Resource-aware Broadcast and Multicast in Multi-rate Wireless Mesh Networks

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Abstract

This paper studies some of the fundamental challenges and opportunities associated with the network-layer broadcast and multicast in a multihop multirate wireless mesh network (WMN). In particular, we focus on exploiting the ability of nodes to perform link-layer broadcasts at different rates (with correspondingly different coverage areas). We first show how, in the broadcast wireless medium, the available capacity at a mesh node for a multicast transmission is not just a function of the aggregate pre-existing traffic load of other interfering nodes, but intricately coupled to the actual (sender, receiver) set and the link-layer rate of each individual transmission. We then present and study six alternative heuristic strategies for computing a broadcast tree that not only factors in a flow’s traffic rate but also exploits the wireless broadcast advantage (WBA). Finally, we demonstrate how our insights can be extended to multicast routing in a WMN, and present results that show how a tree-formation algorithm that combines contention awareness with transmission rate diversity can significantly increase the total amount of admissible multicast traffic load in a WMN.
1 Introduction

Recent experiences with the deployment of Wireless Mesh Networks (WMNs) (e.g., the Roofnet [5] and TFA [8] projects) attest to their significant promise as an alternative, low-cost, fault-tolerant wireless access infrastructure for many novel applications [2]. The traffic capacity of such multi-hop wireless networks, however, continues to be an area of concern. It is becoming increasingly clear that individual WMN nodes should utilize the link/MAC layer multi-rate capability, especially as commodity 802.11-based cards already dynamically adjust the transmission rate on any wireless link by varying the modulation technique. However, most research on exploiting this rate-diversity feature in WMNs, such as contention-aware channel assignment [25] or channel diversity-aware routing metrics [12], has focused on unicast flows.

As part of our ongoing work on the Aiolos project [1], we are investigating how this multi-rate capability of wireless radios can be exploited to better support broadcast and multicast traffic. We believe that high-speed WMNs will eventually serve as the transport network in many communities for several broadcast/multicast consumer applications (such as IP-TV or local content delivery, streaming of rich sensor feeds from security/traffic cameras, and multi-player games); it is thus necessary to devise traffic routing algorithms that maximize the volume of broadcast/multicast traffic that may be supported by a WMN. In previous work [11], we had considered the simplistic case of a single broadcast flow and demonstrated how link-layer rate diversity could be exploited to reduce the broadcast latency (defined as the maximum delay between the transmission of a packet by the source node and its eventual reception by all receivers) in a single-channel WMN. While this work helped establish the importance of exploiting rate-diversity for link-layer broadcasts, it did not consider the question of how such rate diversity affects the total admissible network load.

In this paper, we consider the more practical case of having multiple broadcast or multicast flows present in a single-channel WMN1 and address the following two questions:

1The study of multicast flow capacity optimization in multi-channel WMNs, with multiple radios per node, is
• How does the potential transmission rate diversity impact the notion of how much broadcast traffic load can be feasibly accommodated on a specific data path?

• What sort of routing strategies can increase the amount of broadcast/multicast traffic loads that a WMN can accommodate, and what benefit (if any) does the use of link-rate diversity offer over the conventional approach of performing link-layer broadcasts at the base rate?

In particular, we shall devise algorithms that consider (a) the multi-rate operation of an individual WMN, (b) the offered traffic load of a new multicast flow, and (c) the load from prior existing flows, to construct the corresponding broadcast/multicast tree. We assume that each mesh node can alter its link-layer broadcast transmission rate, which implicitly alters the transmission range (or its set of ‘covered’ receivers). While the current 802.11a/b/g standards mandate the broadcast transmission of control frames (e.g. RTS/CTS/ACK) at the lowest possible rate (e.g., 1 Mbps for 802.11b and 6 Mbps for 802.11a), broadcast transmission rates are currently implementation-specific.

Not surprisingly, the creation of an individual broadcast/multicast tree depends directly on the interplay between the choice of a node’s transmission rate, the resulting packet routing topology, and the consequent level of channel contention among neighboring nodes. For a broadcast flow, where a sender generates $t$ bits/sec of traffic, to be feasibly transmitted over a given tree, an intermediate node on the tree must be able to access the shared channel for a sufficient time to transmit at least $t$ bits/sec. A rate choice of $R$ bits/sec not only implies that node $X$ must access the channel for the fraction $\frac{t}{R}$, but also implicitly alters the degree of contention. For example, a faster rate $R$ may reduce the ‘airtime held’ by node $X$, but also reduces the coverage area of the broadcast, implying the need for additional transmissions (thus raising the level of contention) by a larger subset of downstream neighbors. Besides incorporating the impact of such link-layer rate diversity, a distribution tree for a newly arriving multicast flow should “route

left to a future paper.
around” existing hot-spots (pockets of high contention from existing transmissions), so as to maximize the total amount of multicast traffic supported on the WMN. This problem is unique and different from prior work on load-aware multicasting in wired networks (e.g., [7]), since the algorithms also need to exploit the wireless broadcast advantage (WBA) [28] (whereby a single transmission reaches multiple one-hop neighboring nodes) to minimize the number of independent transmissions.

1.1 Key Contributions of This Paper

Our principal objective is to maximize the total amount of broadcast or multicast traffic load that may be feasibly supported on a given WMN topology. Accordingly, we should route an individual broadcast or multicast flow to such that it uses the minimally feasible network resources. Given this objective, this paper makes the following three contributions:

1. It shows how the feasibility of a particular link-layer broadcast is a function of not just the existing and incoming traffic load, but also of the chosen link-layer transmission rate. As an important consequence, the true feasibility for a link-layer multicast is not captured by the notion of ‘maximal cliques over contention graphs’ [27], that has been previously devised for unicast flows.

2. For network-wide broadcast traffic, it presents and evaluates six heuristic tree construction algorithms that exploit transmission rate diversity, WBA and the residual capacity of the network (after taking into account the existing traffic load) to increase the amount of total traffic load that a WMN can carry.

3. For the practically important case of multicast traffic, it presents and evaluates a heuristic algorithm for tree construction that exploits transmission rate diversity, WBA and the residual capacity of the network. The proposed algorithm admits 30-40% more traffic than algorithms that use the base rate for all its link-layer transmissions.
The rest of this paper is organized as follows. Section 2 reviews the relevant related work. Section 3 details the unique interference-related capacity constraints for multicast link-layer transmissions, highlighting the inadequacy of the contention-graph based constraints formulated for unicast traffic in previous work and suggesting the use of an alternative, practical, ‘worst-case’ constraint. Section 4 describes the heuristic algorithms and performance results for broadcast traffic, both via analytical techniques (using Matlab) and discrete event simulations (using Qualnet [23], and then establishes how these results validate recent results on ‘broadcast network capacity’. Subsequently, Section 5 describes and evaluates the heuristic algorithm for resource-aware multicast in WMNs. Finally, Section 6 concludes the paper with the important observations and discussion of open work.

2 Related Work

A significant body of research in MANETs (Mobile Ad Hoc Networks) has researched efficient network layer multicast and broadcast, typically focusing on metrics such as energy consumption [9][28], the number of transmissions (which is equivalent to energy consumption if transmission power cannot be adjusted) [20] or the overhead in route discovery and management [14]. For WMN, where the mesh nodes are largely static (e.g., rooftop or electric pole mounted) and may often be powered from AC outlets, the total acceptable traffic load is a more critical performance metric than routing overhead or energy. QoS-aware MANET multicast routing algorithms have so far focussed on improving the delivery reliability (by either using resource reservation over multiple wireless paths (e.g., [3]), or constructing a delivery mesh instead of a tree (e.g., [26])), rather than focusing on the opportunities and challenges associated with link rate diversity and interference.

The problem of high throughput routing in WMN has been studied only for the case of unicast flows. The authors of [12] proposed a routing metric which can be used for a multi-channel,
multi-hop WMN. The proposed WCETT metric takes different transmission rates into account by having WCETT inversely proportional to the transmission rate. The work in [4] shows that if the interference range is infinity, then the unicast routing path that minimizes the total path delay will also maximizes the throughput between the source and destination. To deal with multi-rate links, [4] defines the rate-dependent medium-time metric (MTM), which measures the time it takes to transmit a packet over a multi-rate links including the transmission delay, overheads of the RTS/CTS/ACK frames and channel contention. In contrast to our focus on the network layer, the problem of maximizing the MAC-layer throughput for multicast transmissions (in the presence of different quality links and stability constraints) has been analyzed in [10].

We have previously studied the problem of low broadcast latency in multirate WMNs, for the single-channel case in [11] and for the multi-radio, multi-channel case in [21]. In particular, we presented an algorithm, based on the concept of weighted connected dominating set (WCDS), that explicitly balances the wireless broadcast advantage (WBA) with rate diversity to achieve low-latency network-wide broadcast. However, [11] focused only on a single broadcast flow and does not address the problem of how individual flows should be routed to maximize the total admissible volume of broadcast/multicast traffic in the presence of inter-flow and intra-flow interference.

3 Interference Modeling and Feasibility Analysis for Rate-Diverse Transmissions

In this section, we present the impact of interference on the feasibility of broadcast flows for a single-channel WMN. The analysis presented here explains how a candidate node on the routing tree for a new broadcast flow $F_j$ (with an associated offered load of $L_j$ bits/sec) can determine if it may feasibly forward the traffic for this flow using a link-layer broadcast rate $\rho$ bit/sec. This feasibility analysis will thus directly affect the formation of the broadcast forwarding tree (to be
presented in Section 4). We make the following assumptions in our study:

- Each node is equipped with a single radio and operates on a single common channel.

- Each node transmits with a fixed maximum power but can transmit with different rate by adjusting the modulation scheme. The transmission range is a decreasing function of the transmission rate. While we use a disc model for the transmission range, the proposed algorithms can be also applied to the case of non-uniform and non-isotropic rate-range relationships. We assume that each node can transmit at one of the available rate $R = \{\rho_1, \rho_2, \rho_3, \ldots, \rho_k\}$, where the rates are arranged in ascending order such that $\rho_1 < \rho_2 < \rho_3 < \ldots < \rho_k$, and $d(\rho_i)$ denotes the transmission range for rate $\rho_i$.

- A node’s “neighbors” are all the nodes that can be reachable using the lowest possible transmission rate.

- Let $\{v_1, ..., v_l\}$ be a subset of the neighbors of a node $v$ and the maximum rates that node $v$ can use to reach these nodes individually are $\rho_1, ..., \rho_l$ respectively. The maximum rate that node $v$ can use to reach $\{v_1, ..., v_l\}$ is $\min(\rho_1, ..., \rho_l)$.

- We assume a binary interference model, where two nodes $v_a$ and $v_b$ mutually interfere if and only if $d(v_a, v_b) < \kappa \times d(\rho_1)$, where $\kappa > 1$. The distance $\kappa \times d(\rho_1)$ is known as the interference range.

- For formulating our feasibility criteria, we assume an ideal MAC layer as follows: Two nodes $v_i$ and $v_j$ can transmit at the same time iff node $v_i$’s transmission does not interfere with the intended recipients of node $v_j$’s transmission and vice versa.

We represent the entire WMN as a graph $G(V, E)$, with the mesh nodes forming the vertices and the edge representing the link between two neighboring nodes. A link $(v_a, v_b) \in E$ exists only if the distance $d(v_a, v_b)$ between nodes $v_a$ and $v_b$ is less than $d(\rho_1)$, and is associated with a
rate \( \rho_{v_a,v_b} \), the fastest feasible rate on \((v_a, v_b)\). We denote multiple incoming point-to-multipoint flows as \(F_1, F_2, F_3, \ldots, F_j, \ldots\), each with traffic load \(L_1, L_2, L_3, \ldots, L_j, \ldots\) (where the traffic of a flow is modeled as a fluid arrival process\(^2\)). Each flow \(F_j\) represents the traffic generated from a given source node \(v_j\) to a set of destination nodes. If the destination set includes all mesh nodes except \(v_j\), it is a \textit{broadcast flow}; otherwise, it is called a \textit{multicast flow}.

\textbf{Definition 1} A link layer multicast transmission \(\tau(v_i, F_j)\) on node \(v_i\) for flow \(F_j\) is a two-tuple:

\[
\tau(v_i, F_j) \triangleq \{\rho(v_i, F_j), N(v_i, F_j)\}
\]

where \(\rho(v_i, F_j) \in R\) denotes the transmission rate used by node \(v_i\) for flow \(F_j\), and \(N(v_i, F_j)\) denotes the set of currently uncovered downstream neighbors (uncovered set consists of nodes that \(v_i\) is trying to reach) that node \(v_i\) covers at rate \(\rho(v_i, F_j)\) (i.e., the set of nodes \(\{v_l : d(v_i, v_l) \leq d(\rho(v_i, F_j))\}\) and \(v_l\) is currently uncovered\).

Note that a network flow (broadcast or multicast) consists of a number of such link layer multicast transmissions, where each link layer multicast transmission is associated with a non-leaf node (or transmitting node) on the flow-specific forwarding tree.

### 3.1 Definition and Properties of Broadcast Interference

Given the broadcast nature of the wireless medium, the transmission \(\tau(v_i, F_j)\) will interfere (or, equivalently, cannot occur simultaneously) with a set of other transmissions. In general, this set of interfering transmissions include transmissions by node \(v_i\) itself (i.e., transmissions for other flows where \(v_i\) is a non-leaf node), as well as transmissions by nearby interfering nodes.

The inter-node interference for transmission \(\tau(v_i, F_j)\) with a transmission \(\tau(v_i, F_j)\) by another

\(^2\)Our analysis, which is aimed at understanding the fundamental issues associated with multi-rate transmissions, assumes that \(L_i\) represents the total traffic load of \(F_i\), such that \(\frac{L_i}{\rho_i}\) represents the total transmission time. For precise computation, \(L_i\) should be adjusted to include the various overheads (network, MAC, PHY) associated with a specific transmission technology.
node \( v_i \) occurs when \( \tau(v_i, F_j) \) interferes with the reception by any of the recipients \( N(v_i, F_j) \), or \( \tau(v_i, F_j) \) interferes with the reception by any of the recipients \( N(v_i, F_j) \). In particular, note that \( F_j \) and \( F_j \) may be the same flow – i.e., there may be intra-flow interference caused by different nodes on the forward tree for \( F_j \).

**Definition 2** For any transmission \( \tau(v_i, F_j) \), the interference set \( \text{Inter}(\tau(v_i, F_j)) \) denotes the set of other transmissions that cannot occur in parallel with transmission \( \tau(v_i, F_j) \).

![Figure 1: Example showing the interference among multiple transmissions.](image)

Figure 1 illustrates the nature of interference among several multicast transmissions, where \( R_I = \kappa \times d(\rho_1) \) represents the interference range. There are three flows \( F_1, F_2, F_3 \), three transmitting nodes \( v_1, v_2, v_3 \), and four multicast transmissions \( \tau_1 = \tau(v_1, F_1), \tau_2 = \tau(v_2, F_1), \tau_3 = \tau(v_3, F_3), \tau_4 = \tau(v_1, F_2) \). The currently uncovered downstream neighbors of these four transmissions are as follows \( N(v_1, F_1) = \{v_7, v_8, v_9\}, N(v_2, F_1) = \{v_4, v_5, v_6\}, N(v_3, F_3) = \{v_{10}, v_{11}, v_{12}\}, N(v_1, F_2) = \{v_{13}, v_{14}\} \). We observe that \( \tau_1 \) interferes with \( \tau_2 \) at node \( v_5 \). This interference is intra-flow interference since they are both transmissions for the same flow \( F_1 \). Note that we model each flow as a fluid process. In a packet network, \( \tau_1 \) and \( \tau_2 \) would be packet transmissions where each packet may have a different sequence number (but originate from the same source node). We further observe that \( \tau_1 \) and \( \tau_4 \) compete for resources (or equivalently, interfere with each other) at the same transmitting node \( v_1 \), and \( \tau_3 \) interferes with \( \tau_1 \) at node \( v_8 \).
These interference constraints imply that the multicast transmission $\tau_1$ cannot happen simultaneously with any of the other three multicast transmissions. Note that the interference effects are not symmetric – e.g., while $\tau_3$ interferes with $\tau_1$ (at node $v_8$), $\tau_1$ does not cause any interference to any of the receivers of $\tau_3$. Also note that different transmissions from a same node can have different relationship with another transmission. For example, $\tau_3$ interferes with $\tau_1$ but $\tau_3$ does not interfere with $\tau_4$.

In order to model the interference relationship more accurately for a given transmission, we construct the conflict graph for this transmission and compute the maximal cliques in the conflict graph. Recall that a clique in a graph is a subset of vertices such that each pair of vertices is connected by an edge, or in other words, the subgraph is a complete graph. A clique that is not contained in any other cliques is defined as a maximal clique. We further define the conflict graph for a given transmission $\tau$ as $CG(\tau)$, whose vertices (including $\tau$) correspond to transmissions that may cause interference with $\tau$ or be interfered by $\tau$, e.g., set $\{\hat{\tau}, \forall \hat{\tau} \in \text{Inter}(\tau)\}$. The conflict graph of $\tau_1$ in Figure 1, $CG(\tau_1)$, as well as the resulted maximal cliques, are illustrated in Figure 2. We can see that there are two maximal cliques in this graph: maximal clique 1 includes $\tau_1$, $\tau_2$, and $\tau_4$; maximal clique 2 includes $\tau_1$ and $\tau_3$. The intuition of Figure 2 is that all transmissions within a given maximal clique cannot happen simultaneously. For example, $\tau_1$, $\tau_2$ and $\tau_4$ cannot happen simultaneously. Similarly, $\tau_1$ and $\tau_3$ cannot happen simultaneously. But note that $\tau_3$ can happen simultaneously with either $\tau_2$ or $\tau_4$.

![Figure 2: Conflict graph using maximal clique.](image-url)
It is important to note that this conflict graph is transmission specific – for example, if the transmission $\tau_1$'s transmission rate is modified such that $N(v_1, F_1) = \{v_7, v_9\}$, then it will no longer interfere with $\tau_3$, resulting in a different conflict graph than Figure 2. This is an important distinction from prior works such as [25] [27] on unicast traffic, where the vertices of conflict graphs represent individual links and the maximal cliques are transmission-independent. In the multicast environment, it is this dependency of the maximal cliques on the specific set of receivers (and thus, implicitly, on the link layer transmission rate) that makes the computation and enforcement of feasibility constraints harder.

To determine if transmission $\tau$ is feasible under this interference model, we transpose rate constraints to an airtime constraint – clearly, given the shared channel, the total fraction of airtime consumed by all the contending transmissions must be less than 1. To explicitly embody this constraint, we formally define the metric transmission time fraction (TTF) as follows:

**Definition 3** Assuming that node $v_i$ is a transmitting node of flow $F_j$, the transmission time fraction (TTF) for $\tau(v_i, F_j)$ is:

$$TTF(\tau(v_i, F_j)) = \frac{L_j}{\rho(v_i, F_j)} \quad (2)$$

where $L_j$ denotes the load of flow $F_j$ and $\rho(v_i, F_j)$ denotes the transmission rate selected by node $v_i$ for flow $F_j$.

Given this definition, we can readily derive the necessary conditions on the airtime for the flows in Figure 2 to be feasible:

$$TTF(\tau_1) + TTF(\tau_2) + TTF(\tau_4) \leq 1 \quad (3)$$

$$TTF(\tau_1) + TTF(\tau_3) \leq 1 \quad (4)$$
In general, these necessary conditions on the airtime for a set of flows to be feasible can be expressed as follows:

\[ \sum TTF(\tau) \leq 1 \]  

(5)

where all \( \tau \)'s form the maximal clique.

The above conditions are necessary because we are using fluid approximation to a discrete problem. Alternatively, given a number of transmissions, it can readily be proved that these transmissions can take place simultaneously if and only if they belong to an independent set \(^3\) of the CG. It was proved in [17] that a set of transmissions is feasible (or schedulable) if and only if it lies in the polytope of the independent sets of the CG. However, it is generally not feasible to apply this result in practice since the complexity to compute all independent sets grows exponentially with the number of nodes (which is already very large for a multicast CG).

In order to be able to determine a feasibility of a set of multicast transmissions, we will instead use a sufficient but not necessary condition. This condition may appear to be restrictive but our discrete event simulation shows that it can accurately predict the number of admission flows.

**Theorem 1** For a wireless mesh network with \( p \) point-to-multipoint flows \( F_1, \ldots, F_p \). Flow \( F_j \) has a load of \( L_j \) and whose forwarding tree is \( T_j \). Let \( N\mathcal{L}(T_j) \) denote the set of non-leaf nodes for tree \( T_j \). A sufficient condition for the flows \( F_1, \ldots, F_p \) to be feasible is

\[
TTF(\tau(v_i, F_j)) + 
\sum_{\tau(v_i, F_j) \in \text{Inter}(\tau(v_i, F_j))} TTF(\tau(v_i, F_j)) \leq 1
\]

for all \( v_i \in N\mathcal{L}(T_j) \) and for all \( F_j \). In other words, the sum of TTF of a node and its neighbors

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\(^3\)Given a graph \((V, E)\) where \( V \) is the set of nodes and \( E \) is the set of edges. An independent set \( I \) is a subset of \( V \) such that no two elements in \( I \) are connected by an edge.
in the conflict graph is no more than 1.

This theorem is a generalisation of Theorem 1 in [16] to the case of multi-rate multicast transmissions. The proof is similar to that in [16] and instead of providing a proof, we will discuss the insight behind the proof. The key idea behind the proof is to show that the vertex colouring problem of a corresponding graph \( \tilde{G} \) can be solved provided that equation 6 holds. In fact, equation 6 implies that the number of colours available in the vertex colouring problem for \( \tilde{G} \) is no less than the maximum node degree of \( \tilde{G} \) plus one. Since the number of colours required for vertex colouring is upper bounded by the maximum node degree plus one, the vertex colouring problem can be solved.

An important consequence of our analysis is that determination of the true feasibility of a particular flow transmission requires maintenance of flow-specific state (knowledge of the conflict graph for each distinct separate transmission). In the next section, we shall see how this complicates the formation of a broadcast tree, by requiring each node to essentially maintain awareness of each distinct transmission that has been already scheduled within its interference region. Subsequently, in Section 5, we shall develop a less accurate node-centric feasibility metric for the case of multicast flows.

4 Heuristic Broadcast Algorithms

We first present the generic principle for the formation of a broadcast tree for a newly incoming flow. We assume that there are \( j - 1 \) \((j \geq 1)\) broadcast trees \( \{T_1, \ldots, T_{j-1}\} \) already defined for the \( \{F_1, \ldots, F_{j-1}\} \) flows in the network and describe the process of constructing the tree \( T_j \) for flow \( F_j \). The broadcast tree formulation is top-down – i.e., we start from the source and selectively add new nodes to the broadcast tree.

The objective of the algorithms is to create efficient delivery trees in order to achieve the maximal broadcast capacity, where we define the broadcast capacity as the total amount of
network load (cumulatively over multiple flows) that can be feasibly admitted into the WMN. As
the load for all the flows are $L_1, L_2, L_3, \ldots, L_j, \ldots$, the metric for evaluating the ‘goodness’ of
an algorithm is given by:

$$\sum_j L_j \quad j \in \{1, 2, 3, \ldots J\},$$

(7)

where $F_j$ is the last flow to be feasibly admitted (satisfies the constraints of Equation 6 at all
forwarding nodes) and $F_{J+1}$ cannot be feasibly admitted.

We defer for now the question of selection metric, i.e., the question of how to pick the next
tree node given an existing set of nodes for the partial tree $T_j$. Rather, we first demonstrate the
process of verifying whether a new node (transmitting at a specific rate to a set of child nodes)
is feasible. Our philosophy is thus to incrementally build a top-down broadcast tree $T_j$ that is
feasible at all times, avoiding the addition of any transmission $\tau$ that violates Equations 6.

Let us assume that a number of nodes have been selected as transmitting nodes for flow $F_j$ in previous tree construction steps. This means for a selected node $v\nu^\prime$, the transmission rate $\rho(v\nu^\prime, F_j)$ and the downstream neighbors $N(v\nu^\prime, F_j)$ for transmission $\tau(v\nu^\prime, F_j)$ have been
determined. We are now trying to determine if node $v_i$ can be selected as next transmitting node,
i.e., if $\tau(v_i, F_j)$ with a transmission rate $\rho(v_i, F_j)$ and downstream neighbor $N(v_i, F_j)$ can be
permitted. To verify this process, we consider all the possible transmissions of $\tau(v_i, F_j)$ with
transmission rates $\rho_1, \ldots, \rho_k$. For any $\rho_l$, $l = \{1, \ldots, k\}$ to be feasible, it is essential that the
corresponding airtime constraint for $\tau(v_i, F_j)$ be satisfied, i.e.,

$$\frac{L_j}{\rho_l} + \sum_{\tau(v_i, F_j) \in \text{Inter}(\tau(v_i, F_j))} \text{TTF}(\tau(v_i, F_j)) \leq 1$$

(8)

Given our desire to try to ‘pack’ as many flows into the WMN, it is natural to prefer nodes
where the residual airtime fraction is higher (nodes whose neighborhood is less busy). Accord-
ingly, we define the metric residual transmission time fraction (RTTF) for rate $\rho_l$ associated with
transmission $\tau(v_i, F_j)$ for flow $F_j$ at node $v_i$ as:

$$RTTF(\tau(v_i, F_j) | \rho_l) = 1 - \frac{L_j}{\rho_l} - \sum_{\tau(v_i, F_j) \in \text{Inter}(	au(v_i, F_j))} TTF(\tau(v_i, F_j))$$  \hspace{1cm} (9)

Note that, as before, the computation of $RTTF(\tau(v_i, F_j) | \rho_l)$ is dependent on not just the choice of the node $v_i$, but also the associate rate $\rho_l$ (as $\text{Inter}(	au(v_i, F_j))$ depends on $\rho_l$). It is also worth to note the difference between $\tau(v_i, F_j)$ in Equation 9 and $\tau$ in Equation 6. $\tau$ in Equation 6 is a fixed transmission and hence its rate and downstream neighbors have been determined. In contrast, $\tau(v_i, F_j)$ in Equation 9 is not fixed and we are in the process of determining if it is feasible on node $v_i$ with a possible rate $\rho_l$. For feasibility of the candidate transmission $\tau(v_i, F_j)$, we need to check that $RTTF(\tau(v_i, F_j) | \rho_l) \geq 0$. Moreover, when selecting among alternative nodes for possible inclusion in the tree, we should clearly prefer “less congested nodes”, i.e., nodes with higher $RTTF(\tau(v_i, F_j) | \rho_l)$. Clearly, the $RTTF(\tau(v_i, F_j) | \rho_l)$ computed in Equation (9) is a worst-case value. To see this, suppose that two transmissions $\tau(v_{i'}, F_{j'})$ and $\tau(v_{i''}, F_{j''})$ both interfere with $\tau(v_i, F_j)$, but not with each other and can thus happen concurrently. $RTTF(\tau(v_i, F_j) | \rho_l)$ however assumes that none of its interfering transmissions can occur in parallel with one another.

### 4.1 Heuristic Metrics for Broadcast Tree Formation

Given our goal of maximizing the amount of admitted broadcast load, we should try to reduce the consumption of airtime by individual transmissions. In general, we thus want that (a) each transmission $\tau(v_i, F_j)$ by $v_i$ uses as high a transmission rate as possible, and (b) the number of transmissions required to complete broadcast or multicast be minimized. Clearly, these two desires are mutually conflicting, since a faster rate implies a smaller coverage area, and consequently a larger number of individual transmissions.

We now present six feasible metrics for computing the tree $T_j$ based on the notion of a connected dominating set (CDS). Recall that for a graph $G(V, E)$, a CDS $Z$ of $G$ is a subset of
V such that (1) Every element (node) of $V \setminus Z$ is in the neighborhood of at least one node in $Z$; (2) The set $Z$ is connected. Among all the CDSs of $G$, constructing the one with the minimum cardinality (the minimum connected dominating set or MCDS) is known to be an NP-hard problem for a unit disk [13]. In this paper, we extend the WCDS (Weighted CDS) algorithm presented in [11] for constructing an MCDS-based broadcast tree in a multi-rate WMN. Note that WCDS itself was suggested purely for an individual flow, does not consider the effects of inter and intra-flow interference, and does not attempt to maximize the total amount of admitted broadcast traffic. Our heuristic algorithms start by making the source node $s$ for $F_j$ eligible to transmit, and setting $Z$ (denoting the set of covered nodes) to $\{s\}$. We say that a node is ‘covered’ if it is within the transmission range of a node $v \in Z$, given $v$’s current link rate. In each round of the algorithm, we choose the $\tau(v_i, F_j)$ combination for a node $v_i \in Z$ that maximizes some objective function $f(\tau(v_i, F_j))$ (and, of course, does not violate the constraints of Equations 6). Algorithm 1 illustrates the overall steps for all the algorithms, with the computation of $f(\tau(v_i, F_j))$ being the sole point of difference among the six heuristics. In all cases, the tree formation process may terminate at an intermediate point if no additional feasible transmission is found. In such a case, we reject the admission of incoming flow $F_j$.

We evaluate six different algorithms, with each has a different cost function. The first algorithm, called the Weighted Coverage Maximization Algorithm (WCMA), calculates the cost of a candidate transmission $\tau(v_i, F_j)$ as follows:

$$f_{WCMA}(\tau(v_i, F_j)) = |N(v_i, F_j)| \times \rho(v_i, F_j) \quad (10)$$

This is identical to the WCDS metric in [11], except for the additional step of verifying that the chosen rate satisfies the feasibility constraints.

The second algorithm considers only the effect of interference on a single transmission. The transmission rate is fixed with the lowest rate (e.g., 6 Mbps for IEEE 802.11a radio). This
Input: $G(V,E)$, $s$ – source node for the given flow $F_j$, 
$R = \{ \rho_1, \rho_2, \rho_3, \ldots, \rho_k \}$.
Output: The broadcast tree $T$ for the given flow $F_j$.
$Z = \{ s \}, T = \emptyset$ ;
while ($V \setminus Z \neq \emptyset$) do
  candidate = $\emptyset$ ;
  for $v_i \in Z$ do
    for each possible $\rho(v_i, F_j) \in R$ do
      $\rho = \rho(v_i, F_j)$ ;
      Compute $f(\tau(v_i, F_j))$ ; /* Different $f(\cdot)$ for WCMA, MRA, WMRA and RCA */
      if $\tau(v_i, F_j)$ is feasible for rate $\rho$ ; then
        candidate = candidate $\cup$ $\tau(v_i, F_j)$
      end
    end
  end
  $\hat{\tau}(v_i, F_j) = \text{argmax}_{\tau(v_i, F_j) \in \text{candidate}} f(\tau(v_i, F_j))$ ;
  if $\hat{\tau}(v_i, F_j) = \emptyset$ (no feasible transmission found) then
    return $\{ T = \emptyset \}$ ; /* The flow cannot be admitted */
  else
    Select $\hat{\tau}(v_i, F_j) = \{ \hat{\rho}(v_i, F_j), N(v_i, F_j) \}$ as next transmission for flow $F_j$
    $Z \leftarrow Z \cup N(v_i, F_j)$ ;
    $T \leftarrow T \cup (\cup_{a \in N(v_i, F_j)} \{ (i, a) \})$;
  end
end

Algorithm 1: The Broadcast Tree Formation Process

The algorithm is called the Maximum RTTF Algorithm (MRA) and tries to select the transmission that results in the maximum residual airtime. Accordingly, the cost of a candidate transmission $\tau(v_i, F_j)$ is given by:

$$f_{\text{MRA}}(\tau(v_i, F_j)) = \text{RTTF}(\tau(v_i, F_j)|\rho(v_i, F_j))$$  \hspace{1cm} (11)

By selecting the (node, rate) combination with the largest $\text{RTTF}$ value, this heuristic tries to maximize the residual airtime, with the expectation that this will eventually allow more future transmissions to be admitted.

The third algorithm considers only the maximal coverage of a transmission. The transmission rate is fixed with the lowest rate. This algorithm is called Maximum Coverage Algorithm (MCA)
and tries to select the transmission that results in the maximum number of uncovered neighbors. The cost function of a candidate transmission $\tau(v_i, F_j)$ is given by:

$$f_{MCA}(\tau(v_i, F_j)) = |N(v_i, F_j)|$$ (12)

The fourth algorithm, called the Weighted Maximum RTTF Algorithm (WMRA) balances the desire to select the transmission with the maximum residual airtime and higher transmission rate. The cost function of a candidate transmission $\tau(v_i, F_j)$ is thus computed as:

$$f_{WMRA}(\tau(v_i, F_j)) = \rho(v_i, F_j) \times RTTF(\tau(v_i, F_j)|\rho(v_i, F_j))$$ (13)

Note that in WMRA, the rate selected must cover at least one uncovered neighbor.

The fifth algorithm, called Weighted Maximum Coverage Algorithm (WMCA) balances the desire to select the transmission with the maximum residual airtime and higher number of uncovered neighbors. The cost function of a candidate transmission $\tau(v_i, F_j)$ is given by:

$$f_{WMCA}(\tau(v_i, F_j)) = |N(v_i, F_j)| \times RTTF(\tau(v_i, F_j)|\rho(v_i, F_j))$$ (14)

Finally, the sixth algorithm, called the RTTF-Aware Coverage Algorithm (RCA), computes the cost of a candidate transmission $\tau(v_i, F_j)$ as follows:

$$f_{RCA}(\tau(v_i, F_j)) = |N(v_i, F_j)| \times \rho(v_i, F_j) \times RTTF(\tau(v_i, F_j)|\rho(v_i, F_j))$$ (15)

Intuitively, the RCA algorithm tries to balance the competing objectives of interference minimization (favoring nodes with larger RTTF), link rate maximization (to reduce broadcast latency), and coverage of currently uncovered nodes (favoring transmissions that cover more nodes in the broadcast tree).
4.2 Idealized Performance Results for Broadcast Heuristics

We first present the results of Matlab-based simulations (essentially assuming an ideal MAC layer) to understand the behavior of the various tree formation heuristics. We use the parameters given in Table 1 and $\kappa = 1.7$ in our study. The transmission rates and minimum sensitivities shown in Table 1 are reproduced from IEEE 802.11a specifications [19]. The transmission range for each given rate is derived from Qualnet [23] with two-ray ground propagation model and fixed transmission power of $16dBm^4$. The results presented in this subsection correspond to means computed over 50 uniformly randomly generated network topologies. All network topologies are connected where the connectivity has been tested against the lowest transmission rate. Each network topology covers an area of $1km^2$.

<table>
<thead>
<tr>
<th>Transmission Rate (Mbps)</th>
<th>Minimum Sensitivity (dBm)</th>
<th>Transmission Range (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>-82</td>
<td>170.62</td>
</tr>
<tr>
<td>9</td>
<td>-81</td>
<td>152.07</td>
</tr>
<tr>
<td>12</td>
<td>-79</td>
<td>120.79</td>
</tr>
<tr>
<td>18</td>
<td>-77</td>
<td>95.95</td>
</tr>
<tr>
<td>24</td>
<td>-74</td>
<td>67.93</td>
</tr>
<tr>
<td>36</td>
<td>-70</td>
<td>42.86</td>
</tr>
<tr>
<td>48</td>
<td>-66</td>
<td>27.04</td>
</tr>
<tr>
<td>54</td>
<td>-65</td>
<td>24.10</td>
</tr>
</tbody>
</table>

The study is conducted with all flows having identical load $L = 0.1Mbps$. The source node of each flow has been selected randomly. The network throughput is calculated as the product of the number of flows feasibly admitted and $L$. Figure 3 shows the comparative results of the six heuristic broadcast algorithms with 95% of confidence interval (All figures are plotted with 95% of confidence interval in the rest of the paper). Clearly, the throughput achieved by RCA outperforms the others. On average, RCA achieves 25.2%, 114.8%, 45.6%, 20.8%, 45.8% of improvement than that of WCMA, MRA, MCA, WMRA, and WMCA. This demonstrates that the broadcast capacity can be enhanced by choosing transmissions that balance the need for high

\[^4\text{This is equivalent to } 40mW\text{, which is the standard maximum power for 5.15-5.25 GHz band [19].}\]
link rates with greater per-transmission node coverage and low channel contention (as measured by the \( RTTF \) metric).

![Figure 3: Total admissible traffic with rate diversity.](image)

4.3 Simulations with IEEE 802.11a

Former Matlab-based studies assume idealized MAC where media contentions can be resolved perfectly. Moreover, the traffic load is modeled as a fluid arrival process. In a real WMN, messages are transmitted in packets. While network throughput is a critical measure for the performance of an algorithm, it makes no sense if the packet delivery ratio is too low. In order to study our proposed algorithms in a more realistic wireless environment, we fed the computed trees (with rate-diversity) into the discrete event simulator Qualnet [23] and observe their performance with IEEE 802.11a radio.

In order to compare the performance of different algorithms, we use a different approach in the Qualnet simulation. We continuously increase the number of flows packed in the network even when Equation 8 exceeds 1 for a particular transmission. Note that in a real WMN, the offered load may exceed the broadcast capacity as constrained by Equation 8. The number of
flows packed into network spans from the smallest 1 flow to the largest 15 flows. With this approach, we can observe and compare the deterioration trends of all the algorithms. Each simulation run is conducted for the period of 100 seconds. The 50 network topologies generated in last section have been used with each network topology has 150 nodes. Three performance metrics—packet delivery ratio, broadcast latency, and network throughput—have been measured and we calculate both the average and worst case values. We further denote $N$ as the number of nodes in network and $M$ the number of flows packed in the network. The performance metrics names, mathematical notations, and explanations have been illustrated in Table 2.

Figure 4 illustrates the packet delivery ratio comparison for both average and worst case scenarios. From Figure 4 (a), we can see that RCA and WCMA achieve similar performance and outperform the other algorithms. For example, the delivery ratio of RCA and WCMA falls over 90% when the number of flows reaches 12, while the delivery ratio of WMCA and MCA drop to 90% when the number of flows is 10. WMRA and MRA performs even worse, where WMRA falls over 90% with 8 flows and MRA drops close to 90% with only 6 flows. This demonstrates that RCA and WCMA can accommodate more traffic compare to other algorithms.
Table 2: Performance Metrics Measured in Qualnet Simulation.

<table>
<thead>
<tr>
<th>Metric Name</th>
<th>Math Notation</th>
<th>Notes and Explanations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average packet delivery ratio</td>
<td>$\frac{\sum_{i=1}^{N} \sum_{j=1}^{M} pdr(v_i, F_j)}{\sum_{i=1}^{N} \sum_{j=1}^{M} I(v_i, F_j)}$</td>
<td>$pdr(v_i, F_j)$ represents the packet delivery ratio on node $v_i$ for flow $F_j$; $I(v_i, F_j) = 1$ if node $v_i$ receives messages for flow $F_j$, $I(v_i, F_j) = 0$ otherwise. $pdr(v_i, F_j)$ is calculated as the ratio between the number of packets received by node $v_i$ for flow $F_j$ and the total number of packets generated by the source node of flow $F_j$.</td>
</tr>
<tr>
<td>Worst packet delivery ratio</td>
<td>$\min(pdr(v_i, F_j)), \forall i \in {1, \ldots, N}, \forall j \in {1, \ldots, M}$</td>
<td>The worst packet delivery ratio equals the worst value of packet delivery ratio for a particular flow on a particular node.</td>
</tr>
<tr>
<td>Average network throughput</td>
<td>$\frac{\sum_{i=1}^{N} \sum_{j=1}^{M} thru(v_i, F_j)}{N}$</td>
<td>$thru(v_i, F_j)$ represents the network throughput on node $v_i$ for flow $F_j$. Note that the denominator equals the number of nodes in the network. The equation in fact gives the per node network throughput with multiple flows packed in the network.</td>
</tr>
<tr>
<td>Worst network throughput</td>
<td>$\min(thru(v_i, F_j)), \forall i \in {1, \ldots, N}, \forall j \in {1, \ldots, M}$</td>
<td>The worst network throughput equals the worst value for a particular flow on a particular node.</td>
</tr>
<tr>
<td>Average broadcast latency</td>
<td>$\frac{\sum_{j=1}^{M} lat(F_j)}{M}$</td>
<td>$lat(F_j)$ denotes the broadcast latency for flow $F_j$. The broadcast latency for a given flow $F_j$ is defined as the largest average end-to-end delay experienced by a receiving node of flow $F_j$. Note that a node may receive multiple packets for a flow. Qualnet calculates an average value for a given flow on each node. We choose the largest average value as the broadcast latency for the given flow.</td>
</tr>
<tr>
<td>Worst broadcast latency</td>
<td>$\max(lat(F_j)), \forall j \in {1, \ldots, M}$</td>
<td>The worst broadcast latency equals the worst value of the broadcast latency among all flows.</td>
</tr>
</tbody>
</table>

with a reasonable packet delivery ratio (e.g., 90%). We also observe that all the measured packet delivery ratios are less than 1 even when there is only one flow in the network. This demonstrates that the reliability of multihop broadcast cannot be guaranteed since hidden node problem can cause packet collisions. Collided packets may be damaged and hence to be discarded. Moreover, discarded packet cannot be re-covered because broadcast does not have an acknowledgement scheme. If a packet is lost in an intermediate transmitting node, all the downstream nodes will
not be able to receive this packet. This problem is illustrated more severely in Figure 4 (b), where we observe that the worst performance of packet delivery ratio can be very poor (e.g., less than 10%), especially when the number of flows increases. Increased number of flows causes significant increases in media contention, which in turn causes significant increases in packet collisions and packet loss. How to improve the reliability of multihop broadcast and provide resilient delivery trees is a challenging problem and we leave it for our future work.

Figure 5 illustrates the network throughput comparison for both average and worst case scenarios. Clearly, RCA and WCMA outperform the other algorithms. For example, RCA and WCMA saturates at around 1.1 Mbps when the number of flows is 15. WMCA and MCA saturate slightly below 1 Mbps. WMRA and MRA saturate even lower than 0.8 Mbps. If we compare Figure 5 (a) and Figure 3, we can see that they do not match with each other. For example, WMRA performs the second best in Figure 3 but unfortunately the second worst in Figure 5 (a). WCMA performs the third in Figure 3 but the best in Figure 5 (a) (WCMA achieves a slightly higher performance compare to RCA). The reason behind these discrepancies lies in the fact that Figure 3 is derived from the assumption that MAC layer is ideal, where Figure 5
(a) derived from a real MAC where packets may be lost. The network throughput as shown in Figure 5 (a) is in fact directly related to the packet delivery ratio as shown in Figure 4 (a). It is easy to observe that algorithm with higher packet delivery ratio can achieve higher network throughput. We also observe that the worst network throughput shown in Figure 5 (b) can be very low (e.g., less than 10 Kbps, remember that the load of each flow is 0.1 Mbps) due to the very poor packet delivery ratio.

Figure 6 illustrates the broadcast latency comparison for both average and worst case scenarios. An interesting observation is that RCA and WCMA still outperform the other algorithms although they in fact admitted more traffic (e.g., higher network throughput) compare to the other algorithms.

![Figure 6: Broadcast latency comparison](image)

We now conduct an in-depth analysis of the Qualnet simulation results for different algorithms. Recall that we have three selection metrics in the algorithm design: 1) Wireless broadcast advantage – expressed as the un-covered downstream neighbors – denotes as \( N \) (refer Equation 1); 2) Residual transmission time fraction – denotes as \( RTTF \); 3) Transmission rate – denotes as \( \rho \). If we do a broad classification of the performance of the packet delivery ratio and
network throughput, we can see that RCA \((N \times \rho \times RTTF)\) and WCMA \((N \times \rho)\) belong to the first category that achieve the best performance. MCA \((N)\) and WMCA \((N \times RTTF)\) belong to the second category that achieve lower performance compare to the RCA and WCMA. MRA \((RTTF)\) and WMRA \((RTTF \times \rho)\) belong to the third category that achieves the worst performance. From this classification, we can derive several inferences for broadcast traffic with a real MAC:

1. Wireless broadcast advantage has the most significant impact on the network throughput and packet delivery ratio.

2. The use of RTTF in the node selection criterion enables better prediction of the number of admissible flows.

3. Too low or too high transmission rate may have negative impact on the network throughput and packet delivery ratio.

The first inference can be derived that all algorithms that consider wireless broadcast advantage \((N)\) in their cost function outperform the algorithms that do not consider it, e.g., RCA, WCMA, MCA, WMCA outperform MRA and WMRA.

Regarding the second inference, consider the RCA and WCMA algorithms. RCA algorithm uses RTTF in choosing the forwarding nodes while WCMA does not. From Figure 3, we see that RCA and WCMA predict, respectively, that only 8 and 6 multicast flows can be admitted into the network. However, we find in Figures 4-6 that both algorithms behave more similarly in discrete event simulation. In fact it can be seen from these figures that RCA gives a much better prediction in the number of admissible flows than WCMA. We therefore conclude that using RTTF in the node selection criterion enables the better prediction of the number of admissible flows.

The third inference can be derived from the performance achieved by all the algorithms. For MRA, MCA, and WMCA, the transmission rate is fixed to the lowest rate. We observe that MRA
achieves the worst performance compare to all the others. This in fact has been demonstrated from both Matlab simulation (Figure 3), where an ideal MAC is assumed; and Qualnet simulation (Figure 4 and Figure 5), where a real MAC is assumed. Although MCA and WMCA outperform MRA, this is because MCA and WMCA consider wireless broadcast advantage, which has the most impact on the network performance. But we observe that MCA and WMCA achieve worse performance compare to RCA and WCMA, where both RCA and WCMA consider the transmission rate in their cost function and tend to use higher rate if possible. This demonstrates that lower transmission rate normally achieves poor network performance because lower rate normally implies higher airtime. The final result is less number of transmissions can be admitted in a potential interference vicinity. On the other hand, WMRA considers transmission rate and biases towards selecting fast-rate transmissions which cover less nodes and consumes less airtime. The end result is a lot of fast-rate but short-range transmissions. But it is precisely the increased number of transmissions that lead to significant increases in media contention (Note that although the transmission range of a higher rate is smaller compare to the transmission range of a lower rate, the interference range is same) which leads to poor network performance in a real MAC. We see that although WMRA achieves the second best in Figure 3, it achieves the second worst in Figure 5 (a). Note that the transmission rate $\rho$ and $RTTF$ do not have direct relationship so that WMRA ($RTTF \times \rho$) tends to select the rate as high as possible presume that the rate selected can cover at least one un-covered neighbor. Also note that this is not true for transmission rate $\rho$ and wireless broadcast advantage $N$, where $\rho$ and $N$ do have direct relationship since a higher (lower) $\rho$ normally implies a smaller (larger) $N$. The performance of WMRA demonstrates that higher rate may also induce poor network performance with a real MAC. We will have further discussions on the rate diversity when we discuss the broadcast capacity in section 4.4.
4.4 The Effect of Transmission Rates on Broadcast Capacity

Since our objective is to maximize the broadcast (and multicast) capacity by exploiting link-rate diversity, it is nature to pursue the relationship between the transmission rate and broadcast capacity. The aim of this section is to study how the network broadcast capacity depends on the choice of transmission rate. This question does not appear to have a trivial answer because of the trade off between transmission time and transmission coverage area. Although a higher rate transmission takes a shorter time, its coverage area is smaller. This means that it will take more higher rate transmissions to cover the same physical area and higher number of transmissions can also mean more contention to the channel.

In order to answer the above question, we used the rate-diversity aware routing algorithms that we have proposed earlier but instead of using multiple transmission rates, each node is restricted to transmit at a single link-layer transmission rate. In our simulation, we used three broadcast algorithms: WCMA, MRA and RCA. For each algorithm, we carried out three sets of simulations where the nodes in each set of simulations used only one single rate. Three different rates were used: 6 Mbps, 9 Mbps and 12 Mbps. Note that network partitioning prevented us from using higher transmission rates. Note also that although reducing the network area can result in a connected network for higher transmission rates, the reduced network area becomes unsuitable for lower rate transmissions because its bigger transmission range means most nodes are covered in a one-hop transmission.

Figure 7 shows the comparative performance of the three algorithms with different transmission rates. The results were obtained from the average of 50 random topologies with 150 nodes in each network. The figure shows that 9Mbps results in the highest network broadcast capacity. This shows that a higher link-layer transmission rate does not necessarily lead to a higher broadcast capacity. It was shown in [18, Theorem 8] that for the broadcast capacity $c$ of a multi-hop wireless network (in a $d$-dimensional cube) whose nodes use a single-link layer transmission rate
Figure 7: Total admissible traffic with a single link-layer transmission rate.

$\rho$ is bounded between:

\[
\frac{c_1 \rho}{\max(1, \Delta^d)} \leq c \leq \frac{c_2 \rho}{\max(1, \Delta^d)}
\]  

(16)

where $c_1$ and $c_2$ are constants independent of the network parameters, and

\[
\Delta = \frac{d_i - d_r(\rho)}{d_r(\rho)}
\]  

(17)

where $d_i$ is the interference range and $d_r(\rho)$ is the transmission range for transmission rate $\rho$.

This shows that in a 2-dimensional cube, the broadcast capacity varies with transmission rate according to

\[
c \sim \frac{\rho}{\max(1, (\frac{d_i - d_r(\rho)}{d_r(\rho)})^2)}
\]  

(18)

By using the transmission rate and transmission range values given in Table 1 and $\kappa = 1.7$, Figure 8 shows how broadcast capacity (18) varies with transmission rates. It shows that the broadcast capacity is highest when the 9Mbps transmission rate is being used. Our simulation
Figure 8: The figure shows how broadcast capacity varies with transmission rate.

results therefore agrees with the prediction derived from [18]. Our study therefore shows that a higher link-layer transmission rate does not necessarily translate to a higher broadcast capacity. If a single link-layer transmission rate is to be used to achieve high broadcast capacity, then equation (16) can be used to predict the best rate to be used. Note that although [18] presented formula (16), the question of how the trade-off between transmission rate and transmission range (which rises from a constant transmission power assumption) affects the broadcast capacity was not discussed. In particular, note that our derivation assumes the same transmission power is used for all transmission rate.

The above discussion shows that some link-layer rates are better in realizing a high broadcast capacity than the others when a single link-layer rate is to be used. What about in the multi-rate case? Will a certain subset of link-layer rates be better in realizing high broadcast capacity? We study this question by performing simulations using three multi-rate broadcast algorithms WCMA, MRA and RCA. Instead of using all the transmission rate available in Table 1, we perform simulations where only rates from 6Mbps to \( x \) Mbps are used where \( x = 6, 9, 12, 18, 24, \ldots \). The results are plotted in Figure 9 where the \( x \)-axis indicates the highest transmission rate being used, e.g. \( x \) equals to 24 means the rates 6, 9, 12, 18 and 24 Mbps are available for the nodes
to use. Figure 9 shows that the inclusion of higher transmission rates increases the broadcast capacity (which is understandable) but the inclusion of some higher rates does not seem to improve the broadcast capacity much. It is interesting to note in Figure 9 that the inclusion of 9Mbps gives the biggest increase in broadcast capacity. This suggests that equation (16) can be used as a guideline to determine which rates are to be included in the multi-rate case.

We note that the effect of link-layer rates on broadcast latency (i.e. the time it takes a source node to reach all nodes) in a multi-hop wireless mesh network was studied in [11, 22]. They show that if a single link-layer rate is to be used, then a higher rate may not result in lower broadcast latency. In fact, they show that the rate-area product (i.e. the product of transmission rate and transmission coverage area) is a good rule-of-thumb in determining how effective a transmission rate is in reducing broadcast latency in a 2-dimensional network. Note that the rate-area product is in fact the first order approximation of the right-hand-side of equation (18) for $\Delta < 1$.
5 Rate and Contention Aware Multicast

We now consider the more practical problem of building similar routing trees for multicast flows. Unlike all earlier work on broadcasting, we aim to build a multicast tree that explicitly factors in three unique WMN features – (a) the ability of nodes to operate at different link rates; (b) the impact of interference on the available (bandwidth) capacity of a WMN node, and (c) the WBA. The key difficulty in extending the accurate interference-aware approach (embodied by the RTTF metric of Equation 9) to multicast flows is that the broadcast tree formation algorithms are greedy – i.e., they compute the tree starting at the source and greedily select add nodes to the tree, corresponding to the “best subsequent” transmission. In contrast, the multicast tree cannot be built greedily, since nodes should only be added if they extend the tree towards one of the receivers. (Most distance-vector algorithms, such as Dijkstra, cannot solely compute the shortest path to a specific destination node \( v_d \) from a source \( v_s \), but instead, reconstruct the shortest path by backward traversal after computing a larger set of shortest paths). While one approach for multicasting may thus be based on pruning (i.e., first create the broadcast tree, and then simply prune all unnecessary edges), this is likely to be unsatisfactory. In particular, by assuming that all neighboring nodes needed to receive a transmission, the broadcast tree formation process may have incorrectly excluded some (link, rate) combinations.

Accordingly, we have devised the Rate and Contention Aware Multicast Algorithm (RCAM) (mathematically outlined in Algorithm 2) with the following intuition. The multicast tree will be constructed incrementally taking into account the rate, time fraction usage, and WBA. We assumed that the set of \( Q \) multicast receivers \( \{mr_1, ..., mr_Q\} \) are known at the start of the tree formation process. In the first step, we find the least-cost unicast path from source \( s \) to any member, say \( mr_\alpha \), of the set of \( Q \) receiver nodes, assuming a link cost \( c(v_a, v_b) \) for any link \( (v_a, v_b) \). In general, the higher the rate for the edge \( (v_a, v_b) \), the smaller should be the link cost. However, to balance the link cost with the level of channel contention, \( c(v_a, v_b) \) needs to also
account for the amount of residual airtime in the neighborhood of \((v_a, v_b)\). The most accurate determination of this contention is given by the metric \(RTTF\) (see Equation 9), which however, depends on the precise receiver set for a specific transmission \(\tau()\). As this is not possible for multicast as the relevant downstream receivers are not known a-priori, we instead define a flow-independent metric \(Cumulative\ Transmission\ Time\ Fraction\ (CTTF)\) for a node \(v_i\) as:

\[
CTTF(v_i) = \sum_{i=1}^{j-1} \sum_{v_m \in V} \frac{L_i}{\rho(v_m, F_i)} I(v_i, v_m, F_i)
\]

(19)

where \(I(v_i, v_m, F_i)\) is an indicator function that equals 1 (otherwise 0) if: \((v_m\) is a transmitting node for tree \(T_i) \land (v_m\) or at least one of the receivers in \(N(v_m, F_i)\) is within the interference range of \(v_i)\). In other words, \(CTTF(v_i)\) defines the cumulative airtime usage (across all prior scheduled transmissions) in the interference range of \(v_i\).

To account for interference, the link cost \(c(v_a, v_b)\) is modified to be a function of both the link speed \(\rho(v_a, v_b)\) and the most critical airtime constraint in \(v_a\’s\) vicinity. Thus,

\[
c(v_a, v_b) = \frac{1}{\rho(v_a, v_b)} \times \frac{1}{1 - \max_{d(v_a, v_i) \leq \kappa \cdot d(v_1)} CTTF(v_i)}
\]

(20)

Moreover, if \(CTTF(v_i) + \frac{L_j}{\rho(v_a, v_b)} > 1\), then \(c(a, b)\) should equal \(\infty\) to reflect the fact that this link, although physically present, is unusable due to airtime constraints.

While such a formulation accounts for the rate diversity, RCAM also needs to account for the WBA. In particular, if a node \(v_a\) has already been chosen to as a forwarding node of the tree \(T_j\), it follow that any node \(v_b\) in the neighborhood of \(\tau(v_a, F_j) = \{\rho(v_a, F_j), N(v_a, F_j)\}\), i.e., \(v_b \in N(v_a, F_j)\), can receive the packet for free due to WBA. This is reflected by setting their cost \(c(v_a, v_b)\) to 0 (label 1 in Algorithm 2). After this adjustment, the RCAM algorithm proceeds iteratively by selecting the next receiver node having the least-cost unicast path among the re-
Input: $G(V, E)$, source node $s$, list of receivers $\{mr_1, mr_2, \ldots, mr_Q\}$, load $L$, cumulative transmission time fraction $\{CTTF(v)\}$

Output: The multicast tree $T$

$T = \emptyset$, $A = \{s\}$;

for $v \in V$ do
  $CTTF_{\text{max}}(v) = \max_{u: (v, u) \text{ interfere}} CTTF(u)$;
end

for $(a, b) \in E$ do
  maxCont = $\max(CTTF_{\text{max}}(a), CTTF_{\text{max}}(b))$;
  if $\text{maxCont} + \frac{L}{\rho(a, b)} < 1$ then
    $c(a, b) = 1/(\rho(a, b) \times (1 - \text{maxCont}))$
  else
    $c(a, b) = \infty$
  end
end

for $p = 1$ to $Q$ do
  /* $SP$ computes the shortest path from the set $A$ to $m_p$, using $c(a, b)$ as the cost function. */
  minpath = $SP(A, m_p, \{c(a, b)\})$;
  $T = T \cup \{(v, u) \in \text{minpath}\}$;
  for $(v, u) \in E : v \in T \&\& d(v, u) \leq d(p(v))$ do
    label 1: $c(v, u) = 0$; $A = A \cup u$;
  end
end

if $T$ is valid i.e., if $T$ does not violate airtime constraints then
  Return $T$;
else
  Return $\emptyset$; //No valid multicast tree found
end

Algorithm 2: RCAM Algorithm.

remaining receivers (e.g., selecting $mr_\beta$ next) and grafting this path onto the existing multicast tree (the set $A$ in Algorithm 2). To perform this grafting, RCAM selects the least-cost feasible path from the receiver to any member of $A$. Note that due to the inaccurate formulation of $CTTF(v_i)$, it is possible that the final computed tree $T_j$ may actually be infeasible (i.e., it may violate one of the constraints of Equations 6. Accordingly, RCAM performs a final feasibility check on the whole tree $T$; if it is found to be infeasible, the entire multicast flow is rejected.
5.1 Idealized Performance Results for Multicast Heuristics

We first used Matlab-based studies (assuming an idealized MAC) to compare the multicast capacity achieved by RCAM in a WMN with two alternative algorithms that do not consider interference effects: (a) The Pruning algorithm, where the broadcast tree is first constructed (using Equation 10) and all unnecessary nodes are subsequently pruned. (b) The conventional shortest path tree (SPT) algorithm, where the tree is formed by merging the shortest unicast path (with a link’s cost being the inverse of its transmission rate) from source to each individual multicast receiver.

Our primary metric of interest is the amount of multicast traffic that the algorithms can feasibly admit. To study the dependence of capacity on the number of multicast receivers per group ($Q$), $Q$ is chosen from the lowest 5 to the highest 30. For each flow, the source and receiver nodes are selected randomly from the set of WMN nodes. All flows have identical load $L = 0.1$Mbps. Simulations are performed with 50 network topologies where each network topology has 400 nodes uniformly randomly distributed on a $1.5\text{km} \times 1.5\text{km}$ area. Figure 10 illustrates the simulation results.

![Figure 10: Total admissible multicast traffic load.](image-url)
From Figure 10, we see that, by considering both link rate diversity and interference-induced contention, RCAM outperforms WCMA and SPT. More importantly, the performance gains for RCAM are much greater when the number of multicast receivers are sparse (83% improvement over SPT for $Q = 5$) compared to dense receiver sets (64% improvement over WCMA for $Q = 30$). Clearly, sparse multicast groups allow multicast trees to be routed around WMN ‘hotspots’ or bottlenecks, allowing more flows can be ‘packed’. As $Q$ increases, it is more likely that receivers for a newly arriving flow will be located in an existing ‘hotspot’, leaving little choice to the routing protocol. Finally, we see that the total network throughput (measured purely as the sum of the sender load, and not weighted by the number of receivers per flow) decreases as $Q$ increases, since a larger value of $Q$ implies a greater overall use of airtime resources per flow.

5.2 Simulations with IEEE 802.11a

Similar to the broadcast study, we conduct discrete event simulations via Qualnet for all the multicast algorithms and observe their performance. Similar parameters (e.g., packet delivery ratio, network throughput, multicast latency) have been measured according to the equations in Table 2 but only the multicast group member associated values have been used in the final average calculation. This means that the performance values for intermediate routing nodes (not belong to the multicast group member) have not been used in the calculation. The number of multicast receivers per group ($Q$) is set to 5 and we continuously increase the number of flows packed into the network. The number of flows spans from the smallest 1 flow to the maximum 25 flows. We then observe the deterioration trend of each algorithm.

Figure 11 illustrates the packet delivery ratio comparison for both average and worst case scenarios. Figure 11 (a) reveals that RCAM achieves much better packet delivery ratio compare to WCMA and SPT when more flows are ‘packed’ in the network. We can see that when the number of flows is less than 15, all of the three algorithms can maintain the delivery ratio above
90%. When the number of flows is over 15, both WCMA and SPT start to deteriorate significantly. This trend indicates that the offered load reaches the full network capacity for WCMA and SPT when the number of flows reaches 15. This result clearly mirrors our former Matlab simulation in Figure 10, where the maximum achievable network throughput of both WCMA and SPT is less than 1.5 Mbps. On the other hand, RCAM can still maintain delivery ratio above 90% even when the number of flows reaches 25. For similar reasons as broadcast scenario, none of the multicast algorithms can achieve 100% of packet delivery ratio. This has been demonstrated in both Figure 11 (a) and (b). We also observe that RCAM achieves better packet delivery ratio compare to WCMA and SPT in the worst case scenario.

Figure 12 illustrates the network throughput comparison for both average and worst case scenario. We observe that the network throughput for all three algorithms increases linearly when the number of flows is less than 15. But both WCMA and SPT start to saturate when the number of flows is over 15 while RCAM still shows a linear increasing trend. Clearly, these results also match the results as shown in Figure 10.

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5The network throughput as shown in Figure 10 is computed as the product of the average number of flows packed and the traffic load for each flow (e.g., 0.1 Mbps).
Figure 12: Multicast network throughput comparison

Perhaps the most significant performance difference is the multicast latency, which is illustrated in Figure 13. The multicast latency of WCMA and SPT increases sharply when the number of flows is over 15. Figure 13 (a) shows that the average multicast latency of WCMA and SPT is well above 10s when the number of flows is 25. This behavior reveals that when the offered load exceeds the network capacity, queueing delay will play a dominant role in the multicast latency. On the contrary, RCAM achieves a much better performance where the average multicast latency is only 0.053s and the worst multicast latency is only 0.087s (compare to WCMA and SPT’s 10s) when the number of flows is 25.

Similar to broadcast study, we conduct an in-depth analysis of the performance of different multicast algorithms. Unlike broadcast flow, each multicast flow is only associated with a small number of multicast group members, and the flow traffic are only required to be delivered to this small number of multicast group members (instead of all nodes in the network). By considering $CTTF$ (Note that $(1 − CTTF)$ used in Algorithm 2 has similar meaning as $RTTF$), RCAM achieves significant improvements in network performance compare to WCMA and SPT. This means that consideration of $CTTF$ can have significant impact on the network performance (as
Figure 13: Multicast latency comparison

illustrated from Figure 11 – Figure 13) for multicast traffic. By considering $CTTF$, RCAM can fairly distribute the traffic load across the whole WMN and allow the multicast tree to be routed around the ‘hotspots’ or bottlenecks of the WMN. The consequence is that less network congestions which leads to more flows can be admitted without significant performance degradation. On the contrary, WCMA and SPT cannot route around the network ‘hotspots’ or bottlenecks during the multicast tree formation process because they do not consider $CTTF$ (or more accurately, $(1 - CTTF)$). The resulted consequence is serious network congestions in the ‘hotspots’ or bottlenecks of WMN when the number of flows increases. Network congestion induces significant media contentions and packet collisions, which in turn causes significant packet loss and increased packet latency.

5.3 The Effect of Transmission Rates on Multicast Capacity

Similar to study carried out in Section 4.4, in this section, we study how the choice of link-layer transmission rate impacts on multicast capacity. Figure 14 illustrates the comparative results of the three algorithms with different combinations of transmission rate and number of multicast
receivers per group. Transmission rate that is higher than 12Mbps cannot be used since the network connectivity cannot be guaranteed. Figure 14 shows that algorithms with fixed transmission rate of 9 Mbps achieve the highest performance, which is very close to the results as shown in Figure 10. But it is clear that our proposed heuristic RCAM algorithm outperforms the peak performance by exploiting the rate-diversity. An interesting observation of Figure 14 is that the algorithms with fixed transmission rate of 12 Mbps achieves worse performance than that of 9 Mbps. This means that the employment of higher rate may have negative impact on the network throughput. This result matches our former studies for broadcast capacity in Section 4.4.

6 Concluding Remarks and Future Work

We have demonstrated that the combined consideration of link-rate diversity and channel interference can significantly increase the amount of broadcast/multicast traffic load that may be feasibly admitted and routed within a WMN. For network-wide broadcast traffic, the RCA heuristic algorithm provides up to 78% of improvement in the total broadcast capacity (total feasible load) by choosing transmissions that balance between high link rates, greater node coverage and low
channel contention. For multicast flows, the RCAM algorithm is able to significantly enhance the amount of admissible multicast traffic on a WMN by exploiting both the transmission rate and the available (contention-free) airtime at individual nodes. In particular, results demonstrating large (e.g., 59% for $Q = 5$) capacity gains for relatively sparse multicast groups are of great practical significance to many real-life applications (e.g., games, video-conferencing). In addition, our discrete events simulations with 802.11a radio via Qualnet shows decent “matching results” compare to the idealized studies.

While these are fundamental results, we are currently working to develop more practical, *distributed tree formation* algorithms based on the heuristics presented in this paper. The design of rate-diversity aware multicast tree formation algorithms for multi-radio, multi-channel wireless mesh nodes remains an open question for future research.

**References**


