formulate an optimization problem to minimize the system delay with the individual delay bound for each multicast group. Given the NP-hardness of the problem, we design two algorithms to solve the optimization problem effectively. The first one is an efficient resource algorithm by using geometric programming (GP), and the second one is a low complexity heuristic by allocating subcarriers and power separately. Numerical results show that the proposed algorithms are able to allocate subcarriers and power efficiently and effectively, and also achieve the low system delay.

1 Introduction

The next generation broadband wireless networks are expected to enable a whole new range of exciting multimedia services, such as IP-TV, on-line games, audio/video conference. These applications can significantly benefit from multicasting from the networks, which provides an efficient method to transmit the same data to multiple receivers. Especially in the resource-limited wireless scenario, multicasting can benefit from the wireless broadcast advantage (WBA) [14] since every transmission from the base station (BS) can be received by all nodes which lie within its communication range.

Orthogonal frequency division multiplexing (OFDM) is one of the key promising techniques for the next generation wireless broadband networks, such as WiMAX, due to its ability to provide high data rate in multipath fading environments [5]. The multiuser OFDM system is referred as orthogonal frequency division multiple access (OFDMA). In OFDMA systems, the entire available frequency band is subdivided into subcarriers and allocated to different users. How to allocate the available resources by exploiting multiuser diversity gain to improve the system performance has attracted much attention [6, 12, 15, 18]. Generally, the resource allocation problem is cast into two categories: 1) to minimize the total transmission power with the constraints on user's data rate or bit error rate [15], and 2) to maximize system throughput with the constraints on total transmission power or user's achievable data rate [6, 12, 18].

Apart from maximizing system throughput or minimizing power in multicast resource allocation, researchers have started looking at minimizing the delay experienced by the packets. More specifically, this is an important area as many applications that require multicasting tend to be delay sensitive, e.g. video streaming and Voice over IP (VoIP) [1]. Delay in packet arrivals can cause the video player to drop packets and create artifacts during real-time viewing. In addition, delay in VoIP can result in unacceptable user experience. Therefore, our objective is to develop a resource allocation algorithm that is suitable for delay sensitive multicasting in OFDMA-based systems. As far as we know, we are the first to look at addressing the delay constraints of multicasting in an OFDMA-based system.

The major contributions of this paper are:

- A new mathematical formulation is presented for the efficient power and subcarrier allocation problem in OFDMA-based multicast system. The objective is to minimize the average transmission delay, on the premise that it can guarantee the transmission delay bound for each multicast group.
- Given the NP-hardness of the problem, we propose an algorithm to transform the objective function and constraints to the approximated posynomial form, so that it can be solved by geometric programming (GP) [3] efficiently and reliably.
- We also propose a low-complexity heuristic to optimize resource allocation. The subcarriers are allocated based on the equal power assumption. According to the determined subcarrier allocation, the heuristic assigns the power distribution on a water filling manner [2].

The rest of this paper is organized as follows: The related works are discussed in Section 2. Section 3 formulates the resource allocation problem in OFDMA-based multicast system with the delay requirement. Section 4 proposes a method to transform the optimization problem to the form which is solvable by GP. Section 5 presents a low-complexity resource allocation heuristic. The simulation results are discussed in Section 6. Finally, we conclude this paper in Section 7.

2 Related Work

Most resource allocation algorithms for OFDMA systems are designed for unicast. Researchers in [17] propose a maximum sum rate (MSR) algorithm to maximize the sum rate of all users, given a total transmit power constraint. A maximum fairness algorithm is proposed in [11] to allocate the subcarriers and power such that the minimum user's data rate is maximized with the equal rate constraint, so that it is called "maximum fairness". For resource allocation for multicast in OFDMA systems, Liu et al. studied dynamic power and subcarrier allocation by formulating an optimization problem to maximize the system throughput [8]. The study in [10] extended the maximum throughput problem by introducing one more bandwidth constraint. Three low-complexity heuristics based on genetic algorithm (GA) were proposed to improve downlink capacity of the system. In these two works, each of the available subcarrier is assigned to the group with the best channel condition and equal transmission power. The research in [13] considered the resource allocation problem for multicast services over multicarrier systems using the assumption of multiple description coding (MDC). A resource allocation strategy is introduced in [7] to minimize the number of OFDM symbols users can receive, thereby resulting in the saving of power.

Apart from maximizing system throughput or minimizing power in multicast resource allocation, researchers have started looking at minimizing the delay experienced by the packets. In [16], the authors addressed the issue of delays in multicast systems that use network coding, while the capacity of mobile ad hoc networks with delay-constrained multicast requirements was investigated in [19]. However, to the best of our knowledge, there is existing algorithms on resource allocation for multicast in OFDMA-based systems given the delay constraints.

3 System Model

In this paper, we consider multicast in a one-cell WiMAX system that uses OFDMA, similar to the example shown in Figure 3.1. The total system bandwidth can be subdivided into M number of subcarriers, to be allocated to different users. To utilize the wireless broadcast advantage (WBA), such that all users in a particular multicast group receive multicast data in a single transmission, we require that the subcarriers to be allocated to the multicast groups in a way that all members in a group use the same set of subcarriers. Let the total number of users in the system be K and the number of downlink multicast flows be G. This also means that there are G multicast groups in the

system. We assume that each user receives only one flow at any given period, which equivalently means that each user is a member of exactly one multicast group. Let $\mathcal{K}_i, i \in \{1, \ldots, G\}$ denote the user set of group *i* receiving the *i*-th traffic flow, and $|\mathcal{K}_i|$ represents the cardinality of \mathcal{K}_i . In the special case where $|\mathcal{K}_i| = 1$, the *i*-th traffic is unicast. Our model is applicable to both unicast and multicast scenarios. Finally, let the user set containing all *K* users is denoted by $\mathcal{K} = \bigcup_{i=1}^G \mathcal{K}_i$.



Figure 3.1: A multicast OFDMA network consisting of a base station (BS) and user stations where the colours represent the multicast groups.

In our model, the BS allocates the power and subcarrier resource in a centralized manner, based on the perfect Channel State Information (CSI) of all the users in the system. The upper bound for the transmission rate of any user in a particular subcarrier is determined by the channel condition experienced in that subcarrier and the transmit power of the BS. Given a user $k \in \mathcal{K}_i$ on subcarrier m, this can be represented as

$$r_{k,m} = \frac{B_0}{B} \log_2 \left(1 + \frac{|h_{k,m}|^2 P_m}{B_0 N_0}\right), \quad \forall k \in \mathcal{K}_i$$
(3.1)

where B_0 is the bandwidth of one subcarrier, B is the total system bandwidth, P_m denote the transmission power allocated on subcarrier m, N_0 is the onesided power spectral density of white Gaussian noise, and $|h_{k,m}|$ is the channel coefficient of user k on subcarrier m. In this paper, we assume the users are stationary such that the channel conditions do not change for the time duration of interest.

WBA allows all users in a multicast group to receive multicast data from the BS through a single transmission. However, it also means that all users in the multicast group receive data at the same rate. In order for all the users to decode the transmitted data successfully, the BS must transmit data at a rate no more than the maximum transmission rate of the worst-off member of the group. That is, the transmission rate received by all the members in a group is the minimum rate in equation (3.1).

$$r_{\mathcal{K}_i,m} = \min_{k \in \mathcal{K}_i} r_{k,m} .$$
(3.2)

Accordingly, the equivalent channel gain of the i-th multicast group on subcarrier m is denoted by

$$\alpha_{i,m} = \min_{k \in \mathcal{K}_i} |h_{k,m}|^2 . \tag{3.3}$$

Therefore, we have

$$r_{\mathcal{K}_i,m} = \frac{B_0}{B} \log_2\left(1 + \frac{\alpha_{i,m}P_m}{B_0 N_0}\right) \,. \tag{3.4}$$

In this paper, we assume that the same channel gain $\alpha_{i,m}$ is experienced by each multicast group across all subcarriers. This could mean that the entire channel is non-frequency selective. However, this could also represent the case where, even though the channel coefficient $|h_{k,m}|$ for a user k is different on different subcarrier (i.e., frequency selective channel), the channel coefficient of the user with the worst channel condition in a group is approximately the same. Since the transmission rate is the minimum experienced by any member of a group, it is clear that to a given multicast group, the subcarriers are identical with respect to the transmission rate. Therefore, our subcarrier allocation problem can be interpreted as one of allocating the number of subcarriers to each group. We let the number of subcarriers allocated to group *i* is denoted by s_i . The transmission delay of L bits data sent from the BS to group *i* can then be calculated as

$$d_{\mathcal{K}_{i},m} = \frac{M}{s_{i} \cdot \log_{2}\left(1 + \alpha_{i,m}P_{m}/B_{0}N_{0}\right)}$$
(3.5)

The objective of this paper is to minimize the average delay by choosing the optimal power and subcarrier combination, on the premise of the delay guarantee. The optimization problem is formulated as follows

$$\min_{s_i, P_m} \frac{1}{K} \sum_{i=1}^G |\mathcal{K}_i| \cdot \frac{M}{s_i \cdot \log_2 \left(1 + \alpha_{i,m} P_m / B_0 N_0\right)}$$
(3.6)

subject to

$$\sum_{m=1}^{M} P_m \le P_{tot} \tag{3.7}$$

$$P_m \ge 0, \qquad m = 1, \dots, M \tag{3.8}$$

$$\sum_{i=1}^{G} s_i \le M \tag{3.9}$$

$$s_i \ge 0 \tag{3.10}$$

$$\frac{1}{s_i \cdot \log_2 \left(1 + \alpha_{i,m} P_m / B_0 N_0\right)} \le \tau_i, \qquad i = 1, \dots, G$$
(3.11)

Equations (7) and (8) corresponds to the transmission power limitation. Equation (11) represents the transmission rate bound required for each multicast group, where τ_i is the maximum delay required for group *i*. We believe this minimum delay problem is NP-hard. Therefore, exhaustive searching algorithm

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at the BS within a given time can hardly be applied in practice. To solve this problem, a suboptimal algorithm with low complexity and good performance is preferred for practical implementation. In the following two sections, we provide an efficient algorithm by geometric programming and a low-complexity heuristic to allocate the subcarriers and power.

4 Resource Allocation by Geometric Programming

4.1 Overview of geometric programming

Geometric programming is an efficient method to solve the optimization problems designed to determine the minimum value while the objective function and the constraints follow a special form:

minimize
$$f_0(x)$$

subject to $f_i(x) \le 1, \quad i = 1, \cdots, m$
 $h_j(x) = 1, \quad j = 1, \cdots, n$ (4.1)

where f_0 and f_i are posynomials and h_j are monomials. A monomial is defined as a function $h : \mathbf{R}_{++}^n \to \mathbf{R}$:

$$h(\mathbf{x}) = c x_1^{a_1} x_2^{a_2} \cdots x_n^{a_n} , \qquad (4.2)$$

where the multiplicative constant $c \ge 0$ and the exponential constants $a_i \in \mathbf{R}, i = 1, 2, \dots, n$. A posynomial is a sum of monomials:

$$f(\mathbf{x}) = \sum_{k=1}^{K} c_k x_1^{a_1 k} x_2^{a_2 k} \cdots x_n^{a_n k} .$$
(4.3)

Geometric programming in the above form is not a convex optimization problem, because posynomials are not convex functions. However, with a logarithmic change of variables: $y_i = \log x_i$ and $b_k = \log c_k$, the optimization problem can be converted to:

minimize
$$p_0(y) = \log \Sigma_k \exp(a_{0k}^T y + b_{0k})$$

subject to $p_i(y) = \log \Sigma_k \exp(a_{ik}^T y + b_{ik}) \le 0$ (4.4)
 $q_i(y) = a_i^T y + b_i = 0$

This converts the optimization problem to a convex form, which can be solved globally and efficiently through the interior point primal dual method [9].

GP is a nonlinear, nonconvex optimization problem with many useful computational properties. Although GP theory and very efficient GP algorithms are already well-developed [3], researchers interested in using GP still need to model or approximate engineering problems as GP. Therefore, in the next part of the paper, we transform the minimum delay problem which is not composed of posynomials to a geometric program.

4.2 Transformation process

The optimization problem (6) - (11) cannot be solved by GP directly, since both the objective function and constraint (11) are not posynomials. In [3], the authors introduce several extensions (e.g. *fractional powers of posynomials*, *maximum of posynomials* and *generalized posynomials*) to transform a function to a generalized posynomial. An optimization problem composed of generalized posynomials is called generalized geometric programming (GGP), which can be converted to equivalent GP. In this part, we use the techniques to transform the minimum delay problem to a GP. First, we let

$$r_{i,m} = 1 + \gamma_{i,m} P_m$$
 (4.5)

Then the objective function can be represented as

$$\min_{s_i, P_m} \frac{M}{K} \sum_{i=1}^G |\mathcal{K}_i| \cdot {s_i}^{-1} \frac{1}{\log_2 r_{i,m}} , \qquad (4.6)$$

where $|\mathcal{K}_i|$ is constant, s_i and r_m are variables. Here the objective function is still not a posynomial. To convert $\frac{1}{\log_2 r_m}$ to a posynomial form, we use the approximation

$$\log u \approx a(u^{1/a} - 1) , \qquad (4.7)$$

valid for large a. Now we introduce a new variable t_m along with the inequality constraint

$$\frac{1}{\log_2 r_m} \approx \frac{\log 2}{a(r_m^{1/a} - 1)} \le t_m \ . \tag{4.8}$$

Equation (4.8) can be converted to

$$\frac{\log 2}{a} \cdot t_m^{-1} \cdot r_m^{-1/a} + \frac{1}{a} \cdot r_m^{-1/a} \le 1 , \qquad (4.9)$$

which is a valid posynomial inequality. Therefore, the objective function can be converted to

$$\min_{s_i, t_m} \sum_{i=1}^{G} |\mathcal{K}_i| \cdot {s_i}^{-1} \cdot t_m , \qquad (4.10)$$

with one additional constraint (4.9). Also, the equation (11) is converted to

$$\frac{1}{\tau_i} s_i^{-1} \cdot t_m \le 1 . (4.11)$$

According to equation (4.5),

$$P_m = \frac{r_m - 1}{\gamma_{i,m}} . \tag{4.12}$$

Therefore, equation (8) is equal to

$$\sum_{m=1}^{M} \frac{r_m - 1}{\gamma_{i,m}} \le P_{tot} .$$
(4.13)

After transformation, Equation (4.13) is presented as a valid posynomial inequality:

$$\sum_{m=1}^{M} \frac{1}{\gamma_{i,m} \cdot (P_{tot} + \sum_{m=1}^{M} 1/\gamma_{i,m})} \cdot r_m \le 1 .$$
(4.14)

According to the above transformation, we obtain a new optimization problem in which the objective function and constraints are in valid posynomial forms:

$$\min_{s_i, t_m} \frac{M}{K} \sum_{i=1}^G |\mathcal{K}_i| \cdot s_i^{-1} \cdot t_m \tag{4.15}$$

subject to

$$\frac{\log 2}{a} \cdot t_m^{-1} \cdot r_m^{-1/a} + r_m^{-1/a} \le 1$$
(4.16)

$$\sum_{m=1}^{M} \frac{1}{\gamma_{i,m} \cdot (P_{tot} + \sum_{m=1}^{M} 1/\gamma_{i,m})} \cdot r_m \le 1$$
(4.17)

$$\sum_{i=1}^{G} \frac{1}{M} s_i \le 1 \tag{4.18}$$

$$\frac{1}{\tau_i} s_i^{-1} \cdot t_m \le 1 \tag{4.19}$$

The problem can now be solved using GP.

5 Low-complexity Resource Allocation Heuristic

In this section, a low-complexity resource allocation heuristic is proposed. Instead of allocating power and subcarrier jointly, this heuristic applies a two-step optimization for the two sets of variables separately. The first step is to allocate subcarriers given the equal-power assumption. The power distribution over the allocated subcarriers is achieved using the water-filling heuristic in the second step.

5.1 Step 1: Subcarrier allocation with equal-power assumption

Given the assumption that the transmission power is equally distributed to each subcarrier, the data rate for multicast group g on subcarrier m is represented by

$$r_{g,m} = \frac{1}{M} \log_2 \left(1 + \frac{\alpha_{g,m} P_{tot}}{B_0 N_0 M} \right) \,. \tag{5.1}$$

Given the constraint (11), we can calculate the minimum number of subcarriers required for each multicast group

$$s'_g = \left\lceil \frac{1}{\tau_g \cdot \log_2\left(1 + \frac{\alpha_{g,m}P_{tot}}{B_0 N_0 M}\right)} \right\rceil \,. \tag{5.2}$$

Once a subcarrier is allocated to a multicast group, it cannot be used for another group. Then the remaining subcarriers are assigned to the best multicast group based on the proportional rate constraint strategy, which is expressed as

$$\frac{|K_1|r_{1,m}}{s_1''} \approx \frac{|K_2|r_{2,m}}{s_2''} \cdots \frac{|K_G|r_{G,m}}{s_G''} .$$
(5.3)

The benefit of the proportional rate strategy is that it rewards the multicast group with larger group size $|K_g|$ and better channel quality which contributes less delay. As a result, the number of subcarriers allocated to group g is $s_g = s'_g + s''_g$.

5.2 Step 2: Power allocation with water-filling

Once the subcarrier allocation is accomplished, all s_i are known. Then the optimization problem (6)-(12) becomes

$$\min_{P_m} \sum_{i=1}^G |\mathcal{K}_i| \cdot \frac{M}{s_i \cdot \log_2\left(1 + \alpha_{i,m} P_m / B_0 N_0\right)}$$
(5.4)

subject to

$$\sum_{m=1}^{M} P_m \le P_{tot} \tag{5.5}$$

$$P_m \ge 0, \qquad m = 1, \dots, M \tag{5.6}$$

$$\frac{1}{s_i \log_2 \left(1 + \alpha_{i,m} P_m / B_0 N_0\right)} \le \tau_i, \qquad i = 1, \dots, G$$
(5.7)

Here, the equation (37) equals to

$$P_m \ge \frac{(2^{1/s_i\tau_i} - 1)B_0 N_0}{\alpha_{i,m}} .$$
(5.8)

By using Lagrangian relaxation technique, the power allocation problem (5.4) can be express as

$$L = \sum_{i=1}^{G} |\mathcal{K}_{i}| \cdot \frac{M}{s_{i} \cdot \log_{2} (1 + \alpha_{i,m} P_{m} / B_{0} N_{0})} + \lambda (\sum_{m=1}^{M} P_{m} - P_{tot}) , \qquad (5.9)$$

where λ is the Lagrangian multiplier. Therefore, the problem (5.4) can be solved by solving $\partial L/\partial P_m = 0$:

$$\frac{\partial L}{\partial P_m} = \frac{|K_i|M}{S_i} \cdot \frac{-1}{[\log_2 (1 + \alpha_{i,m} P_m / B_0 N_0)]^2} \\
\cdot \frac{\alpha_{i,m} / B_0 N_0}{\log 2(1 + \alpha_{i,m} P_m / B_0 N_0)} + \lambda \\
\approx \frac{|K_i|M}{S_i} \cdot \frac{-1}{(a((1 + \alpha_{i,m} P_m / B_0 N_0)^{1/a} - 1))^2} \\
\cdot \frac{\alpha_{i,m} / B_0 N_0}{\log 2(1 + \alpha_{i,m} P_m / B_0 N_0)} + \lambda \\
= 0.$$
(5.10)

Once we find a P_m^* that satisfies the equation (5.10), the amount of power P_m allocated to subcarrier m can be represented by

$$P_m = \max\{P_m^*, \frac{(2^{1/s_i\tau_i} - 1)B_0 N_0}{\alpha_{i,m}}\}.$$
(5.11)

It is observed that the solution in (5.11) has the form of water filling, where λ can be found from the constraint (35). By combining the two steps, the efficient resource allocation heuristic has been developed.

6 Performance Evaluation

In this section, we provide some numerical simulation examples to evaluate the performance of the proposed algorithms in different configurations. For the simplicity, the average channel gain $E(|h_{k,m}|^2)$, the noise power in each subcarrier N_0 are normalized to 1, the individual subcarrier bandwidth is 1000 Hz and the data size L is set to 1000 bits. Assume the subcarrier channel undergo flat Rayleigh fading, so the independent channel coefficients $\{h_{k,m}\}$ are randomly generated according to Rayleigh distribution. We compare the performance between the optimal solution (OPT), efficient resource allocation by GP (GP) and low-complexity resource allocation heuristic (LC-HEU) in Matlab. Here the geometric programming is solved by the toolbox GGPLAB [4].

We first consider an OFDMA system with M = 8 subcarriers, K = 6 users belonging to G = 3 multicast groups, where $|\mathcal{K}_1| = 3, |\mathcal{K}_2| = 2, |\mathcal{K}_3| = 1$. All three nodes in group \mathcal{K}_3 are closer to the BS, so it results in average 2dB path loss compared with \mathcal{K}_1 and \mathcal{K}_2 . We increase the total transmission power from 1 dB to 30 dB. Figure 5.1(a) shows the comparison of average delay in this scenario. With the increase of the total transmission power, all algorithms achieve lower transmission delay. There is a slight gap between GP and optimal result so GP can achieve a near optimal solution effectively. The LC-HEU consumes more transmission time than the optimal solution and GP. This is because LC-HEU processes subcarrier and power allocation separately rather than jointly. Figure 5.1(b) compares the average number of subcarriers allocated to each multicast group, where both GP and LC-HEU achieve very similar results to the optimal solution.



(b) Average number of subcarriers

Figure 5.1: Performance comparison when K_1 is closer to BS



(b) Average number of subcarriers

Figure 6.1: Performance comparison when K_3 is closer to BS

In the second scenario, we still keep the previous subcarrier and group settings. The difference is that the largest group \mathcal{K}_1 is closer to \mathcal{K}_2 and \mathcal{K}_3 , so that \mathcal{K}_1 obtains better channel condition. Figure 6.1(a) shows the comparison in average delay. As in the last scenario, GP performs very close to the optimal solution, and LC-HEU results in more delay. Figure 6.1(b) shows the subcarriers distribution for the three algorithms. GP still performs close to the optimal result. However, the number of subcarriers allocated to group \mathcal{K}_1 in LC-HEU is obviously higher than GP and the optimal solution. This is because the nodes in \mathcal{K}_1 have the best channel quality, and the group size of \mathcal{K}_1 is largest. The water-filling strategy in LC-HEU causes more resources to be allocated to \mathcal{K}_1 . It shows that the water-filling strategy results in less fairness in resource allocation.

There is a very special scenario in simulation: when the multicast group \mathcal{K}_3 is very sensitive to delay, such that τ_3 is very small. Additionally, \mathcal{K}_1 and \mathcal{K}_2 have very loose delay bounds and \mathcal{K}_3 experiences the worst channel quality. In this case, the equal-power subcarrier allocation in LC-HEU cannot find a feasible solution for subcarrier allocation, resulting in a failed the resource allocation when LC-HEU is used. However, the GP still can find a solution which satisfies all constraints. This observation shows that the joint power and subcarrier allocation achieves higher success rate than when allocation is done separately.

7 Conclusion

This paper studies efficient resource (radio spectrum and transmission power) allocation in OFDMA-based multicast wireless system under guaranteed QoS to users. Since most multicast applications are delay sensitive (e.g. Voice over IP, video gaming, online conference etc.), this paper takes minimizing average transmission delay with individual delay requirement as the objective of resource allocation. We first formulate an optimization problem to minimize the system delay with the individual delay bound for each multicast group. Given the NPhardness of the problem, we design two algorithms to solve the optimization problem effectively. The first one is an efficient resource allocation algorithm by using GP, and the second one is a low complexity heuristic by allocating subcarriers and power separately. Numerical results show that the resource allocation algorithm by GP performs very close to the optimal results, and the low-complexity heuristic also achieves optimization objective effectively. As future work, we plan to implement the algorithms in larger scale network as well as to analyse the complexity of the proposed algorithms. Also, we intend to implement the efficient resource allocation algorithm by GP in a real testbed for realistic video streaming.

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