Interference Analysis and Channel Assignment in Multi-Radio Multi-Channel Wireless Mesh Networks

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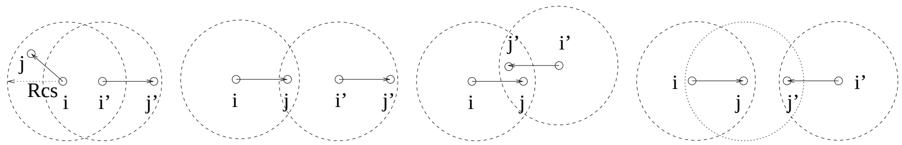
Abstract

In a typical Wireless Mesh Network (WMN), the links that interfere with a particular link can be broadly classified into two categories depending on their geometric relationships: coordinated and non-coordinated links. In this paper, we analytically quantify the impact of both kind of interfering links on transmission losses. Our analysis shows that compared to coordinated links, the non-coordinated links result in significantly lower throughput and an unfair distribution of channel capacity amongst interfering links. We hypothesize that channel assignment in multi-radio multi-channel WMNs can be effective in significantly reducing the interference caused by the non-coordinated links. We prove that the channel assignment problem based on this hypothesis is NP-Hard. We propose a novel two-phase heuristic channel assignment protocol referred as Cluster-Based Channel Assignment Protocol (CCAP). The protocol logically partitions the network into non-overlapping clusters. In the first phase, nodes within a cluster are assigned to a common channel with orthogonal channels being used in adjacent clusters. The inter-cluster links are assigned channels with the aim of minimizing non-coordinated interference. The second phase of CCAP exploits channel diversity to sub-divide each cluster into multiple interference domains, thereby increasing the capacity of individual links. Simulation-based evaluations demonstrate that CCAP can achieve twice the aggregate network throughput as compared to existing channel assignment protocols, while ensuring a fair distribution of capacity amongst the links.



(a) CO (b) IA (c) NH (d) FH

Figure 1.1: Coordinated and non-coordinated interference

1 Introduction

In a typical Wireless Mesh Network (WMN), co-located links frequently interfere with each other, thus reducing the channel utilization. Interfering links can be classified as coordinated or non-coordinated links depending on their geometric relationships [1, 2]. For a given directional link $l(i, j)$ – where i is the transmitter and j is the receiver – the directional link $l'(i', j')$ is a coordinated interfering link (referred to as CO) if the euclidean distance $d(i, i')$ is less than the carrier sensing range (R_{CS}) (see Figure 1.1(a)). On the other hand, the directional link $l'(i', j')$ is a non-coordinated interfering link (Figure 1.1(b)-1.1(d)) if $d(i, i') > R_{CS}$ and $\{d(i, j')$ and/or $d(i', j)$ and/or $d(j, j')\} \leq R_{CS}$. Non-coordinated links can be further classified into three categories depending on their geometric relationships: (i) Information Asymmetric (IA), (ii) Near Hidden (NH) and (iii) Far Hidden (FH). The differences between them are explained later in the paper. The existence of non-coordinated links has three potential drawbacks:

(i) Sub-optimal channel utilization: Coordinated interfering links (such as l and l' in Figure 1.1(a)) tend to synchronise their transmissions when random-access MAC protocols such as CSMA are employed. As a result, packet collisions are reduced, resulting in near optimal channel utilization. On the contrary, non-coordinated links (e.g. l and l' in Figure 1.1(b)-1.1(d)) fail to coordinate their transmissions [1,2], resulting in significantly higher transmission losses and sub-optimal channel utilization. We illustrate the impact of non-coordinated links on the channel utilization using an example. Figure 1.3(a) shows a simple WMN consisting of 7 nodes. The distance between adjacent nodes is equal to the transmission range (R_T). Assume that $R_{CS} = 2 * R_T$. The links are single-hop and directional with traffic flowing in the direction of the marked arrows. Each node uses a separate interface to communicate with each neighbour (e.g., node C has 3 interfaces). All links operate on the same channel. This network contains 5 non-coordinated interfering link pairs: $\{AB, ED\}$, $\{AB, DC\}$, $\{BC, ED\}$, $\{AB, FG\}$ and $\{ED, FG\}$. We simulated this scenario in Qualnet assuming that each transmitter always has a packet to send (i.e. saturation). The value depicted against each link is the achievable goodput (the number in paranthesis is the channel used by the link). The maximum link capacity is set to $1Mbps$. However, the aggregate achievable goodput is just $0.37Mbps$. Observe that link CF, which does not have any non-coordinated relationships, has significantly higher goodput as compared to the rest of the links.

(ii) Unfair capacity distribution: It is well known that certain geometric relations of interfering links result in short term and long term fairness [2–4]. Consider the same example as before (Figure 1.3(a)). In this network, link FG has an information asymmetric relationship with link AB (and link ED). AB experiences significant packet loss while FG suffers no loss. This

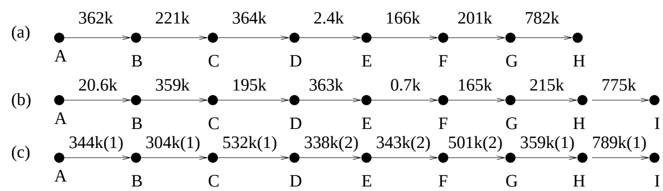


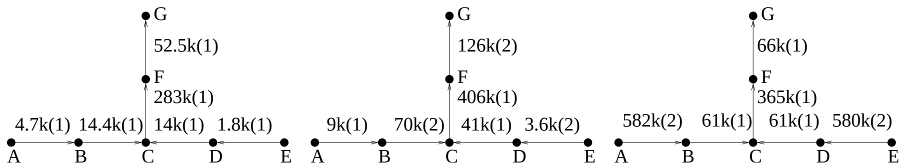
Figure 1.2: Example illustrating the impact of interference dependencies

is because of the asymmetric view of the channel state as perceived by the transmitters (A and F). Consequently, the goodput achieved by AB is negligible in comparison to FG. Near hidden and far hidden non-coordinated links also exhibit similar unfair properties (demonstrated in simulations later). The links which experience significantly lower goodput are very likely to form bottlenecks in the network and thus severely impact the end-to-end throughput of multi-hop flows.

(iii) Dependencies beyond the local interference domain: We have observed that two links in a WMN which are several hops away can indirectly interfere with each other, thereby creating a chain of dependencies. Consider the chain topology shown in Figure 1.2a, which was simulated in Qualnet. The node placements are such that link pairs $\{GH, DE\}$ and $\{DE, AB\}$ have an information asymmetric relationship. Consequently, the goodput of DE is severely impacted by GH. Since DE has a very low goodput, it does not adversely affect AB. Now assume that a new link HI is added to this chain as shown in Figure 1.2b. As a result the goodput of GH drops, causing an increases in the goodput of DE, which in turn throttles AB. This simple example illustrates that the non-coordinated links play a significant role in creating dependencies that span beyond a local interference domain.

In this paper, we argue that *channel assignment* in multi-radio multi-channel WMN can be effective in addressing the aforementioned issues. Typical mesh routers are equipped with multiple interfaces (radios). The channel assignment protocol is responsible for assigning the limited number of orthogonal channels amongst these interfaces. We hypothesize that *the channel assignment protocol can significantly improve the channel utilization by minimizing the number of non-coordinated interfering links operating on a common channel*. Our work is significantly different from the existing body of work. The existing channel assignment protocols use the following metrics to measure the impact of interference: the number of interfering links [5–8], the traffic load [8–13] or the channel utilization [12–14]. However, these metrics do not necessarily capture the impact of non-coordinated links.

The following empirical examples demonstrate the effectiveness of the proposed hypothesis. First consider the simple network in Figure 1.3(a). As before, results presented are from a Qualnet simulation. In Figure 1.3(a), all links are operating on a single channel and the resulting aggregate goodput is $0.37 Mbps$. Now suppose two channels are available. We first evaluate the impact of existing channel assignment protocols. Since the topology is small, metrics such as traffic load and number of interfering links lead to the same assignment. Figure 1.3(b) illustrates the channel assignment which leads to the maximum aggregate goodput of $0.65 Mbps$ (80% improvement over single channel case). However, links ED and AB still experience negligible goodput. This is because



(a) Single channel (b) Existing metrics (2 channels) (c) Proposed hypothesis (2 channels)

Figure 1.3: An empirical example comparing channel assignment protocols.

the non-coordinated link pairs $\{FG, ED\}$ and $\{BC, ED\}$ (ED is the affected link) operate on channel 2, while the non-coordinated links DC and AB operate on channel 1. Now consider Figure 1.3(c) where the two channels are assigned based on the proposed hypothesis, which aims to reduce the non-coordiniated interference. The aggregate goodput significantly improves to $1.7Mbps$ (more than 2.5 times of the existing schemes). Further, the capacities of links ED and AB have also improved and the capacity distribution amongst links is relatively fair. In addition, there is only one non-coordinated interfering link pair $\{AB, ED\}$ operating on a common channel (compared to 3 in Figure 1.3(b)).

Now consider the chain topology in Figure 1.2. This example has five non-coordinated link pairs: $\{AB, DE\}$, $\{BC, EF\}$, $\{CD, FG\}$, $\{DE, GH\}$, and $\{EF, HI\}$. In Figure 1.2b, all links are operating on a common channel. Now assume that two channels are available. Figure 1.2c illustrates the goodput achieved when the channels are assigned in such a way that the non-coordinated links operate on separate channels. It is evident that no link is significantly disadvantaged. The above examples, though simplistic, demonstrate that the notion of assigning channels with the aim of reducing non-coordinated interference has merit.

The paper is divided into two parts. In the first part (Section 3), we analytically prove that non-coordinated interfering links induce higher transmission losses as compared to coordinated links. We first prove the result for two interfering links (Section 3.1) and subsequently extend the analysis to a single interference domain (Section 3.2). We prove that within an interference domain consisting of n links, the transmission losses are minimum (and consequently channel utilization is maximum) when all n links have a coordinated relationship with each other (compared to any combination of coordinated and non-coordinated links). Subsequently, we find the minimum number of coordinated interfering links that induce same level of transmission losses as a single non-coordinated link (Section 3.3). This allows us to quantitatively compare the impact of the two. These proofs cannot however be generalized, i.e., extended beyond a single interference domain. This is due to the complex geometric relationships that exist between the links in a typical WMN. However, the proofs strongly support the hypothesis proposed above.

In the second part of the paper, we formulate the channel assignment problem as a two phase minimization problem. The objective of the first phase is to minimize the number of non-coordinated interfering links that operate on the same channel. The second phase achieves the same for coordinated links without introducing additional non-coordinated links. We prove that this problem is NP-hard (Section 4). We propose a static distributed heuristic referred as *Cluster-based Channel Assignment Protocol (CCAP)* (Section 5), informed by the results of our interference analysis. The first phase of our protocol uses the clique of coordinated links as a unit for channel assignment and logically partitions the network into non-overlapping clusters – each consisting of a clique. The nodes within a cluster operate on common channel while orthogonal channels are used for neighbouring clusters. Inter-cluster links are assigned channels while ensuring that minimum number of non-coordinated interfering links operating on the same channel are introduced. The second phase of CCAP exploits channel diversity to sub-divide each cluster into multiple interference domains, thereby increasing the capacity of individual links.

We evaluate CCAP by comparing it with existing channel assignment protocols through simulations (Section 6). The results demonstrate that CCAP can achieve twice the aggregate goodput as compared to existing protocols and ensures a fair distribution of capacity amongst the links.

2 Background

In this section, we revisit the relevant portions of the analytical models proposed in [15] and [2], which form the foundation of our analysis in Section 3. Bianchi [15] modeled the IEEE 802.11 DCF mode (which is based on CSMA/CA) for a single carrier sensing domain as a Markov model. Note that, all the links in a single carrier sensing domain are coordinated. Bianchi developed the expressions for the conditional packet loss probability p and the transmission probability τ of a node in an n node network as follows,

$$p = 1 - (1-\tau)^{n-1} \tag{2.1}$$

$$\tau = \frac{2(1-2p)}{(1-2p)(W+1) + pW(1-(2p)^m)} \tag{2.2}$$

where W is the size of minimum backoff window and m is the maximum number of backoff stages (for IEEE 802.11b, $W = 32$ and $m = 6$). Note that, the maximum value of τ is achieved when $p = 0$ and is computed as follows,

$$\tau(max) = \frac{2}{1+W} = 0.060606 \tag{2.3}$$

Garetto et al. [2] extended Bianchi's model to a single radio single channel multi-hop wireless network. The authors have derived the expression for the conditional packet loss probability p of the nodes, accounting for both coordinated and non-coordinated links. We now present some of the important results from their model, which are used in our analysis. For detailed derivations of the expressions, the readers are directed to the relevant publication [2].

Garetto et al. have categorized the interfering links into four categories based on their geometric relationship. For a link $l(i,j)$, the interfering link $l'(i',j')$ can be a coordinated (CO), Information Asymmetric (IA), Near Hidden (NH) or Far Hidden (FH) interfering link (these categories are explained in detail in Section 3.1). The impact on the conditional packet loss probability p of link l varies for each of these categories. The authors have derived the following expressions where $p_{(i,i')}$ is the conditional packet loss probability of transmitter

i of link l due to the interference caused by the transmitter i' of link l'.

$$\begin{aligned} p_{CO(i,i')} &= \tau_{i'}\left(\frac{Q_\phi}{\Sigma_{D\in\gamma(i)}Q_D}\right) \\ p_{IA(i,i')} &= 1-\frac{T_{OFF(i')}}{T_{ON(i')}+T_{OFF(i')}}e^{-\frac{d}{T_{OFF(i')}}} \qquad (2.4) \\ p_{NH(i,i')} &= [1-(1-\tau_{i'})^m]\frac{Q_\phi}{\Sigma_{D\in\gamma(i)}Q_D} \\ p_{FH(i,i')} &= \frac{T_{ON(i')}}{T_{ON(i')}+T_{OFF(i')}} \qquad (2.5) \end{aligned}$$

where d is the duration of transmission of the first packet (RTS or DATA), $m = \lfloor \frac{d}{\sigma} \rfloor$ while σ, $T_{OFF(i')}$ and $T_{ON(i')}$ are duration of idle slot, average idle duration and average active duration of the node i' respectively. We do not elaborate on the quantities $\gamma(i)$, Q_D and Q_ϕ because these quantities are all equal to unity (i.e. 1) for a single interference domain. The analysis presented in this paper is limited to a single interference domain, thus reducing the expressions for $p_{CO(i,i')}$ and $p_{NH(i,i')}$ to,

$$p_{CO(i,i')} = \tau_{i'} \qquad (2.6)$$

$$p_{NH(i,i')} = 1-(1-\tau_{i'})^m \qquad (2.7)$$

The conditional packet loss probability $p_{<type>(i,i')}$, where $< type >= \{CO, IA, NH, FH\}$, of all interfering links of a particular category can be combined to obtain the collective conditional packet loss probability $p_{<type>i}$ experienced by node i. For example, $p_{CO(i)}$ denotes the collective packet loss probability of node i due to all coordinated links. The combined conditional packet loss probability experienced by node i due to all types of interfering nodes is computed as,

$$p_i = 1-[1-p_{CO(i)}][1-p_{IA(i)}][1-p_{NH(i)}][1-p_{FH(i)}] \qquad (2.8)$$

3 Interference Analysis

In this section, we analytically compare the impact of coordinated and non-coordinated links in terms of the induced transmission losses. We prove the hypothesis that *within an interference domain, the non-coordinated links induce higher transmission losses as compared to the coordinated links*. The section is divided into three parts. In the first part, we prove this hypothesis for the simple case of two interfering links. In the second part, we extend the analysis to n links within an interference domain. In the third part, we find the minimum number of coordinated interfering links that induce the same level of transmission losses as a single non-coordinated link. Although the proofs are limited to a single interference domain, they offer strong motivation to the credence that reducing the non-coordinated interference in a mult-hop WMN significantly increases the network capacity.

The following terms are repeatedly used throughout the rest of the paper. The *clique of interfering links* is the set of links where every link interferes with every other link in the set. The links with transmitters or receivers located within a carrier sensing domain form such a clique. The *clique of coordinated interfering links* is the set of links where every link has coordinated interference relation with every other link in the set. The links with transmitters located

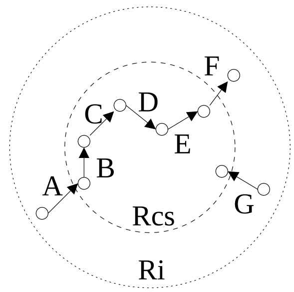


Figure 3.1: Example depicting the difference between the clique definitions

within a single carrier sensing domain form such a clique. Figure 3.1 illustrates an example to clarify the difference between the two. In this example, the set of links $\{B, C, D, E, F\}$ form a clique of coordinated links, since their transmitters are within a single carrier sensing range, R_{CS}. However, all links $A - G$ form a clique of interfering links since either their transmitters or receivers (or both) are within the carrier sensing range.

We make the following assumptions for mathematical tractability:

1. The transmission range and carrier sensing range of the nodes is fixed and all nodes transmit with same transmission power. The ranges form perfect disks.

2. Each transmitter operates in the saturated state, i.e., the transmitter always has a packet to transmit.

3. All nodes transmit fixed size packets.

4. All links are directional with data packets being sent by the transmitter only. The receiver only transmits CTS and acknowledgement packets. A bi-directional link is realized by two directional links.

Note that, we relax most of these assumptions while formulating the channel assignment problem in Section 4 and in all subsequent sections.

3.1 Two Link Case

In this section, we analytically compare the transmission losses of two coordinated interfering links with that of two non-coordinated interfering links. Garetto et al. [1] had presented an empirical comparison of this scenario. However, the results were not proved analytically.

Lemma 1. *The combined conditional packet loss probability of two non-coordinated interfering links is higher than that of two coordinated interfering links.*

Proof. We first compute the combined conditional packet loss probability of two interfering links under all possible topologies and then compare the resulting values. The possible topologies listed below are exhaustive.

Coordinated Links: Two interfering links $l(i, j)$ and $l'(i', j')$ are coordinated interfering links if $d(i, i') < R_{CS}$ (Figure 1.1(a)). Since coordinated interference is symmetric, the conditional packet loss probability of both transmitters is equal and is given by Equation 2.6 (i.e., $p_i = p_{i'} = \tau$). Substituting $p = \tau$

in Equation 2.2, and solving for τ results in $\tau = 0.057044$. The combined conditional packet loss probability P of the two links is computed as $P = 1-(1-\tau)^2$. Substituting the value of τ gives $P = 0.110833$.

Information Asymmetric Non-Coordinated Links: Two interfering links $l(i,j)$ and $l'(i',j')$ are Information Asymmetric (IA) non-coordinated interfering links (Figure 1.1(b)) if the following conditions hold:

- $d(i,i') > R_{CS}$, transmitters are outside CS range
- $d(j,i') < R_{CS}$, receiver of l is within CS range of transmitter of l'
- $d(j',i) > R_{CS}$, receiver of l' is outside CS range of transmitter of l

In this case, the conditional packet loss probability of the transmitter of link l' (i.e., i') is 0. This is because a collision can only occur when j receives a complete RTS/DATA packet during the backoff phase of i' and transmits CTS/ACK exactly when i' starts its next packet transmission. The conditional packet loss probability of the transmitter of link l (i.e., i) is given by Equation 2.4. The values of the variables d, T_{ON} and T_{OFF} are given in Table 3.1. Substituting the values in Equation 2.4, we can compute the value of $p_{IA(i,i')}$ to be 0.878581. Note that, this is the minimum possible value of IA induced conditional packet loss probability for IEEE 802.11b radios. The combined conditional packet loss probability of the two links, $P_{min} = 1-(1-0)(1-0.878581) = 0.878581$.

Table 3.1: Parameter values assuming a minimum 802.11B MAC payload

SIFS, DIFS, σ, PLCP	$10\mu s$, $50\mu s$, $20\mu s$, $192\mu s$
Basic Rate & Data Rate	$2Mbps$, $11Mbps$
MAC Header	$28Bytes$@data rate
RTS,CTS,ACK	$(20,14,14)Bytes$@basic rate
Min. Packet Payload	$28Bytes$ (8 Bytes UDP header + 20 Bytes IP header)@data rate
T_{ON}	(RTS + CTS + MAC Header + Payload + ACK + 4*PLCP + 3*SIFS + 3*σ)$\approx 1091\mu s$
T_{OFF}	DIFS + $16 * \sigma = 370\mu s$
d	RTS + PLCP = $272\mu s$

Near Hidden Non-Coordinated Links: Two interfering links $l(i,j)$ and $l'(i',j')$ are Near Hidden (NH) non-coordinated interfering links (Figure 1.1(c)) if the following conditions hold:

- $d(i,i') > R_{CS}$, transmitters are outside CS range.
- $\{d(j,i'), d(j',i)\} < R_{CS}$, receivers of l and l' are within CS range of transmitters of l' and l respectively.

The NH relationship is symmetric and the conditional packet loss probability of the transmitter of each link is given by Equation 2.7. Simultaneously solving Equation 2.7 and Equation 2.2 gives $\tau = 0.031442$. Substituting the value of τ in Equation 2.7 gives the conditional packet loss probability of each link as,

$$p_i = p_{i'} = 1-(1-\tau)^m = 0.339864 \tag{3.1}$$

The combined conditional packet loss probability of the two links is computed as $P = 1 - (1 - p_i)(1 - p_{i'}) = 0.564220$.

Far Hidden Non-Coordinated links: Two interfering links $l(i, j)$ and $l'(i', j')$ are Far Hidden (FH) non-coordinated interfering links (Figure 1.1(d)) if the following conditions hold:

- $\{d(i, i'), d(j, i'), d(j', i)\} > R_{CS}$,
- $d(j, j') < R_{CS}$, only the receivers are within CS range.

FH is also a symmetric relationship. The conditional packet loss probability of the transmitter of each link is given by Equation 2.5. The values of the variables d, T_{ON} and T_{OFF} are given in Table 3.1. Substituting the values in Equation 2.5 gives $p_i = p_{i'} = 0.746748$ which is the minimum possible value of FH induced conditional packet loss probability for IEEE 802.11b radios. The combined conditional packet loss probability of the two links is $P_{min} = 1-(1-0.746748)(1-0.746748) = 0.935863$.

Table 3.2 summarizes the results from the analysis above. Note that in case of IA and FH flows, the least possible value is listed. It is obvious that the combined conditional packet loss probabilities for all the non-coordinated link cases (rows $2-4$ of Table 3.2) are significantly higher as compared to the coordinated link case. Even though the probability values may change if we consider a different transmission rate or a different radio (IEEE 802.11g/a) due the corresponding change in parameters, the above relationship continues to hold. □

Table 3.2: Combined conditional packet loss probabilities for the two link case

Topology type	**Value of P**
Coordinated Links	0.110833
Information Asymmetric (Min.)	0.878581
Near Hidden	0.564220
Far Hidden (Min.)	0.935863

3.2 n Link Case

We now extend the above analysis to a topology consisting of a clique of n interfering links. We separately treat the comparisons of Information Asymmetric, Near Hidden and Far Hidden non-coordinated links in Lemmas 2-4.

Lemma 2. *Given n WMN links forming a clique of interfering links, the combined packet loss probability is minimum when all n links share a coordinated relationship with each other as compared to topologies having a combination of coordinated and Information Asymmetric non-coordinated links.*

Proof. We prove the Lemma for the minimum value of combined conditional packet loss probability of Information Asymmetric non-coordinated links where only one link has IA relationship while remaining $n-1$ links are coordinated links. Any other combination of coordinated and IA non-coordinated links will have higher combined conditional packet loss probability. We first derive the expressions for combined conditional packet loss probability of n coordinated links and that of $n-1$ coordinated and one IA non-coordinated link and then compare the resulting expressions.

Clique of coordinated links: The conditional packet loss probability of a single transmitter within the clique is given using Equation 2.6 as:

$$p_i = 1 - (1-\tau)^{n-1} \tag{3.2}$$

where τ is the transmission probability of any node. The combined conditional packet loss probability of all nodes is given as:

$$P = 1 - [1-p_i]^n \tag{3.3}$$
$$P = 1 - [(1-\tau)^{n-1}]^n \tag{3.4}$$

Clique of IA non-coordinated links: Consider the IWMN node deployment such that $n-1$ links form the clique of coordinated links while the nth link is located such that it has IA non-coordinated relation with the remaining $n-1$ links. Note that the nth transmitter is outside the carrier sensing range of the remaining $n-1$ transmitters, therefore, the conditional packet loss probability for the transmitter of this link is $p'_n = 0$. The conditional packet loss probability of the transmitters of remaining $n-1$ links is same for all links and consists of two components – the CO component induced by the transmitters of $n-2$ coordinated links and IA component induced by the transmitter of nth link. Assume that the transmission probability of any of the $n-1$ transmitters is τ'. The CO component of conditional packet loss probability for a single transmitter is given as:

$$p_{CO(i)} = 1 - (1-\tau')^{n-2} \tag{3.5}$$

The IA component of the conditional packet loss probability of any of the $n-1$ transmitters is given by equation 2.4. The minimum value of IA component has been computed in proof of Lemma 1 and turns out to be $p_{IA(i)(min)} = 0.878581$. Substituting the values of $p_{IA(i)}$ and $p_{CO(i)}$ (Equation 3.5) in equation 2.8 we obtain the minimum conditional packet loss probability of one transmitter out of the $n-1$ transmitters as:

$$p'_{i(min)} = 1 - [1-p_{CO(i)}][1-p_{IA(i)(min)}]$$
$$p'_{i(min)} = 1 - [(1-\tau')^{n-2}][1-0.878581]$$
$$p'_{i(min)} = 1 - [(1-\tau')^{n-2}][0.121419] \tag{3.6}$$

The minimum value of combined conditional packet loss probability of n transmitters under the placement shown in Figure 1.1(b) is given as:

$$P' = 1 - [\Pi_{i=1}^{n-1}(1-p'_i)][1-p'_n]$$
$$P'_{(min)} = 1 - [\Pi_{i=1}^{n-1}(1-p'_{i(min)})][1-p'_n] \tag{3.7}$$
$$P'_{(min)} = 1 - [((1-\tau')^{n-2})(0.121419)]^{n-1}[1-0]$$
$$P'_{(min)} = 1 - [((1-\tau')^{n-2})(0.121419)]^{n-1} \tag{3.8}$$

We now want to prove that the combined conditional packet loss probability P (Equation 3.4) of n coordinated links is lesser than the minimum combined conditional packet loss probability $P'_{(min)}$ (Equation 3.8) of $n-1$ coordinated and one IA link. This is proved in two steps. In first step, we prove by contradiction that conditional packet loss probability p_i (Equation 3.2) of a single transmitter in all coordinated links placement is less than minimum conditional packet loss probability $p'_{i(min)}$ (Equation 3.6) of a single transmitter out of the $n-1$ coordinated links transmitter. i.e., $p_i < p'_{i(min)}$. In second step, we use the result $p_i < p'_{i(min)}$ and prove by induction that $P < P'_{(min)}$.

PROOF: $p_i < p'_{i(min)}$

Assume by contradiction that $p_i \geq p'_{i(min)}$. This implies that $\tau \leq \tau'$ because τ is continuous decreasing function of p (see Equation 2.2).

$$\begin{aligned} &\tau \leq \tau' \\ \Rightarrow &(1-\tau)^{n-2} \geq (1-\tau')^{n-2} \\ \Rightarrow &x * (1-\tau)^{n-2} = (1-\tau')^{n-2} \qquad 0 < x \leq 1 \end{aligned} \tag{3.9}$$

Now replacing the expressions for p_i and $p'_{i(min)}$ in the inequality $p_i \geq p'_{i(min)}$ we get:

$$\begin{aligned} \Rightarrow &1-(1-\tau)^{n-1} \geq 1-[(1-\tau')^{n-2}][0.121419] \\ \Rightarrow &(1-\tau)^{n-1} \leq [(1-\tau')^{n-2}][0.121419] \\ &\text{Substituting the value from Equation 3.9} \\ \Rightarrow &(1-\tau)^{n-2}(1-\tau) \leq [x*(1-\tau)^{n-2}][0.121419] \\ \Rightarrow &(1-\tau) \leq x * 0.121419 \end{aligned}$$

Considering the maximum value of $\tau = 0.060606$, the above inequality only hold for $x > 1$ which violates the condition of Equation 3.9. Therefore the assumption $p_i \geq p'_{i(min)}$ does not hold. This proves that $p_i < p'_{i(min)}$.

PROOF: $P_i < P'_{(min)}$

Substituting the values from Equation 3.3 and Equation 3.7 in the expression $P < P'_{(min)}$, we get:

$$\begin{aligned} &1-[1-p_i]^n < 1-[\Pi_{i=1}^{n-1}(1-p'_{i(min)})][1-p'_n] \qquad p'_n = 0 \\ \Rightarrow &[1-p_i]^n > (1-p'_{i(min)})^{n-1} \end{aligned}$$

We prove above result by induction.

Induction Base Case: $(1-p_i)^m > (1-p'_{i(min)})^{m-1}$ for $m = 2$

Lemma 1 proves the result for $m = 2$.

Induction Hypothesis: Suppose the inequality $(1-p_i)^m > (1-p'_{i(min)})^{m-1}$ hold for all $m = 3 \rightarrow r$ where $r < n$.

To Prove: The inequality $(1-p_i)^m > (1-p'_{i(min)})^{m-1}$ holds for $m = r+1$.

Proof. We have to prove that:

$$\begin{aligned} &(1-p_i)^{r+1} > (1-p'_{i(min)})^r \\ \Rightarrow &(1-p_i)^r(1-p_i) > (1-p'_{i(min)})^{r-1}(1-p'_{i(min)}) \end{aligned} \tag{3.10}$$

We divide the above inequality into two separate inequalities

$$(1-p_i)^r > (1-p'_{i(min)})^{r-1}\text{AND}(1-p_i) > (1-p'_{i(min)})$$

First inequality holds based on the induction hypothesis while the second inequality has been proved above (i.e. $p_i < p'_{i(min)} \Rightarrow (1 - p_i) > (1 - p'_{i(min)})$). This implies that the inequality 3.10 holds proving by induction that the inequality $P < P'_{(min)}$ holds for all values of n. □

This proves the Lemma. □

Lemma 3. *Given n WMN links forming a clique of interfering links, the combined packet loss probability is minimum when all n links share a coordinated relationship with each other as compared to topologies having a combination of coordinated and Near Hidden non-coordinated links.*

Proof. We prove the Lemma for the minimum combined conditional packet loss probability of Near Hidden (NH) non-coordinated links where only one link is located to have NH relationship while remaining $n - 1$ links are coordinated links. Any other combination of coordinated and NH non-coordinated links will have higher combined conditional packet loss probability. The expressions for conditional packet loss probability of single transmitter and the combined conditional packet loss probability of n transmitters for the clique of coordinated interfering links is given by Equation 3.2 and Equation 3.4 respectively.

Clique of 1 NH and $n - 1$ *coordinated links:* Consider the WMN deployment such that $n - 1$ links form a clique of coordinated links while the *nth* link is located such that it has a NH non-coordinated relation with the remaining $n-1$ links. Let the transmission probability of each of the $n-1$ transmitters of the CO links be τ_1 and that of NH link transmitter be τ_2. The conditional packet loss probability experienced by the transmitter of the NH link due to the existence of one of the other $n - 1$ transmitters is computed using Equation 2.7 as,

$$p'_{(n,i)} = 1 - (1 - \tau_1)^m$$

Since each of the $n - 1$ CO transmitters has a similar effect on the NH link transmitter, the values $p'_{(n,i)}$ can be combined for the $n - 1$ transmitters to obtain the expression for the conditional packet loss probability p'_n of the NH link transmitter.

$$p'_n = 1 - [(1 - \tau_1)^m]^{n-1} \tag{3.11}$$

The conditional packet loss probability of the transmitters of the $n-1$ CO links are equal and consist of two components: (i) the CO component induced by the transmitters of the other $n - 2$ coordinated links, (ii) the NH component induced by the NH link. The CO component is obtained from Equation 2.6,

$$p_{CO(i)} = 1 - (1 - \tau_1)^{n-2} \tag{3.12}$$

The NH component is computed using Equation 2.7,

$$p_{NH(i)} = 1 - (1 - \tau_2)^m \tag{3.13}$$

Substituting the values of $p_{NH(i)}$ and $p_{CO(i)}$ in Equation 2.8 we obtain the conditional packet loss probability of one of the $n - 1$ transmitters as,

$$p'_i = 1 - [1 - p_{CO(i)}][1 - p_{NH(i)}]$$
$$p'_i = 1 - [(1 - \tau_1)^{n-2}][(1 - \tau_2)^m] \tag{3.14}$$

The combined conditional packet loss probability of the n transmitters is given as,

$$P' = 1 - [\Pi_{i=1}^{n-1}(1 - p'_i)][1 - p'_n] \tag{3.15}$$
$$P' = 1 - [(1 - \tau_1)^{n-2}(1 - \tau_2)^m]^{n-1}[(1 - \tau_1)^m]^{n-1} \tag{3.16}$$

We now prove by contradiction that the combined conditional packet loss probability P (Equation 3.4) is less than P' (Equation 3.16). The inequality $P < P'$ holds if: (i) $p_i \leq p'_i$ and $p_i < p'_n$ OR (ii) $p_i < p'_i$ and $p_i \leq p'_n$. We assume by contradiction that $p_i \geq p'_i$ and $p_i \geq p'_n$. This implies that $\tau \leq \tau_1$.

$$\Rightarrow 1 - \tau \geq 1 - \tau_1$$
$$\Rightarrow x * (1 - \tau) = 1 - \tau_1 \qquad 0 < x \leq 1$$

Substituting the expressions of p_i and p'_n in inequality $p_i \geq p'_n$, we get:

$$1 - (1 - \tau)^{n-1} \geq 1 - [(1 - \tau_1)^m]^{n-1}$$
$$\Rightarrow (1 - \tau)^{n-1} \leq [(1 - \tau_1)^m]^{n-1}$$
$$\Rightarrow (1 - \tau) \leq (1 - \tau_1)^m$$
Substituting the value of $1 - \tau_1$ we get
$$\Rightarrow (1 - \tau) \leq x^m * (1 - \tau)^m$$

However, the above expression is not true for all values of τ and x as $\tau < 1$ and $x < 1$. Therefore the assumption does not hold. Note that, the contradicting assumption eliminates all contradicting possibilities. This proves that $P < P'$. □

Lemma 4. *Given n WMN links forming a clique of interfering links, the combined packet loss probability is minimum when all n links share a coordinated relationship with each other as compared to topologies having a combination of coordinated and Far Hidden non-coordinated links.*

Proof. We prove the Lemma for minimum combined conditional packet loss probability of Far Hidden (FH) non-coordinated links where only one link is placed to have FH relationship while remaining $n-1$ links are coordinated links. The expressions for conditional packet loss probability of single transmitter and the combined conditional packet loss probability of n transmitters for the clique of transmitters is given by Equation 3.2 and Equation 3.4 respectively.

Clique of FH non-coordinated links: Consider the IWMN node deployment such that $n - 1$ links form the clique of coordinated links while the nth link is located such that it has FH non-coordinated relation with the remaining $n - 1$ links. The conditional packet loss probability of nth transmitter experienced because of one of the $n - 1$ transmitters is given as (Equation 2.5):

$$p'_{(n,i)} = \frac{T_{ON(i)}}{T_{ON(i)} + T_{OFF(i)}}$$

The minimum value of $T_{ON(i)}$ is given in table 3.1. The minimum value of $p'_{(n,i)}$ is obtained when $T_{OFF(i)}$ is maximum. The maximum possible average value for $T_{OFF(i)}$ is given as $T_{OFF(i)} = DIFS + 512\sigma = 10290\mu s$ which is achieved when a transmitter always selects the maximum backoff window size ($W_{(max.)} = 1024$). Therefore, minimum possible value of $p'_{(n,i)}$ is $p'_{(n,i)} = 0.095861$. We will use the minimum value of $p'_{(n,i)}$ because we are interested in proving that p'_n is greater than p_i (Equation 3.2). Note that the minimum value of conditional packet loss probability of two flows in far hidden topology computed in proof of Lemma 1 is not applicable here because the transmission probability of the interfering links is different in two link topology and n link topology. The values $p'_{(n,i)}$ can be combined for $n-1$ transmitters to obtain the expression for conditional packet loss probability p'_n of nth transmitter.

$$p'_{n(min.)} = 1 - [1 - 0.095861]^{n-1} \tag{3.17}$$

The conditional packet loss probability of the transmitters of remaining $n-1$ links is same for all links and consists of two components – the CO component induced by the transmitters of $n-2$ coordinated links and FH component induced by the transmitter of nth link. Assume that the transmission probability of any of the $n-1$ transmitters is τ'. The CO component of conditional packet loss probability for a single transmitter out of the $n-1$ transmitters is given as:

$$p_{CO(i)} = 1 - (1 - \tau')^{n-2} \tag{3.18}$$

The minimum possible value of FH component of the conditional packet loss probability of any of the $n-1$ transmitters is given as $p_{FH(i)(min)} = 0.095861$. Substituting the values of $p_{FH(i)}$ and $p_{CO(i)}$ (Equation 3.18) in equation 2.8 we obtain the conditional packet loss probability of one transmitter out of the $n-1$ transmitters as:

$$\begin{aligned}
p'_i =& 1 - [1 - p_{CO(i)}][1 - p_{FH(i)}] \\
p'_{i(min)} =& 1 - [1 - p_{CO(i)}][1 - p_{FH(i)(min)}] \\
p'_{i(min)} =& 1 - [(1 - \tau')^{n-2}][1 - 0.095861] \\
p'_{i(min)} =& 1 - [(1 - \tau')^{n-2}][0.904138] \qquad (3.19)
\end{aligned}$$

The combined conditional packet loss probability of n transmitters is given as:

$$P'_{(min)} = 1 - [\Pi_{i=1}^{n-1}(1 - p'_{i(min)})][1 - p'_{n(min)}] \tag{3.20}$$

$$P'_{(min)} = 1 - [(1 - \tau')^{n-2})(0.904138)]^{n-1}[0.904138]^{n-1} \tag{3.21}$$

To prove that the combined conditional packet loss probability P of n coordinated links is less than the minimum combined conditional packet loss probability $P'_{(min)}$ of $n-1$ coordinated and one far hidden link, it is sufficient to prove that the conditional packet loss probability p_i (Equation 3.2) of single transmitter is less than conditional packet loss probabilities $p'_{i(min)}$ (Equation 3.19) and $p'_{n(min)}$ (Equation 3.17). We prove by contradiction that $p_i < p'_{i(min)}$ and $p_i < p'_{n(min)}$.

Assume by contradiction that $p_i \geq p'_{i(min)}$. This implies that $\tau \leq \tau'$.

$$\begin{aligned}
&\tau \leq \tau' \\
\Rightarrow& 1 - \tau \geq 1 - \tau' \\
\Rightarrow& x * (1 - \tau) = (1 - \tau') \qquad 0 < x \leq 1
\end{aligned}$$

Now Consider;

$$\begin{aligned}
&p_i \geq p'_i \\
\Rightarrow &1 - (1-\tau)^{n-1} \geq 1 - [(1-\tau')^{n-2}][0.904138] \\
\Rightarrow &(1-\tau)^{n-2}(1-\tau) \leq [(1-\tau')^{n-2}][0.904138] \\
&\text{Substituting the value of } 1-\tau' \text{ we get} \\
\Rightarrow &(1-\tau)^{n-2}(1-\tau) \leq [x^{n-2}(1-\tau)^{n-2}][0.904138] \\
\Rightarrow &(1-\tau) \leq x^{n-2}[0.904138]
\end{aligned}$$

Which is not possible because maximum value of τ is 0.060606 (see Equation 2.3). Therefore, the assumption do not hold and $p_i < p'_{i(min)}$.

Now Assume by contradiction that $p_i \geq p'_{n(min)}$.

$$\begin{aligned}
&p_i \geq p'_{n(min)} \\
\Rightarrow &1 - (1-\tau)^{n-1} \geq 1 - [0.904138]^{n-1} \\
\Rightarrow &1 - \tau \leq 0.904138
\end{aligned}$$

Which is not possible because maximum value of τ is 0.060606 (see Equation 2.3). Therefore, the assumption do not hold and $p_i < p'_{n(min)}$. As $p_i < p'_{i(min)}$ and $p_i < p'_{n(min)}$, this implies that $P < P'_{(min)}$. Which proves the Lemma. □

Theorem 1. *Given n WMN links forming a clique of interfering links, the combined packet loss probability is minimum when all n links share a coordinated relationship with each other as compared to topologies having any combination of coordinated and non-coordinated links.*

Proof. The theorem is readily proved using Lemmas 1-4 based on the following two facts; Firstly, any topology within the clique of links is the combination of the basic topologies used in the Lemmas. Secondly, the Lemmas have been proved for the minimum values of the conditional packet loss probability due to Information Asymmetric, Near Hidden or Far Hidden non-coordinated links. Any other combination of non-coordinated links within the clique of interfering links will result in higher conditional packet loss probability. □

3.3 Quantitative Comparison of Non-Coordinated and coordinated Links

The theorem above proves that within an interference domain, the non-coordinated links induce higher transmission losses as compared to coordinated links. However, it does not provide any insights into quantitative comparison between the two. In an attempt to quantify the severity of the interference caused by the non-coordinated links, this section seeks to answer the following question: *For a given link, how many coordinated interfering links will induce the same transmission losses as a single non-coordinated link?*

Table 3.2 summarizes the combined conditional packet loss probability of two links under different geometric placements. Note that, a pair of Near Hidden (NH) links have the least combined conditional packet loss probability ($p = 0.564220$) amongst all kinds of non-coordinated links. Hence, we seek to answer the above question for the NH relationship. Recall that the NH relationship is symmetric. The conditional packet loss probability of each link in a pair of NH links was computed in Equation 3.1 to be 0.339864. We now turn our attention to a clique of n coordinated links. Our goal is find the value of n that induces the same packet loss probability as the NH case (i.e., 0.339864) Simultaneously solving Equation 2.1 and Equation 2.2 for p using different values of n, we observe that for $n = 14$, the conditional packet loss probability $p \approx 0.339864$. This implies that 13 coordinated links located within the carrier sensing range of a particular link will induce the same level of transmission losses as a single NH link. Note that, for IA and FH links the value of n would be even higher.

Clearly for a given link, just one non-coordinated interfering link is far more worse than having a number ($< n$) of coordinated interfering links. This result lends support to the notion of minimizing the number of non-coordinated links by assigning them to operate on different channels. Further, it motivates the formation of cliques of coordinated links in the network. We now formalize the channel assignment problem.

4 Channel Assignment Problem

The results from the interference analysis in Section 3 show that non-coordinated interfering links induce significantly higher transmission losses as compared to the coordinated interfering links. We argue that the impact of interference induced by non-coordinated links can significantly be reduced in multi-radio multi-channel WMN using channel assignment. Typical mesh routers are equipped with multiple interfaces. The number of orthogonal (i.e. non-interfering) channels available in the 802.11 family of protocols are limited (3 in 802.11b and 12 in 802.11a/g). Given the limited number of channels, in a typical WMN topology, it can be difficult to completely eliminate interference. Since we have proved that non-coordinated interference significantly increases the transmission losses, it makes sense to prioritize eliminating this category of interference. Based on this argument, we formulate the channel assignment problem as a two phase minimization problem. The objective of the first phase is to minimize the number of non-coordinated interfering links operating on the same channel. Consequently, this eliminates the problems associated with non-coordinated links (discussed in Section 1). In the second phase, the channel diversity is exploited to minimize the number of coordinated links operating on common channel, thereby increasing the capacity of individual links. We prove that the channel assignment problem based on this hypothesis is NP-hard. In the subsequent section, we propose a heuristic protocol as a solution. For the remainder of this paper, we make the following simplifying assumptions:

1. The transmission range (R_T) and carrier sensing range (R_{CS}) of the nodes is fixed and all nodes transmit with the same transmission power.
2. All nodes are equipped with multiple radio interfaces (i.e., radios per node ≥ 2). Each node can have different number of interfaces. Let $n(v)$ be the number of interfaces of node v.
3. Let $C = \{1, 2, 3, ..., k\}$ be the set of available orthogonal channels. We assume that $k \geq 3$. Note that, IEEE 802.11b has 3 orthogonal channels while 11a/g have 12.

Problem Formulation: It is a common practice to model the network as a graph and we follow a similar approach here. Let undirected graph $G_u(V, E_u)$ represent the WMN, V being the set of nodes and E_u being the set of edges where an edge exists between two nodes $(v_i, v_j) \in V$ if $d(v_i, v_j) \leq R_T$. We transform the graph G_u into a directed graph $G(V, E)$, where each undirected edge $e_u(v_i, v_j) \in E_u$ is replaced by two directed edges $e(v_i, v_j)$ and $e(v_j, v_i)$. For all $e \in E$, let $c(e) \in C$ be the channel used by e. Let $n'(v), v \in V$ be the number of edges incident on vertex v and operating on different channels.

Channel Assignment Phase 1: Let $N_{nco}(e) \subset E, (e \in E)$ be the set associated with e where $N_{nco}(e) = \{e' \mid e' \in E$ and e' is non-coordinated interfering link of the link $e\}$. Let $N_{nco}(e, c) \subset N_{nco}(e)$ be the set where $N_{nco}(e, c) = \{e' \mid e' \in N_{nco}(e), c \in C, c(e') = c\}$. Clearly, $N_{nco}(e, c_i) \cap N_{nco}(e, c_j) = \phi$ and $\cup_{c \in C} N_{nco}(e, c) = N_{nco}(e)$. Note that the set $N_{nco}(e, c(e))$ contains the non-coordinated interfering links of e operating on the channel used by link e. For all $e \in E$, let $n(e)$ be the number of subsets $N_{nco}(e, c)$ of $N_{nco}(e)$. The primary objective of channel assignment is to minimize the number of non-coordinated interfering links operating on a common channel by utilizing minimum number of channels. This can be achieved by minimizing the following sum, while ensuring that for all $e \in E$, $n(e)$ takes on its minimum possible value.

$$\begin{aligned}&\textit{minimize} \quad \Sigma_{e \in E} |N_{nco}(e, c(e))| \quad \textit{s.t.}\\&\forall v \in V, n'(v) \leq n(v)\end{aligned}$$

Note that the condition $n'(v) \leq n(v)$ ensures that the number of channels assigned to a particular node should not exceed the number of interfaces available at that node.

Channel Assignment Phase 2: Suppose that the first phase of channel assignment (stated above) is completed and a channel is assigned to each $e \in E$. The second phase can then be formulated as follows. Let $N_{co}(e) \subset E, \forall (e \in E)$ be the set where $N_{co}(e) = \{e' \mid e' \in E$ and e' is a coordinated interfering link of $e\}$. For all $e \in E$, let $c(e) \in C$ be the channel assigned to e in the first phase. Let $N_{co}(e, c) \subset N_{co}(e)$ be the set where $N_{co}(e, c) = \{e' \mid e' \in E, c \in C, c(e') = c\}$. The objective of this phase is to minimize the number of coordinated interfering links operating on a common channel while ensuring that any pair of non-coordinated interfering links operating on orthogonal channels after first phase maintains the channel diversity. This is achieved as follows:

$$\begin{aligned}&\textit{minimize} \quad \Sigma_{e \in E} |N_{co}(e, c(e))| \quad \textit{s.t.}\\&\forall e \neq c(e') \text{ if } e' \in N_{nco}(e) \;\&\; e' \notin N_{nco}(e, c(e))\\&\forall v \in V, n'(v) \leq n(v)\end{aligned}$$

We prove that this channel assignment problem is NP-Hard.

Theorem 2. *The two phase channel assignment problem described above is NP-Hard.*

Proof. Suppose that there is an algorithm that can solve the first phase of the channel assignment problem stated above. This means that for a certain minimum number of channels k, the algorithm is capable of reducing the non-coordinated interfering links operating on a common channel to zero. This implies that the algorithm will also be able to solve the proper graph vertex coloring problem for non-planner graphs. However, the vertex coloring problem is known to be NP-Hard [16]. This proves that the first phase of the channel assignment problem is NP-Hard. It is easy to show along similar lines that the list coloring problem is a special case of the second phase of the channel assignment problem. As the list coloring problem is known to be NP-Hard [16], it follows that both phases of the channel assignment problem are NP-Hard. □

In the following section, we propose a heuristic protocol as a solution to the channel assignment problem.

5 Channel Assignment Protocol

In this section we present a novel channel assignment heuristic, referred to as *Cluster-based Channel Assignment Protocol (CCAP)*. Informed by the findings of our interference analysis (Section 3), CCAP aims to minimise non-coordinated interference by partitioning the network into non-overlapping clusters, each consisting of a clique of coordinated links. Recall that, such a clique induces significantly lower transmission losses as compared to a single non-interfering link. CCAP works in two phases, with the first phase responsible for the clustering process. The nodes within a cluster operate on a common channel, with orthogonal channels being used for neighbouring clusters. The channel assignment of the inter-cluster links aims to minimize the introduction of non-coordinated links, which may interfere with the clusters. The second phase of CCAP exploits channel diversity to sub-divide each cluster into multiple interference domains, thereby increasing the capacity of individual links. The two phases of CCAP correspond to the two steps of the channel assignment problem formulation introduced in Section 4.

Note that, on the surface, the approach taken by CCAP may appear to be similar to that employed in cellular networks, wherein the network is divided into multiple clusters (i.e., cells) and channels are assigned for communication within each cell. However, there are significant differences between the two. First and foremost, in cellular networks, the channel assignment problem is a bipartite graph coloring problem [17] which is known to have a tractable solution. On the contrary, we have proved in the previous section that the channel assignment problem in a WMN is NP-Hard. CCAP is a heuristic solution to this problem. Secondly, in cellular networks, nodes (i.e. mobile phones) in adjacent cells do not communicate with each other directly. They do so only via their respective base-stations. As a result, there is no need to establish direct connectivity between neighbouring cells. On the other hand, just partitioning the WMN into disconnected clusters is not enough, since a typical WMN will have several multi-hop flows spanning across multiple clusters. Hence, as in CCAP, it is necessary to establish inter-cluster connectivity.

We now present a simple example to illustrate the rationale behind the clustering approach employed in our protocol. Consider the labeled directional links in the network in Figure 5.1. We first describe the interference relationships that exist between the links. Links AB, BC, CD, CK form a set of coordinated links. $\{EF, BC\}$ and $\{FG, CD\}$ represent two sets of non-coordinated links. Similarly, DE and DH are non-coordinated interfering links of AB. Further, links DE, DH, EF, FG form another set of coordinated links. Suppose that we as-

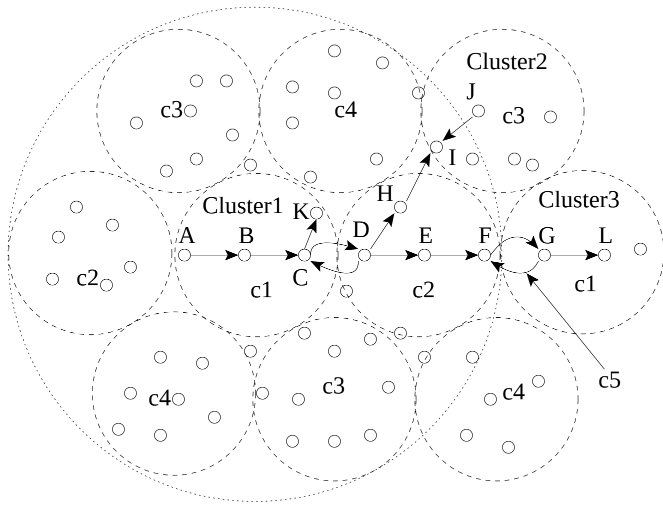


Figure 5.1: Example illustrating CCAP. c_1, c_2, c_3, c_4, c_5 are orthogonal channels used for channel assignment. 4 channels are used a default channel while channel c_5 is used to realize certain inter-cluster connectivity links.

sign channels based on a greedy approach where the objective is to minimize the non-coordinated interference for each link. A simple way to achieve this would be to assign links AB, BC, CD, CK to operate on channel $c1$ and the rest of the links DE, DH, EF, FG to another channel $c2$. Observe that this channel assignment has resulted in the formation of two clusters ($\{AB, BC, CD, CK\}$ and $\{DE, DH, EF, FG\}$), each comprising of coordinated interfering links only. If this approach is repeatedly followed for individual links, assuming that sufficient channels are available, the entire network can be partitioned into clusters as shown in Figure 5.1. The links within a cluster operate on a common channel. To avoid neighbouring clusters from interfering with each other, the channel used within a cluster is only reused by clusters that are separated by the carrier sensing range (for example, $Cluster1$ and $Cluster3$ in Figure 5.1). However, if this approach is followed, the resulting protocol can take a significantly long time to reach completion. This is further exacerbated by the delay and message passing overheads involved in determining all non-coordinated relationships. A simpler approach, which is employed by CCAP, involves partitioning the network into non-overlapping clusters, comprising of a clique of coordinated links and using the cliques as a unit for the first phase of channel assignment.

Algorithm 1 outlines the step-by-step operation of CCAP. It should be highlighted that CCAP is a static scheme, which is executed when the WMN is initialized. Once the channel assignment is complete, no further changes are made during the network lifetime. Further, CCAP executes distributively without the need of a controller node. One interface of each node is designated as the *default interface* while the remaining interfaces are referred to as *non-default interfaces*. We will use the example in Figure 5.1 to explain the operation of CCAP. In the first step, the network is partitioned into non-overlapping clusters, where each cluster is a clique of coordinated links. The clustering technique by Althofer et al. [18] is used to distributively partition the network into equal sized clusters.

Algorithm 1 CCAP

1: Partition the WMN into non-overlapping clusters – each comprising of nodes within one carrier sensing domain (i.e. clique of coordinated links).
2: **PHASE 1**: Select a *default channel* for each cluster such that outside this cluster, the channel is not used within the carrier sensing range. A minimum of 4 channels are required to ensure this. (For 3 channels, we only ensure that the default channel is not assigned to adjacent clusters).
3: Assign the default channel of the cluster to the default interface of all nodes that are part of the cluster. {Till the end of step 3, there are no non-coordinated interfering links.}
4: For all $e \in E$ such that e is an inter-cluster link, select channel $c | \forall c \in C, min. |N_{nco}(e, c)|$. Assign c to link e.
5: **PHASE 2**: For every link $e(i, j) \in E$, $List = C \setminus \{\{\bigcup_{e_k \in N_{nco}(e)} c(e_k)\} \bigcup \{\bigcup_{e_k \in N_{co}(e)} c(e_k)\}\}$. If $List \neq \phi$ and an interface exists in nodes i and j which has not yet been assigned a channel, then assign a channel to link e from set $List$. {Notations used in above step are explained in Section 4.}

The first phase of the protocol comprises of Steps 2-4. In Step 2, a channel is assigned to each cluster, such that the default interface of each node is tuned to this common channel, which is referred to as the *default channel*. In Figure 5.1, channels $c1, c2, c3$ and $c4$ are used as the default channels. Notice that, a cluster is assigned the channel that is not being used by any other cluster within the carrier sensing distance from this cluster. For example, consider $Cluster1$ in Figure 5.1. The carrier sensing distance around this cluster is shown by a large circle. Observe that other clusters within this distance are using channels $c2$, $c3$ and $c4$ as their default channels. Hence, these cannot be reused by $Cluster1$. However, $c1$ is not being used and hence it can be used as the default channel for $Cluster1$. Also notice that $c1$ can be reused by $Cluster3$ but not by $Cluster2$. Note that, all non-coordinated links of the links within $Cluster1$ will have their transmitters or receivers located within the large circle. If no cluster within this region is assigned the channel used by $Cluster1$ then, $Cluster1$ will not experience any non-coordinated interference. Note that, a minimum of 4 non-overlapping channels are required to ensure that this holds true for all 2-dimensional WMN. This is because the clustered network can be compared to a planner map, which is known to be 4-colorable [16]. If only 3 channels are available as in the case of IEEE 802.11b, then CCAP relaxes the channel reuse condition, such that only the adjacent clusters are assigned orthogonal channels. Assume that only 3 channels are available in the example in Figure 5.1. In this case, $Cluster2$ can be assigned channel $c1$ because it is not adjacent to $Cluster1$. Clearly, such an assignment cannot ensure the elimination of non-coordinated interference. Step 3 assigns the default channel to the default interfaces within the cluster. Up to this point, the network is still partitioned.

The fourth step establishes inter-cluster connectivity by assigning channels to the inter-cluster links. The non-default interfaces of the nodes are used in forming these links. We follow a greedy approach. Individual inter-cluster links are visited sequentially and assigned a channel, such that any non-coordiniated interference introduced as a result of this channel assignment is kept to a minimum. For example, consider links CD, DC, FG, GF in Figure 5.1. Link CD can be assigned channel $c1$ while links FG and DC can both be assigned channel $c2$ without introducing non-coordinated interference in the network. However, link GF cannot be assigned channel $c1$ or $c2$ because it would result in GF forming a non-coordinated relationship with CD or DC respectively. Therefore, link

GF is assigned channel $c5$, which is not used as a default channel by any neighbouring cluster. Note that, at most 2 interfaces are required per node to realize the first phase of CCAP. This requirement is in line with our assumptions in Section 4.

The second phase of CCAP seeks to enhance the network capacity by subdiving each cluster into multiple interference domains. In other words, the goal is to minimize coordinated interference by exploiting channel diversity. All links are examined sequentially and a channel is selected for the link if none of its coordinated and non-coordinated interfering links are using the channel. This channel is assigned to the non-default interfaces of the nodes forming the link only if an unassigned interface exists at both nodes. Consider link AB in Figure 5.1. AB can be assigned channel $c5$ because no coordinated or non-coordinated interfering link of AB is operating on channel $c5$. Thus, this step also ensures that the resulting changes do not introduce non-coordinated interference in the network.

6 Evaluation

In this section, we present a thorough simulation-based evaluation of CCAP using Qualnet. We first demonstrate that non-coordinated interfering links induce higher transmission losses and result in an unfair distribution of capacity amongst links. Next, we compare the performance of CCAP with the protocol proposed by Ramachandran et al. in [14] (The authors refer to their scheme as BFS-CA). We select BFS-CA for comparison because like CCAP this protocol also aims to preserve the WMN connectivity. Furthermore, this scheme has been shown to outperform other existing channel assignment protocols [14]. Note that BFS-CA is a dynamic protocol, where channel assignments are changed at regular intervals based on certain channel quality measurements. On the contrary, CCAP is a static protocol. To ensure a fair comparison, we disable the dynamic channel assignment functionality of BSF-CA when the stable channel assignment is reached during execution. We conduct three set of experiments. In the

Table 6.1: Simulation Parameters

Simulation time	1500 sec.
Number of nodes	36
Number of interfaces	3/node
Terrain Dimensions (b,a) radio	$1000 \times 1000m^2, 325 \times 325m^2$
Node Placement	Uniform Random
Radio Propagation model	Two ray
Propagation shadowing model	Constant (mean=4.0)
Basic rate (b, a) radio	$2Mbps, 6Mbps$
Data rate (b, a) radio	$11Mbps, 54Mbps$
Application traffic	Poisson Traffic Gen. (App. Layer)
Packet inter-arrival mean (b, a)	$1.6msec, 0.33msec$
Packet size (fixed)	512 Bytes

first set of experiments, we compare the per link goodput and the fairness (using

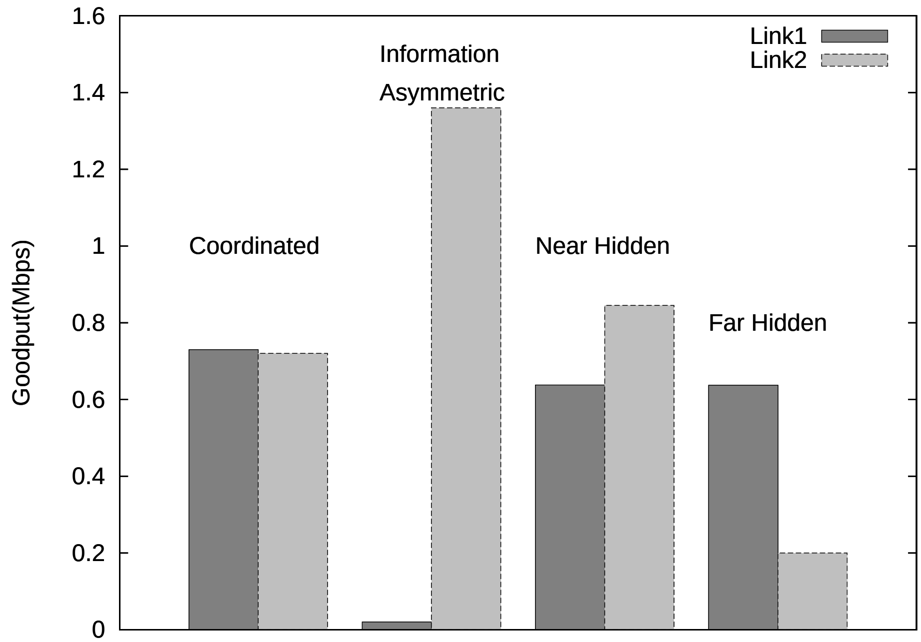
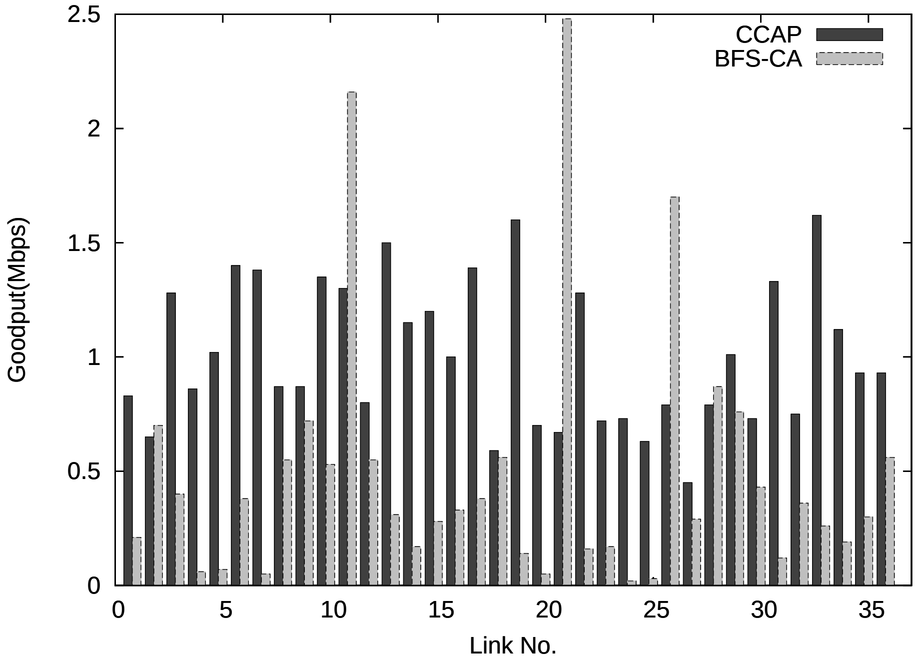


Figure 6.1: Comparison of CO, IA, NH and FH

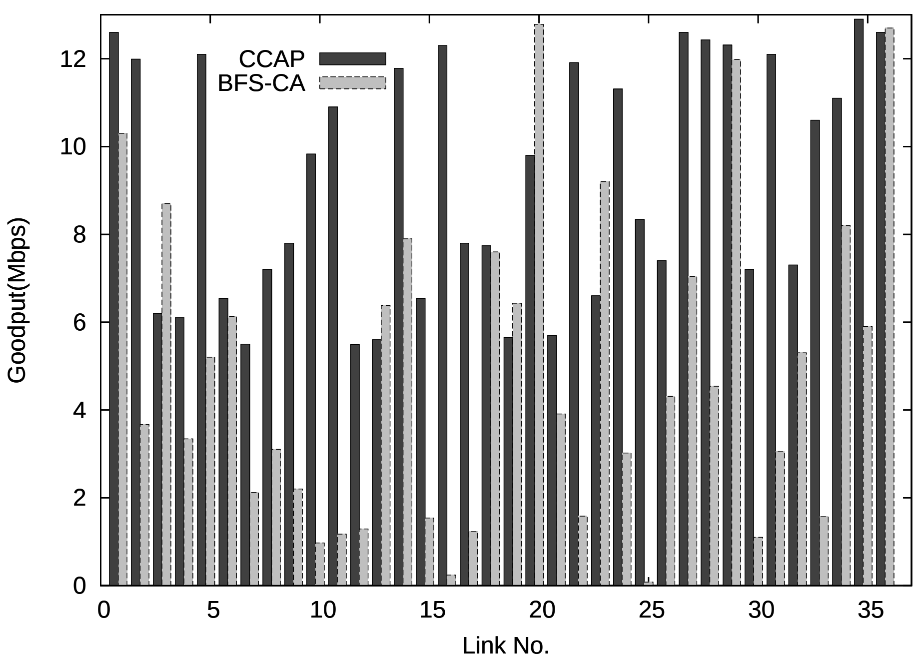
Jain's fairness index) achieved by the two protocols for a network of single-hop flows using both IEEE 802.11 a & b radios. The second set of experiments explores the impact of increasing the number of channels on the aggregate goodput for the same scenario. In the last part, we investigate the performance achieved by end-to-end multi-hop flows as opposed to the single-hop flows used in the previous experiments. Table 6.1 summarizes the simulation parameters that are used in all the experiments unless otherwise stated.

(i) Impact of Non-coordinated Links: Figure 6 shows the achievable goodput for two links placed under different geometric relations. Each link supports a data rate of $2Mbps$. It is clear from the graph that two coordinated (CO) links equally share the channel capacity. On the contrary, when two links share an Information Asymmetric (IA) relationship, one of the link receives the lion's share of the capacity while the other can barely transmit any data. Near Hidden (NH) and Far Hidden (FH) links also result in unfair capacity distribution. The combined channel utilization of the two CO links is 73% (max capacity is $2Mbps$ within one carrier sensing range). On the other hand, the utilization of IA, NH and FH links is $0.34\%, 0.37\%$ and 0.2% respectively. Clearly, the coordinated links result in fair share and better channel utilization.

(ii) Goodput and Fairness for Single Hop Flows: In this set of experiments, we use a scenario with 36 random single-hop flows. We first use IEEE 802.11b radios with 3 orthogonal channels and compare the per link goodput distribution and fairness for CCAP and BFS-CA protocols. This allows us to study if the limited number of channels (3 in this case) have an adverse impact on the

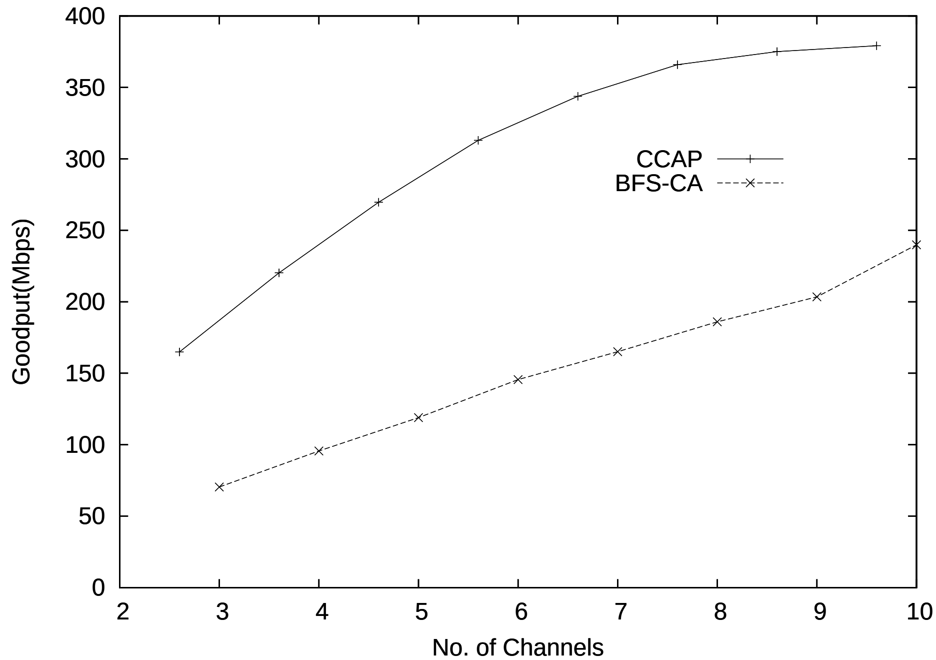


(a) Goodput for single-hop flows (IEEE 802.11b, 3 channels)

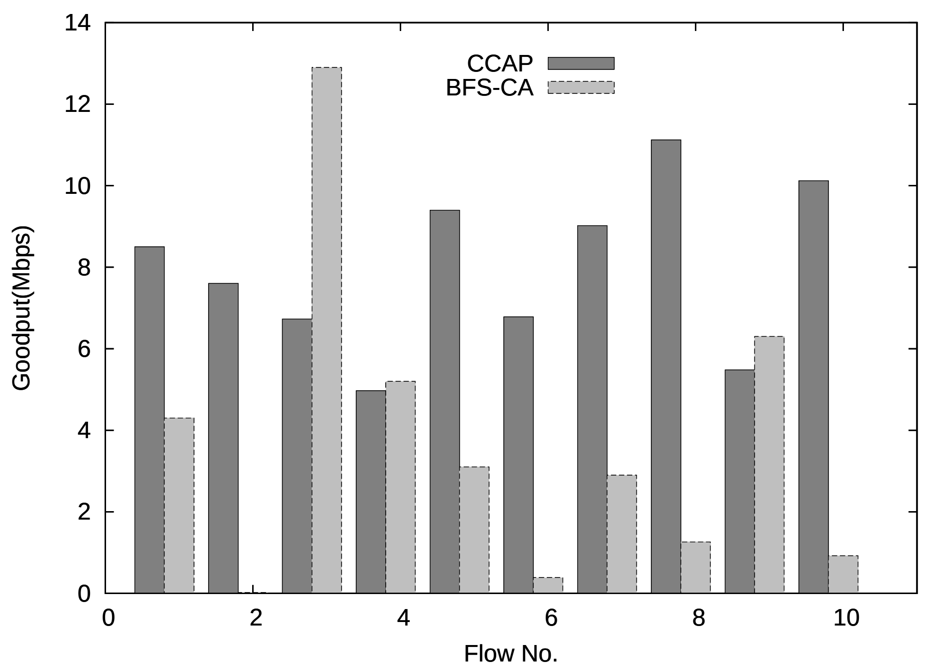


(b) Goodput for single-hop flows (IEEE 802.11a, 6 channels)

Figure 6.2: Evaluation Results Part 2



(a) No. of chnls vs. aggregate goodput (802.11a)



(b) Goodput for multi-hop flows (802.11a)

Figure 6.3: Evaluation Results Part 3 & 4

performance of CCAP. The per-flow goodput for the two schemes from one run of the simulation are shown in Figure 6.2(a). One can readily observe that most flows experience higher goodput with CCAP. The aggregate goodput across all flows is $36.22 Mbps$ and $17.3 Mbps$ for CCAP and BFS-CA, respectively. Clearly, the former achieves better channel utilization. Further, the limited availability of channels has not had a major impact on the performance of CCAP. We use Jain's fairness index to determine, which protocol is more fair in distributing the channel capacity amongst the interfering flows. This metric takes on a value from 0 to 1, with 1 implying equal allocation to all flows. The values of the index for CCAP and BFS-CA are 0.9137 and 0.4361 respectively, implying that CCAP is significantly more egalitarian when it comes to sharing the channel capacity. Even though, the results presented above are from a single simulation run, we have observed a similar trend across multiple simulation runs (excluded for reasons of brevity).

The above experiment is now repeated for IEEE 802.11a radios. We utilize 6 out of the 12 orthogonal channels available (6 are sufficient due to the limited number of links). This allows us to compare the performance of the protocols when the number of channels available is not limited. Figure 6.2(b) illustrates that a large proportion of the flows achieve significantly higher goodput with CCAP as compared to BFS-CA. The aggregate throughput for CCAP and BFS-CA are 331.86 and 175.77, respectively. Jain's fairness index for CCAP is 0.9225 and for BFS-CA is 0.6542. This shows that for both the limited channel case and otherwise, CCAP achieves fairer distribution of capacity and greater aggregate goodput.

(iii) Impact of Number of Channels on Aggregate Goodput: We now observe the impact of increasing the number of available orthogonal channels on the aggregate goodput. This gives us insight into the channel utilization achieved by the two protocols. We use the same scenario as before (i.e., 36 random single-hop flows). We use IEEE 802.11a radios due to the large number of orthogonal channels available. The results have been averaged over 20 executions. Figure 6.3(a) shows the achievable aggregate goodput of 36 single-hop flows as a function of the number of channels. CCAP consistently outperforms BFS-CA by a factor of at least 2. Note that, for 9 channels, with CCAP the network capacity is nearly saturated and any further increase in the number of channels does not significantly improve the channel capacity. This is because with 9 channels, CCAP has nearly eliminated the interference in this particular network. On the other hand, the capacity of BFS-CA keeps increasing even beyond 9 channels but still remains lower than that of CCAP. This highlights the efficient channel utilization achieved by CCAP.

(iv) Goodput of Multi-hop Flows: In our evaluations so far, we have employed single-hop flows. However, typical WMN traffic consists of multi-hop flows. Hence, we now evaluate the impact of the two protocols on the end-to-end goodput of multi-hop flows. We generate 10 traffic flows between randomly selected source-destination pairs. The experiment was repeated 20 times with a random selection of source-destination pairs and the results are averaged. Figure 6.3(b) shows the average end-to-end throughput achieved by the individual flows. The aggregate throughput with BFS-CA and CCAP is $37.3 Mbps$ and $79.7 Mbps$ respectively. Note that, the throughput experienced by each flow is largely dependent upon its location. Although BFS-CA results in considerably better performance for certain flows $(3, 4, 9)$, the aggregate throughput for all

flows is significantly greater with CCAP.

7 Related Work

A number of channel assignment algorithms have been proposed in recent years [5–14]. Use of multiple radios and multiple channels in WMN was first proposed by Raniwala et al. [9] and a centralized channel assignment scheme was outlined. In a subsequent publication [10], the authors proposed a dynamic distributed channel assignment and routing algorithm. Alicherry et al. [11] proposed a centralized load-aware link scheduling, channel assignment and routing protocol. The authors propose the division of fixed duration time frames into slots where a specific set of nodes can transmit within each time slot on specific channels, which are assigned by a channel assignment algorithm. These schemes use traffic load to measure interference. Ramachandran et al. [14] have proposed a centralized channel assignment algorithm where channel utilization and channel quality is used as the metric to quantify the effect of interference. Marina et al. [8] have proposed a static centralized greedy heuristic channel assignment algorithm for finding the connected low-interference topologies using number of interfering radios to measure interference. However, none of these metrics can effectively capture the impact of non-coordinated links. On the contrary, in this paper, we use transmission losses as a measure of interference. Transmission losses are dependent upon the geometric relation between WMN nodes and hence can explicitly isolate the effect of non-coordinated links. We propose a channel assignment protocol, which seeks to minimize the non-coordiniated interference and thus improve the network capacity.

8 Conclusion

In this paper, we analytically proved that non-coordinated links induce significantly higher transmission losses as compared to coordinated links. We formulated the channel assignment problem for multi-radio multi-channel WMN with the primary objective of minimizing the non-coordinated interference and proved that the problem is NP-Hard. We proposed a novel two phase distributed *Cluster-based Channel Assignment Protocol (CCAP)* as a solution to the problem. The first phase of CCAP minimizes the non-coordinated interference in the network while the second phase exploits channel diversity to minimize the coordinated interference. Simulation based experiments showed that CCAP outperforms existing channel assignment protocols by a factor of at least 2.

Recall that CCAP is a static protocol. We are currently working on a dynamic enhancement of CCAP, where the second phase assigns the channels dynamically, based on the current level of interference in the network. We expect that this will enable CCAP to adapt to the current network dynamics and consequently further improve the channel utilization. In the future, we also intend to explore the idea of minimizing the non-coordinated interference through topology control.

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