Low Latency Broadcast in Multi-Rate Wireless Mesh Networks

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Abstract

In a multi-rate wireless network, a node can dynamically adjust its link transmission rate by switching between different modulation schemes. For the current IEEE802.11a/b/g standards, this rate adjustment is limited to unicast traffic. In this paper, we consider a novel type of multi-rate mesh networks where a node can dynamically adjust its link layer multicast rates to its neighbours. In particular, we consider the problem of realising low latency network-wide broadcast in this type of multi-rate wireless meshes. We will first show that the multi-rate broadcast problem is significantly different from the single-rate case. We will then present two algorithms for achieving low latency broadcast in a multi-rate mesh which exploits both wireless broadcast advantage and the multi-rate nature of the network. Simulations based on current 802.11 parameters show that multi-rate multicast can reduce broadcast latency by 3–6 times compared with using the lowest rate alone. In addition, we show the significance of the product of transmission rate and transmission coverage area in designing multi-rate wireless mesh networks for broadcast.

1 Introduction

Wireless mesh networks increasingly appear to be a cheap and easily deployable extended-area access technology in a variety of environments. In suburban and urban community-based networking scenarios, the nodes in the wireless mesh often act as both *relays*, forwarding traffic to or from other mesh nodes, and *access points* providing localized first-hop connectivity to mobile or pervasive wireless devices, such as laptops and PDAs. In fact, one popular use of the wireless mesh is to extend the benefits of wide-area connectivity to a larger community, by using the multi-hop wireless mesh to funnel traffic from an extended area to a much smaller set of gateway nodes, that connect to the Internet backbone over a wired access medium (such as DSL/cable modem

Given the extensive body of research of mobile, multi-hop wireless networks (MANETs), there exists an understandable impression that wireless meshes are merely an embodiment of the MANET paradigm, with many MANET techniques and approaches directly applicable to wireless meshes. We believe that such a view is simplistic, and that wireless meshes introduce many fundamentally new challenges, at both the protocol and architectural level. Two aspects of wireless mesh research seem to be especially popular at present:

a) Use of Multi-Channel, Multi-Radio Mesh Nodes: Recent research demonstrates that the use of multiple radios on a single node, each tuned to possibly distinct channels, can significantly improve the spatial reuse of an individual channel, and result in higher overall capacity, by increasing the degree of concurrent transmissions in the network.

b) Multi-rate MAC Protocols: Researchers are beginning to move away from 802.11-based single-rate MACs for wireless meshes, and studying the throughput and fairness issues that arise from multi-rate MAC protocols, where adaptive modulation is used to dynamically modify the data rate on a particular link in response to the perceived signal-to-noise ratio (SNR).

Such advances should solve one of the fundamental problems of existing MANET-oriented networks, namely the sharp drop in multi-hop throughput to a few Kbps, even though individual wireless links are operated at progressively higher speeds (such as 54Mbps or 108Mbps). Most research on both these aspects have, however, focused on the *unicast* traffic scenario, where each traffic flow is defined between a particular node pair. For example, [9] demonstrates the use of new unicast routing metrics for multi-channel multi-rate mesh environments, to account for both the intra-flow contention and the differential transmission rate on different links.

In this paper, we introduce the problem of efficient routing and packet distribution for *broadcast* (or, in general, multicast) traffic flows in such multi-rate, multi-channel, multi-radio wireless meshes. Our primary goal will be to show that the presence of multiple radios, or multi-rate adaptive modulation schemes, opens up new possibilities for broadcast traffic distribution that do not seem to have been explored before. Indeed, metrics and routing strategies defined for unicast traffic scenarios do not capture the interesting effects that broadcast traffic introduce in a wireless mesh. We believe that our focus on developing algorithms for high-throughput, low-latency forwarding of multicast traffic is important for many practical wireless mesh applications. For example, wireless meshes may be used to broadcast community-specific content (such as a video feed of a neighborhood soccer game or video feeds from multiple video sensors) or even wide-area content (such as TV feeds received at a particular gateway node) to a group of receiver nodes. While routing algorithms for multicasting traffic have been well studied in multi-hop wireless networks, their focus has been largely limited to the efficient dissemination of control traffic, rather than the support of high bit-rate "data". For example, routing techniques (e.g., [7]) to avoid the broadcast storm problem [21] were motivated by a desire to limit the impact of route-discovery packets broadcast by many popular proactive ad-hoc routing protocols. There appears to be little work on the impact of such multicast techniques on the achievable throughput or latency bounds. For our target applications, such as broadcasting camera feeds or providing peer-to-peer multiplayer games over wireless meshes, bounding the packet distribution latency (without causing unncessary use of channel capacity), however, is of critical importance.

Efficient algorithms for multicasting data in multi-hop wireless networks exploit the natural *wireless broadcast advantage*: due to the broadcast nature of the wireless medium, a single transmitting node can reach multiple one-hop neighboring nodes with a single transmission. Most work on broadcast in MANETs has focused on *energy-efficiency*, and aims to reduce the number of distinct transmissions needed to reach the entire set of receivers. Examples of such energy-efficient broadcasting algorithms include the BIP algorithm [22] for incremental construction of a broadcast tree and the EWMA algorithm [6] etc. While energy efficiency is an important metric for battery-constrained MANET scenarios, it is less relevant in many mesh network scenarios, where the nodes are relatively static (e.g., mounted on rooftops) and directly connected to regular power outlets. In mesh environments designed to support potentially high-bit rate multicast multimedia streams, it is necessary to develop routing techniques that allow for low-latency, high-throughput multi-hop wireless packet broadcast. This is precisely the goal of our current research effort.

1.1 The Main Questions With Broadcasting in Multi-Rate, Multi-Channel Meshes

We can formalize the issues with high-performance multi-hop broadcast by first defining a new metric of interest: *the broadcast latency*, computed as the maximum delay between the transmission of a packet by a source node and its eventual reception (over multi-hop paths) by all the intended receivers. In many cases, our goal is to minimize this worst-case path latency, since this not only indirectly appeals to a notion of more efficient packet delivery, but also translates into lower latency variation among the receivers. Constraining such latency variation may be especially important in interactive environments (e.g., to preserve temporal fairness among players in interactive multi-player games). Given such a metric, our overall research effort addresses the following questions:

a) **Effect of Multi-Rate Links on Efficient Broadcasting:** Is it true that multicast-tree based distribution techniques outperform unicast-based strategies for broadcast traffic in such multi-rate meshes? Or, can one do better by using alternative packet distribution topologies and algorithms? How does one modify practical tree-based routing protocols to better exploit higher-rate links as opposed to slower-rate links in broadcast scenarios?

b) **Sensitivity of Broadcast Topology to Traffic Generation Rate** Since we are no longer confined to low bit-rate, sporadic control traffic, does the variation in the source traffic generation rate affect the nature or topology of efficient packet broadcasting techniques? How does the choice of the broadcast distribution topology depend on the existing "traffic load" on individual nodes or links?

c) **Effect of Multiple Radios and Channels on Efficient Broadcasting** How do we modify the multicast routing protocols to exploit the existence of multiple channels or radios on each node? What are the appropriate routing metrics for multicast/broadcast traffic, and do channel assignment strategies need to be modified to better support multicast flows?

d) **Architectures for Efficient Multicast Route Establishment** Unlike MANETS, mesh networks have relatively stable topologies. In such a scenario, can effective centralized or quasi-distributed route establishment protocols and architectures be designed for multicast flows? How are computed source-specific routes, and/or scheduling strategies, communicated to the individual nodes?

1.2 Contributions of This Paper

Given space limitations and the ongoing nature of our research, we shall tackle only the first question (effect of multi-rate links on broadcasting topologies) in this paper. Accordingly, the analytical and numerical results presented in this paper are restricted to a mesh network, where each node has a single radio, with all radios tuned to a common channel. However, due to adaptive modulation, the data rate on a link between a particular node pair will vary based on the link distance, or more accurately, the SNR characteristics on transmission over this link. Our principal contributions include:

1. Demonstrating that the broadcast latency is not necessarily minimized by tree-based packet distribution topologies, where each intermediate node forwards a packet by broadcasting it to its set of child nodes. Rather, optimal or efficient packet broadcasting is often achieved by having an intermediate node perform *multiple broadcasts*, each of which is directed towards a different subset of child nodes.

2. Formulated the minimum broadcast latency network-wide broadcast problem in a multi-rate wireless mesh networks and showed that this optimisation problem is NP-hard. Proposed two heuristic algorithms, which exploits both wireless broadcast advantage and the multi-rate nature of the networks, to solve this problem. With these algorithms, our simulations using standard 802.11 parameters show that multi-rate multicast can reduce broadcast latency by 3-6 times compared with using the lowest rate alone.

3. Proposed using the product of transmission rate and transmission coverage area to measure the efficiency of using a particular transmission rate in achieving low network-wide broadcast latency in the design of multi-rate systems.

To our knowledge, our work is the first to illustrate the sub-optimality of a strategy that solely exploits the wireless broadcast advantage by sending a packet to all child nodes in a single transmision, and to present heuristic wireless broadcasting techniques that incorporate the multi-rate nature of the wireless links.

2 Impact of Multi-Rate Links on Efficient Broadcasting

Effective packet broadcasting in a multi-rate multi-hop wireless environment depends strongly on the interaction between the routing and MAC layers. Intuitively, a pure flooding strategy, where each intermediate node re-broadcasts a received packet, might be most robust, but can lead to significantly high broadcast latency, as the high number of redundant transmissions at the MAC layer lead to contention-induced backoffs (a.k.a, the broadast storm problem). Accordingly, efficient broadcast strategies typically aim to build a distribution tree (or sometimes a "mesh" [16] for robustness), where redudant transmissions are eliminated or minimized. Given such a distribution tree, the simple strategy of treating each link in the forwarding tree as distinct, and thus having each intermediate node forward a packet to each of its downstream neighbors via individual unicasts, is also wasteful. By failing to exploit the wireless broadcast advantage, the all-unicast approach not only maximizes the forwarding latency at each intermediate node, but can induce additional backoff-based delay at the MAC layer due to the increased number of distinct transmissions at each node. Based on these observations, the natural solution implicit in most multicast routing protocols is that each intermediate node will *transmit its packet only once*, reaching all of its immediate downstream neighbors in a single link-level broadcast.

We first attempt to show how this central premise (i.e., that each intermediate node reaches all its neighbors in a single broadcast transmission) can lead to sub-optimal behavior in multi-rate wireless mesh environments. We implicitly assume that the MAC layer of future wireless meshes will provide some form of multicast support, where the transmitter may be able to specify the transmission rate of the MAC-layer broadcast, and either explicitly or implicitly (based on the range within such a broadcast is correctly received) the recipients of the broadcast. At present, technologies such as 802.11a or 802.11b do not offer this ability and mandate that all link-layer control broadcasts proceed at the lowest possible rate (e.g., 1 Mbps for 802.11b and 6 Mbps for 802.11a). However, the transmission rates for link layer data multicasts is not specified. Clearly, future MAC protocols may permit more flexibility. For example, relatively simple techniques have been proposed (e.g., [8]) to support such selective-broadcast at the wireless link layer, while the IEEE 802.16a group is considering the support of multicast traffic at the MAC layer.

2.1 Illustrating the Role of Differential Link Rates on the Broadcast Latency

To understand the closely coupled nature of the broadcast tree formation and the MAC layer scheduling, consider the topology shown in Figure 1 with five nodes, labelled as Nodes 1 to 5, arranged in a straight line. For simplicity, we will refer Node 1 as N_1 etc. in the text. In Figure 1, the d value between 2 nodes indicates the physical distance in meters between them. We assume each node is equipped with an IEEE 802.11b radio tuned to the same channel. By using the Qualnet simulator [20] as a reference, we find the transmission rate versus transmission range relationship in Table 1 assuming a two-ray propagation model. Note also that the interference range in Qualnet is 520m. Thus, given the network configuration in Figure 1, there are 4 links in the network. Link (1,2) has a maximum capacity of 11Mbps while the other three links have a maximum capacity of 1 Mbps. Since our concern is packet delivery latency, we indicate the relative time required to send a packet for each link using the t value indicated in the Figure.

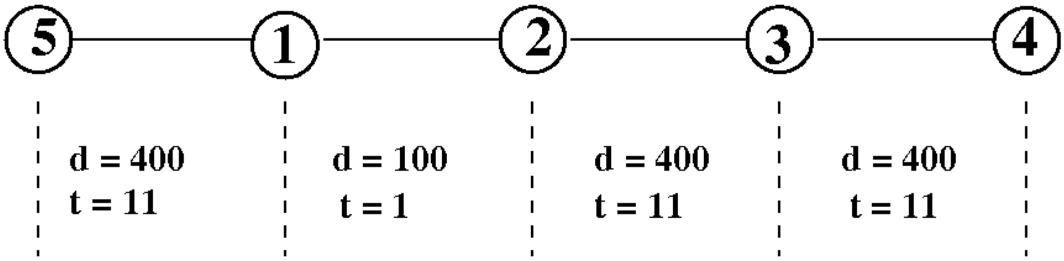

Figure 1: Motivating example for the multi-rate network-wide broadcast problem.

Transmission rate (Mbps)	Transmission range (m)	RAP (Mbps-km^2) (For Sec. 7)
1.0	483	0.73
2.0	370	0.86
5.5	351	2.13
11.0	283	2.76

Table 1: This table shows the maximum transmission range in meters for different IEEE802.11b transmission rates. The ranges are obtained from Qualnet [20] assuming a two-ray model.

We assume that N_1 (i.e. Node 1) is the source node and it wants to send a packet to all the nodes in the network. Since the network is not fully connected, some nodes will need to act as a relay. We consider two different forwarding alternatives. In the first approach, which we call Alt_1, each node is only allowed to broadcast the packet once. Due to this restriction, N_1 (the source node) must broadcast at the lower rate of 1Mbps to both N_2 and N_5, taking a time of 11 units to transmit the packet. Note that N_1 could not possibly use other transmission rates because N_5 will not receive the packet otherwise. This results in the transmission schedule depicted in Figure 2, and leads to a broadcast latency of 33 time units.

In the second approach, which we call Alt_2, we allow each node to broadcast the same packet more than once. Figure 3 depicts the transmission schedule. It shows the source N_1 transmitting the same packet two times. It first transmits to N_2 at 11Mbps (at time $t = 0$), taking 1 time unit. It then transmits the same packet again at time $t = 12$ to N_5 at a lower rate of 1Mbps. Note that the transmissions ($N_1 \rightarrow N_5$) and ($N_2 \rightarrow N_3$) cannot take place at the same time because of interference. In contrast to the first approach, the whole network-wide broadcast latency is now 23 time units.

This examples illustrates the following important feature of broadcasting in multi-rate wireless meshes:

Property 1 *If a node is to multicast to a number of its neighbouring nodes simultaneously,*

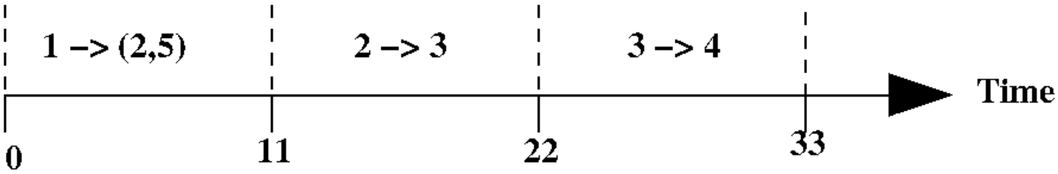

Figure 2: *Alt_1:* Transmission schedule if each node can only broadcast a packet at most once.

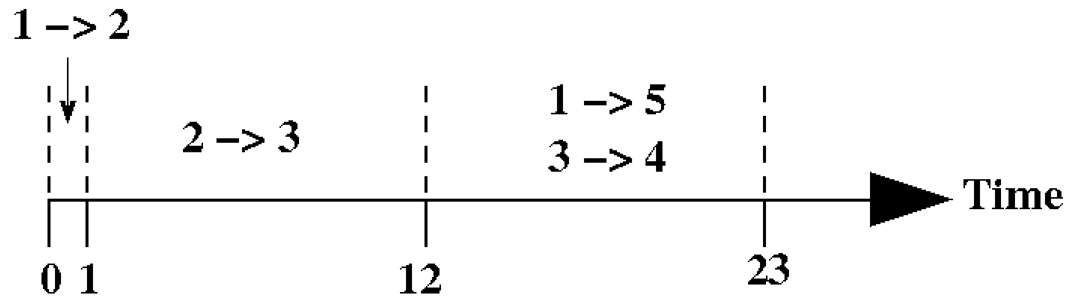

Figure 3: *Alt_2:* Transmission schedule if each node can broadcast a packet more than once.

the maximum broadcast rate that can be used is constrained by the lowest rate to reach all these nodes independently. Accordingly, if the objective is to improve the broadcast latency, a new degree-of-freedom that can be used is to allow a node to transmit the same packet more than once, to different subsets of its immediate downstream neighbors.

By exploiting this degree-of-freedom, an intermediate node can transmit the packet at a higher rate to children that lie along the "more critical" sub-trees (i.e., those that might take longer to forward the packet) to their leaf nodes, and subsequently use a lower-rate transmission to a subset of the "less critical" sub-trees.

To the best of the authors' knowledge, the degree-of-freedom of allowing a node to transmit the same packet more than once has not been pointed out before. Note that this new degree-of-freedom can be combined with others that have already been proposed, namely multi-radio, multi-channel [9] and network coding [24]. Of course, if the objective is instead to minimize the total energy consumption, then transmitting the same packet more than once will always result in worse performance.

3 Related Work

Much work has been done in achieving *efficient* network layer multicast and broadcast in multi-hop wireless networks and wireless ad hoc networks. The majority of the work measures efficiency in terms of energy consumption [6, 7, 22], the number of transmissions (which is equivalent to energy consumption if broadcast power cannot be adjusted) [18] or the amount of overhead in route discovery and management [13, 16]. However, all of these work is based on single-rate wireless networks.

The work that is most similar to ours is [11] which considers the problem of achieving minimum broadcast latency in a single-rate wireless ad hoc network. They show that their optimisation problem is NP-hard and provided a polynomial time algorithm to solve the problem. If each node is allowed to multicast at most once, then our problem is a generalisation of that in [11] to the multi-rate case. However, as we have argued in Section 2, the multi-rate problem has a number of unique properties not present in the single-rate case.

The problem of routing in multi-rate multi-hop wireless networks has previously been studied in [3, 5, 9] but all of them focused on unicast routing. The authors of [9] proposed a routing metric which can be used for a multi-channel multi-hop wireless network. Their metric takes different transmission rates into account by having the metric inversely proportional to the transmission rate. The authors in [5] used simulation to study the end-to-end UDP and

TCP throughput of a multi-rate multi-hop path. Their simulation study revealed a number of interesting findings, for example, they found that a 2-hop path of 11 Mbps links can have very different throughput from that of 4-hop paths of 5.5Mbps links. The work in [3] shows that if the interference range is infinity, then the unicast routing path that minimises the total path delay will also maximises the throughput between the source and destination. In order to deal with multi-rate links, [3] defines the medium-time metric (MTM) for each transmission rate. MTM essentially measures the time it takes to transmit a packet over a multi-rate links and it takes into account transmission delay (i.e. frame size divided by transmission rate) and the overheads which in the case of IEEE802.11 includes RTS/CTS/ACK frames and channel contention. Note that the inclusion of channel contention is needed to account for intra-flow interference.

The problem of rate adjustment at the MAC layer has been considered in [14, 19]. The authors in [14] propose a method to adjust the MAC layer unicast transmission rate by using feedback information from the receivers. The authors in [19] show how MAC layer rate adjustment and retransmission strategies can be jointly optimised to obtain high throughput. Again, the work is focus on MAC layer unicast. This paper proposes a novel type of wireless mesh networks where a node can multicast at different rate at the MAC layer to different subsets of neighbours.

4 Optimal network-wide broadcast in a multi-rate wireless mesh networks

In this section we formulate the problem of finding the optimal network-wide broadcast topology that results in the minimum broadcast latency in a multi-rate multi-hop wireless mesh network. We first discuss the modelling assumptions.

4.1 Transmission and interference models

The IEEE 802.11 Standards for multi-rate transmissions specify that a packet is received correctly if the packet error rate (PER) for a 1000-byte frame is less than 10%. This means that, in order for a packet to be received correctly at a particular transmission rate, the signal-to-interference and noise (SINR) ratio at the receiver must be greater than a threshold. The SINR threshold is different for different transmission rate. Since the modulation scheme for higher transmission rate employs a denser signal constellation, the SINR threshold for higher transmission rate is higher and vice versa.

We consider a multi-rate system with b different rates $r_1 > ... > r_b$. The SINR threshold for rate r_i is σ_i with the property that $\sigma_i > \sigma_j$ if $r_i > r_j$. We assume that a constant transmission power P_t is used for all transmission rates. Further, we assume that the receive power P_r at a distance d be given by the following propagation model:

$$P_r = P_t \frac{1}{d^\theta} \quad (1)$$

where θ is the path loss exponent which takes a value between 2 and 4. Following [25], we define the interference-free transmission range $\bar{d}_i$ of a transmission rate r_i as the maximum distance that a packet is received correctly in the absence of interference. Let N denote the thermal noise power in the system. It can readily be shown that

$$\bar{d}_i = \frac{1}{\sigma_i^{\frac{1}{\theta}}} \left(\frac{P_t}{N} \right)^{\frac{1}{\theta}}. \tag{2}$$

This shows that the interference-free transmission range is a decreasing function of transmission rate.

Consider a situation where we have a transmitter, a receiver and an interferer. Let d be the distance between the transmitter and receiver, and u be the distance between the receiver and the interferer. In order for the receiver to receive a packet correctly at rate r_i, the distances d and u must satisfy

$$\frac{\frac{P_t}{d^\theta}}{\frac{P_t}{u^\theta} + N} \geq \sigma_i \tag{3}$$

$$\Rightarrow \quad u \geq \left(\frac{\frac{P_t}{N}}{(\frac{\bar{d}_i}{d})^\theta - 1} \right)^{\frac{1}{\theta}} \tag{4}$$

This shows that the amount of interference a receiver can tolerate is dependent on distance between the transmitter and receiver. In particular, if the receiver is at the interference-free transmission range $\bar{d}_i$ and intends to receive at rate r_i, then it cannot tolerate any interference at all since u will be infinity. The above interference model, which is similar to the physical model used in [15], has good spatial reuse property but will not be easy to use to study the problem of minimum broadcast latency in a multi-rate network.

In this paper, we will use packet reception model where a packet at rate r_i is received correctly if both of the following conditions are satisfied:

1. The distance between the transmitter and receiver is less than s_i; and,

2. No transmitter within a (finite) distance u_i from the receiver is transmitting concurrently.

This model is similar to the receiver based model in [15]. Since our aim is to minimise the packet delivery latency from the source to all receivers, a receiver based model will be able to tell us the best possible achievable latency. In this paper, we will refer the ranges s_i and u_i as, respectively, the transmission range and interference range of transmission rate r_i. Moreover, we expect that for a well designed broadcast scheme, the number of concurrent transmissions in the same local area will be low and this will be confirmed by simulation in Section 6. Thus, the one interferer model used earlier will be applicable. In this case, the relationship between s_i and u_i will be governed by Equation (4). In order to improve spatial reuse, we require s_i to be strictly less than $\bar{d}_i$ so that u_i is finite. A possible choice is to require that

$$s_i = \frac{1}{\xi^{\frac{1}{\theta}}} \bar{d}_i \tag{5}$$

where $\xi > 1$. The corresponding value of u_i will then be

$$u_i = \left(\frac{1}{\xi} \frac{P_t}{N} \right)^{\frac{1}{\theta}} \tag{6}$$

Note that for this particular choice, the interference radius is independent of the transmission range. It is interesting to note that for $\xi = 2$, the interference radius given above is equal to the maximum interference radius derived in [25] assuming that the thermal noise term is negligible compared with the interference. The above derivation therefore justifies the use of a constant interference radius in our reception model. This is consistent with the fact that for a high transmission rate link, the receiver is closer and therefore the signal strength is higher but at the same time a higher SINR is required for correct packet reception, thus the amount of interference that each transmission rate can tolerate is almost the same.

4.2 The modelling assumptions

We formulate the optimization problem under the following assumptions:

1. Each node in the network is equipped with one radio, with all radios tuned to a common channel.

2. By adjusting the modulation scheme, a node can multicast at different data rates, with the transmission range a decreasing function of the data rate. Let $s_{\max}$ denote the maximum transmission range. Also, we use a disc model for the transmission range.

3. A node's neighbours are all the nodes that can be reachable using the lowest possible transmission rate.

4. Let $i_1, ..., i_k$ be the neighbours of a node k and the maximum rates which node k can use to reach these nodes independently are $r_1, ..., r_k$ respectively. If node k wants to multicast to $i_1, ..., i_k$ in one go, this can only be performed at a rate of $\min(r_1, ..., r_k)$ or lower.

5. We assume a binary interference model, as follows: If while a node k is receiving a frame, a node j within a radius $\kappa s_{\max}$ from node k transmits a frame, then the frame that k is receiving is assumed to be corrupted and lost. Based on the analysis in Section 4.1, we assume that the interference range $\kappa s_{\max}$ is a constant independent of the transmission rate. We call κ the normalised inference range.

6. We assume an ideal MAC layer, as follows: Two nodes i and j can multicast at the same time if and only if node i's multicast does not interfere with the intended recipients of node j's multicast and vice versa.

7. We assume a centralised entity which schedules these multicasts so that, under the ideal MAC layer assumption, no two multicasts will interfere with each other.

8. Each node can multicast the same packet up to $m_{\max}$ times, clearly to different subsets of its neighbors. $m_{\max} = 1$ corresponds to the conventional use of broadcast trees, where each node reaches all its child nodes in a single transmission.

Note that the basic building block of achieving the network-wide broadcast is a sequence of link layer multicasts instead of link layer broadcasts. The use of link layer multicasts is necessary especially when a node is to transmit the same message multiple times to different set of neighbours as illustrated in the motivating examples in section 2.

4.3 Optimisation problem

The actual formulation of the optimization as an integer programming problem is provided in Appendix A. The key decisions in this optimisation problem are: (1) Whether a node should multicast and if so, to which of its neighbours; (2) The timing of these multicasts. To determine the timings of these multicast, we must make sure that a node can only multicast a packet after it has received it. Also, some multicasts cannot take place at the same time because they interfere with each other, then these multicasts must be scheduled in such a way that the minimum latency is achieved. Not surprisingly, this multi-rate broadcast problem is NP-hard.

Theorem 1 *The minimum latency network-wide broadcast problem with possibly multiple number of transmission per node in a multi-rate wireless mesh network is NP-hard.*

Proof: This follows from the fact that the minimum latency network-wide broadcast problem in a single-rate wireless mesh network where each node can transmit at most once, which is a special case of this problem, is NP-hard. The NP-hardness result for the single-rate case is given in [11]. □

The single-rate broadcast problem has been well studied, for other NP-hardness results (e.g. inapproximability) see [10]. Since the single-rate case is a special case of the multi-rate case, these results also apply.

4.4 Sample Performance Results

Given the hardness of the problem, the optimization tool can be executed only for relatively small and simple toplogies. However, it would clearly be useful to compare the relative maximal delivery latency achieved by the optimization technique with that achieved by a conventional tree-formation algorithm that does not exploit the rate diversity of different links. This will provide a sense of how much the development of a multi-rate aware algorithm may be expected to improve the broadcast latency in mesh environments. In Section 5, we will also propose a set of heuristic measures to solve this problem. Comparisons of the integer programming formulation with existing algorithms and new proposed heuristics are provided in Section 6.1.

5 Heuristic Algorithms for Low Latency Broadcast Tree

In this section, we will present two heuristic algorithms to create 'efficient' delivery trees for broadcast packets in a variable-rate mesh network. Broadly speaking, any heuristic algorithm must make three important decisions. Firstly, it has to decide whether a node should multicast. Secondly the algorithm must decide the number of transmissions at each transmitting node and determine the neighboring nodes covered in each of these transmissions. Lastly, the multicast transmissions of all nodes must be scheduled and their transmission time decided while taking radio interference into account. It should be noted that that these decisions are closely coupled, since a multicasting node can only multicast after it has received the packet and radio interference means that the multicasts must be scheduled so that interfering multicasts do not take place at the same time. With the hardness of the problem in mind, each algorithm is decomposed into three logically independent steps:

1. Topology Construction: In this step, the aim is to compute a broadcast tree (or a spanning tree) T. This step decides the make-up of the broadcast tree, i.e. it identifies all nodes that would transmit and the children/parent relation between different nodes. We assume that a node can transmit multiple times at different rates regardless of the number of radio resources available at this stage. The actual decision on the number of distinct-rate transmissions at each node is deferred to the next step. The topology construction algorithm should take into account the multi-rate nature of our problem and must exploit the *wireless multicast advantage (WMA)* [22] afforded by the wireless medium.

2. Downstream Multicast Grouping: The tree construction stage proceeds by assuming that each node can perform multiple distinct-rate transmissions to cover its children, the grouping algorithm can improve performance by realizing that due to limited channel and transceiver resources and interference concerns, it is often prudent to suppress these extra transmissions unless these extra transmissions can actually reduce delay even with resource constraints. The aim of the multicast grouping algorithm is to determine the rates and number of distinct-rate transmissions that each node should be making. Intuitively, the rationale behind multiple transmissions is to allow faster transmission to the more *'critical'* child nodes (i.e., those nodes that have leaf nodes with larger delivery latencies), at the expense of larger transmission latency to the other child nodes.

3. Transmission Scheduling: While we have determined the number of transmissions at each node and the parent/child relationship amongst different nodes in the first two steps. The exact timing of the various multicasts (especially relative to different branches of the tree) still needs to be determined. The final step schedules all transmissions taking into account the precedence constraint and interference constraints present in our problem. These constraints mean that a node can only multicast after it has received the packet, and interfering multicast transmissions cannot occur concurrently. We are conceptually assuming a centralized scheduler in our current work.

Clearly, this decomposition of the overall optimization problem is not optimal. For example, it is only after the grouping phase that we obtain the multicast transmission sets, as well as the transmission rate associated with each link-layer multicast. Similarly, the choice of the grouping would also depend on the scheduling strategy. Ideally, the construction and grouping phases should be performed together. However, as already noted, a joint optimization is computationally infeasible, except for trivially small mesh topologies.

We present two different heuristics, called BIB and WCDS, for the construction phase (in Section 5.1). We follow this up with the broad algorithmic approach for the grouping phase (in Section 5.2), which conceptually takes a tree as input and determines the partitioning of child nodes into different subsets, each corresponding to a separate link-layer multicast. In the subsequent subsection (Section 5.3), we present the scheduling heuristic, that takes into account the conflict graph of the underlying tree topology. Once again, this heuristic is independent of the choice of the construction algorithm.

Let us first introduce some common mathematical notation. The entire wireless mesh is represented as a graph (V, E), with the mesh nodes forming the vertices and the edges representing the direct link between any two nodes. Accordingly, $(i, j) \in E$ denotes the direct unicast link between nodes i and j. Based on the distance between such a node pair, each link (i, j) can be associated with a transmission rate R_{ij}. The transmission rate $R_{ij} = 0$ if i

and j are not one-hop neighbors, i.e. if j cannot correctly receive a packet from i even if i transmits at the slowest rate and maximum power.

5.1 Topology Construction Algorithms

We present two alternative approaches for computing the underlying broadcast tree. The algorithms compute trees centered at the source node s to all the receiver nodes $V \backslash \{s\}$.

5.1.1 Broadcast Incremental Bandwidth (BIB)

The BIB algorithm is very similar to the BIP algorithm [22] in that both use a modified version of Prim's algorithm, greedily adding links to an existing tree such that the incremental cost is minimized. However, while BIP focuses on the development of low-energy packet distribution trees, the focus of BIB is to build distribution trees which minimize broadcast latency. BIB aims to choose high-rate links, since the transmission of a packet from a transmitter to N neighbors is constrained by the slowest of the point-to-point links between the transmitter node and each of the N individual neighbors.

We first compute the tree from a source node s to all the other nodes $V \backslash \{s\}$ in the wireless mesh. Any candidate algorithm should obviously exploit the wireless multicast advantage [22] to reach multiple neighbors in a single transmission. The algorithm must also take into account the multi-rate nature of the problem, for example, considering if a node should reach another neighbor using their direct hop (at a lower transmission rate) or via two hops of higher rates. In addition, the algorithm should be aware of the interference between neighboring multicasts. For example, if a number of multicasts are within the interference range of each other, they can only take place one after another. As a special case, if all the transmissions interfere with one other (i.e. the interference radius is infinity) then only one multicast can take place at a time. In such as case, minimizing the broadcast latency is identical to minimising the total transmission time of all the multicasts, i.e., the resutling tree should be a radio analogue of the wired minimum spanning tree.

The Broadcast Incremental Bandwidth (BIB) algorithm is very similar to the BIP algorithm [22] in that both use a modified version of Prim's algorithm, greedily adding links to an existing tree such that the incremental cost is minimized. However, while BIP focuses on the development of low-energy packet distribution trees, BIB primarily aims to choose high-rate links, since the transmission of a packet by a transmitter to its neighbors is constrained by the slowest of the point-to-point links between the transmitter node and each individual neighbor.

For any particular packet forwarding topology, let $N(x)$ denote the one-hop neighbors of node x and $Neigh(x)$ ($\subset N(x)$) the designated downstream neighbors. In other words, node x must broadcast the packet so that it is correctly received by all nodes $y: \ y \in Neigh(x)$. Clearly, the transmission rate $R(x)$ of node x is given by the slowest downstream link, i.e.,

$$RN_{Neigh(x)} = \min R_{xk} \quad k \in Neigh(x). \tag{7}$$

To apply a minimum-cost tree construction algorithm such as Prim's, the *transmission latency cost* TL_{ij} for a link (i,j) between two nodes i and j is initially set to be the inverse of the transmission rate, i.e., $TL_{ij} = \frac{1}{R_{ij}}$. The algorithm is initiated with a tree T that initially contains only the source node s as the root, with the cost of any other node x, $C(x)$, set to TL_{sx}. Each node in the tree has a *tree node cost* $TC(.)$ reflecting the cost of forwarding

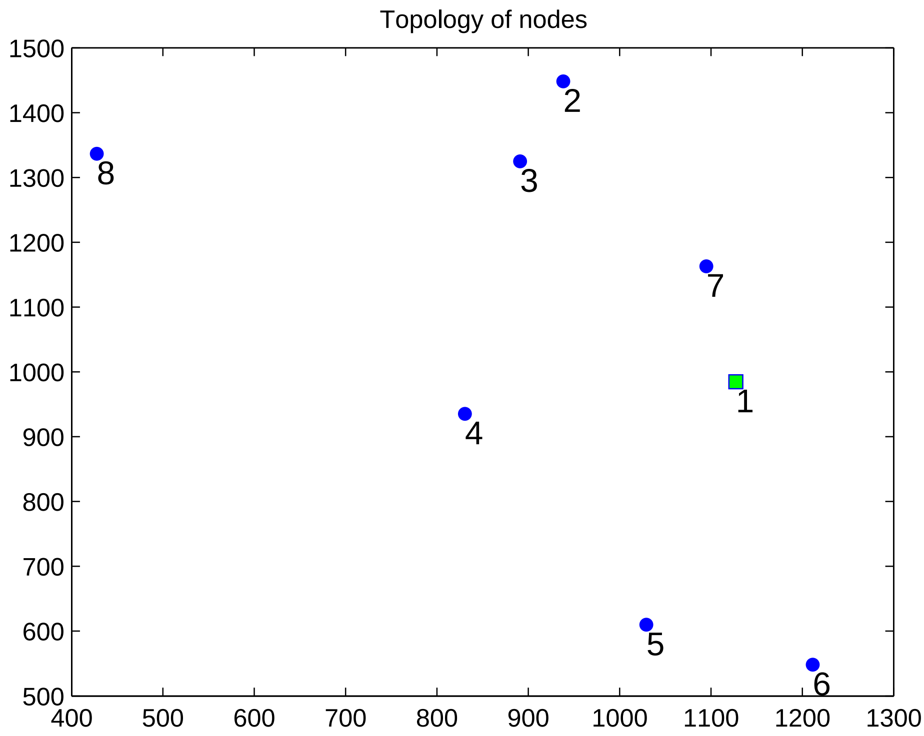

Figure 4: The sample topology.

the packet to the "slowest" child node; at the beginning, $TC(s) = 0$. In each subsequent step, the node x with the current minimum cost is added to the tree. Let P_x denote the parent of the chosen node x; clearly, P_x is already part of the tree. The tree node cost for P_x, $TC(P_x)$, is then incremented by the cost associated with node x. Additionally, for each neighbor y of P_x that is not already in the tree, its cost is dynamically updated to be the difference between the transmission rate cost $TL_{P_x y}$ (which does not change) and the tree node cost $TC(P_x)$ (which might change with each iteration) if it is more favorable to reach y by using incremental broadcast from P_x. This dynamic modification of the link cost at each iteration distinguishes this approach from the basic Prim's algorithm and is designed to reflect the wireless broadcast advantage. The pseudocode for the BIB algorithm is presented in Algorithm 1.

We will use a simple network shown in Fig. 4 to illustrate the algorithm. For clarity sake, we employ 8 nodes in a small network distributed in an area of 1000×1000 m^2. We will use this topology to illustrate different steps of our solution subsequently as well. The nodes are represented by markers with the node number as the label, the source node being shown by a square marker. Fig. 5 shows the tree calculated at the end of tree-construction phases (Step 1). In the tree, the vertices represent the nodes and the edges represent links chosen by the BIB algorithm. The algorithm starts from the source node and adds nodes whilst reducing the incremental bandwidth. The nodes are added in the order of 7,3,2,4,5,6 and 8. These nodes are added because the incremental bandwidth cost of adding them to the tree i.e. 1,1,1,1,6,0 and 7 respectively is the minimum cost at the end of successive rounds.

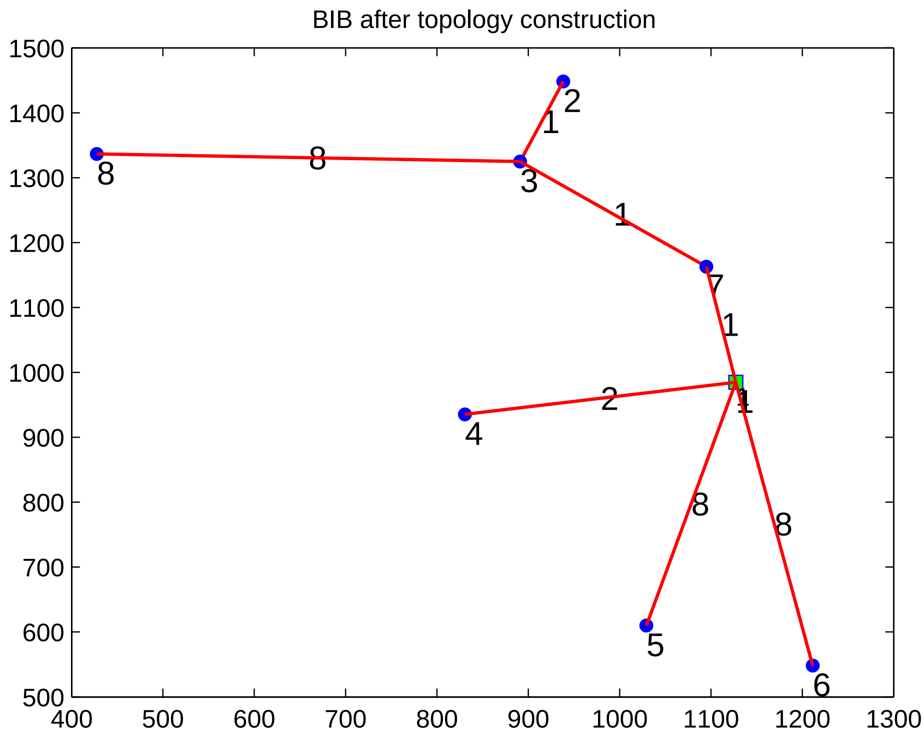

Figure 5: BIB after topology construction

The basic BIB algorithm may also be enhanced (using ideas presented in [4]) for high-performance *reliable* broadcasting trees, where the tree construction process considers the quality of each link (and the resulting retransmissions needed) in addition to the link rate. Since our focus is primarily in exploring the tradeoffs in constructing high-performance packet broadcasting mechanisms, we do not explore this aspect further in this paper.

5.1.2 Weighted connected dominating set (WCDS)

In this section, we present an alternative algorithm based on the concept of weighted connected dominating set (WCDS). Let us first recall that for a graph $G = (V, E)$, a connected dominating set (CDS) Z of G is a subset of V such that firstly every element (node) of $V \backslash Z$ is in the neighborhood of at least one node in Z; and secondly, the set Z is connected. Among all CDSs of graph G, the one with minimum cardinality is called a minimum connected dominating set (MCDS). Computing an MCDS in a unit graph is NP-hard [12]. The use of MCDS to achieve optimal flooding in a single-rate multi-hop wireless networks has been explored in [17] where the authors prove that the size of the optimal flooding tree (measured by the number of nodes performing broadcasts, not by broadcast latency) differs from the size of the MCDS by at most one. However, MCDS performs poorly in multi-rate mesh environments because it does not account for multi-rate links in the tree construction.

To extend MCDS to our multi-rate setting, we assume there are k different rates given by $r_1, r_2, ..., r_k$. Let $N(x, r_i)$ denote the nodes that are reachable from node $x \in V$ using rate r_i. We define the minimum WCDS problem whose aim is to find a subset $Y = \{y_1, y_2, ...\}$ in V

Algorithm 1 The BIB algorithm

$T = \{s\}, S = V \backslash \{s\}$
for $(x \in S)$ **do**
  $C(x) = \frac{1}{R_{sx}}$
end for
while $S \neq$ **do**
  $x \leftarrow MinCostNode(S)$
  $S \leftarrow S \backslash \{x\}$
  $T \leftarrow T \cup \{(P_x, x)\}$
  $s \leftarrow S\{x\}$
  for $(y \in \{N(x) \cap S\})$ **do**
    if $(C(y) > \frac{1}{R_{xy}}$ **then**
      $C(y) \leftarrow \frac{1}{R_{xy}}; P_y \leftarrow x$
    end if
  end for
  for $(y \in \{N(P_x \cap S\})$ **do**
    $c \leftarrow \frac{1}{R_{P_x,y}} - \frac{1}{RN_{Neigh(P_x)}}$
    if $(C(y) > c)$ **then**
      $C(y) \leftarrow c$; $P_y \leftarrow P_x$.
    end if
  end for
end while

and the broadcast rate w_i (which are chosen from $r_1, r_2, ..., r_k$) for node $y_i \in Y$ such that

1. Every element of $V \backslash Y$ is in $\cup_{y_i \in Y} N(y_i, w_i)$
2. The set Y is connected.
3. The weighted sum $\sum_{y_i \in Y} \frac{1}{w_i}$ is minimal.

Note that when there is only one transmission rate, the minimum WCDS is equivalent to the MCDS. We expect the solution to the minimum WCDS problem to be similar to optimal broadcast tree for the multi-rate scenario. We use a greedy algorithm, depicted in Algorithm 2, to obtain an approximation of the minimum WCDS. The algorithm starts by making the source node s eligible to transmit. It does this by moving s to the set C which keeps track of the nodes which have received the message already and are eligible to transmit. We say that a node is covered if it has already received a packet and is in the set C. Also, the set R denote the set of all possible k transmission rates. For an eligible node c and rate $r \in R$, the quantity $|N(c, r) \backslash C|$ is the number of "not-yet-covered nodes" that are reachable by a broadcast by node c at rate r. Thus, in each round of the algorithm, we choose the (c, r) combination that maximizes the rate of increase of not-yet-covered nodes, as measured by $|N(c, r) \backslash C| \times r$. This metrics reflects our desire to both include as many nodes as possible in a single transmission, yet keep the transmission rate high (even though a higher transmission rate implies a smaller range, and thus, a smaller set of covered nodes). The algorithm returns T which is the set of directed links in the broadcast tree. At the end of topology construction step, the tree constructed might have some nodes performing multiple transmissions at different rates.

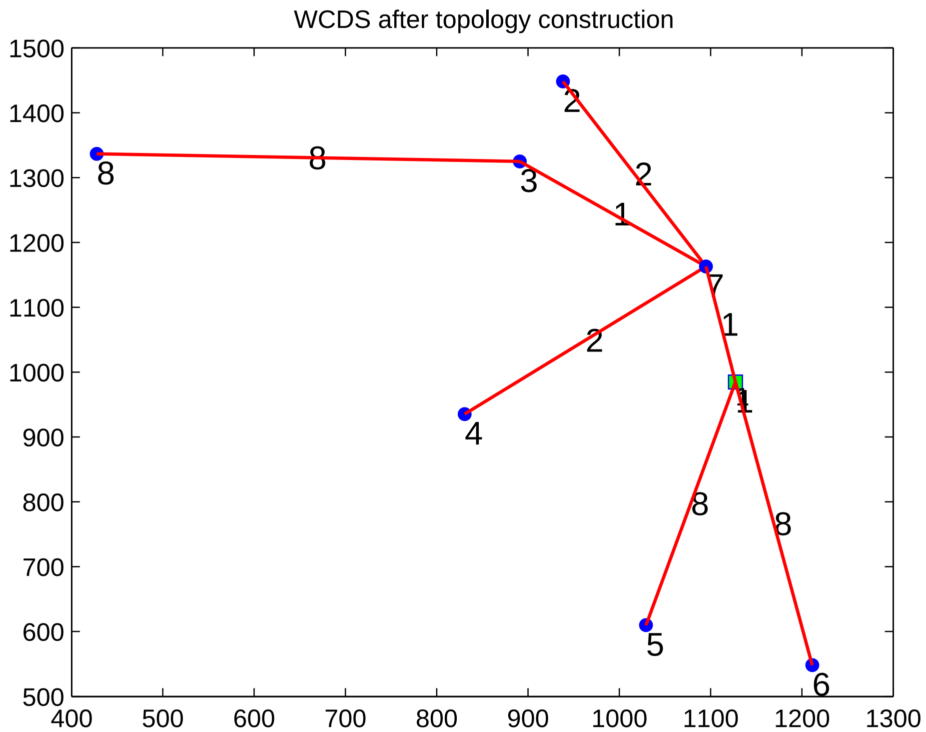

Figure 6: WCDST after topology construction

We re-visit our sample network in Fig. 4. For our sample topology, the choice of the (c, r) combination at the end of each successive round is (1,1), (7,1), (7,2), (1,8) and (3,8) in the order in which they are added. These combinations (c, r) are drafted to the tree because the metric $f(c, r)$ for these combinations (i.e. 1, 1, 1, 0.25 and 0.125 respectively) is the maximum at the end of their respective round.

Our algorithm does not place any restrictions in this step to stem extra transmissions on any node, and we choose any number of transmissions at a node which our greedy algorithm deems fit. The limited resources available i.e. the fact that we have only a single-channel and single-transceiver is accounted for in the next two steps.

5.2 Multicast Grouping Algorithm

Recall from the discussion at the beginning of this Section, in order to find the topology which minimises the broadcast latency, we must make a number of decisions, including which node is to multicast, and if so, how many times it is to multicast, whom the recipients are and its timing. The result of the topology construction is a broadcast tree which specifies that the non-leaf nodes of the broadcast tree will multicast to its child nodes, in possibly multiple transmissions. However, the number of times a transmitting node (i.e. non-leaf node of the broadcast tree) will multicast and the recipients of each multicast still have not been decided. In case where a node multicasts only once, then the recipients will be all its child nodes. For the case where a node is to multicast more than once, a different subset of child nodes will be reached in each multicast such that these subsets together form a partition of the set of child

Algorithm 2 WCDS tree construction

Input: $G, s, R = \{r_1, \cdots, r_k\}$
$C = \{s\}, T = \emptyset$
while $(V \backslash C \neq \emptyset)$ **do**
  for $(c \in C)$ **do**
    for $(r \in R)$ **do**
      $f(c, r) = |N(c, r) \backslash C| \times r$
    end for
  end for
  $(\hat{c}, \hat{r}) = \arg\max_{c \in C, r \in R} f(c, r)$
  $A \leftarrow N(\hat{c}, \hat{r}) \backslash C$
  $C \leftarrow C \cup A$
  $T \leftarrow T \cup (\cup_{a \in A} \{(\hat{c}, a)\})$
end while

nodes. The aim of the multicast grouping stage is to determine the number of multicasts to be made and their recipients.

We begin by defining the concept of valid transmission sequence at a transmitting (i.e. non-leaf) node of the broadcast tree. Consider for example a transmitting node n which has two child nodes c_1 and c_2, which can be reached using a minimum latency of $d_1(= 1)$ and $d_2(= 2)$ time units respectively. Node n can reach these nodes in a number of valid transmission sequences. For example, it can first multicast to c_1 (with latency 1) followed by another multicast to c_2 (with latency 2). We will denote this valid transmission sequence as (d_1, d_2). An alternative valid transmission sequence for node n is (d_2) which reaches both nodes in one multicast. These two are the only two valid transmission sequences for this example. The sequence (d_1) is invalid because it does not reach all the child nodes. In addition, (d_2, d_1) is invalid because the second transmission is unnecessary since both nodes are already reached by transmission d_2 whose coverage area is greater. In general, consider a transmitting node n which has m child nodes $c_1, ..., c_m$ that are reachable using minimum latency of $d_1, .., d_m$ respectively. Let k denote the number of distinct latencies in $d_1, .., d_m$ and let us denote these distinct latencies as $\mathcal{L} = \{l_1, ..., l_k\}$. Without loss of generality, we assume that $l_k \geq ... \geq l_1$. A valid transmission sequence is a r-tuple $(1 \leq r \leq k)$ whose entries are drawn from $\mathcal{L}$ such that

1. Each latency in $\mathcal{L}$ appears in the r-tuple at most once.
2. The latencies in the r-tuple appear in a strictly increasing order.
3. The last entry of the r-tuple must be l_k.

Let $T_V(n)$ be the set of all valid transmission sequences for node n. Since node n uses k distinct rates to reach its child nodes, T_V contains 2^{k-1} valid transmission sequences.

Since our goal is to minimise the broadcast latency, we are interested to find the valid transmission sequence at all the transmitting nodes such that they together will minimise the broadcast latency. For ease of reference, we will refer to the optimal valid transmission sequence at a transmitting node as the *Cardinal Sequence* (CS). Also, if a transmitting node n and all its descendants use their cardinal sequences for transmission, the delay it takes a packet to reach all n's descendants will be called node n's *Cardinal Value* (CV). The aim of the multicast grouping stage is to find the CS and CV at each transmitting node of the network.

Since the choice of CS and CV at a transmitting node n depends on the CS's and CV's of all the transmitting nodes who are descendants of n, the grouping algorithm should proceed from the leaf nodes of the broadcast tree back to the root. For the rest of the description, we will show how the CS and CV of an arbitrary transmitting node n can be determined. We assume that the CS's and CV's of all the transmitting nodes who are descendants of n are already known. Also, for initialisation, we define the CV of all leaf nodes to be zero.

Let us assume that node n uses k distinct transmission rates to reach its child nodes, then the set of all valid transmission sequences at node n, denoted by $T_V(n)$, has 2^{k-1} valid transmission sequences S_q $(1 \leq q \leq 2^{k-1})$. The CS at node n is determined by comparing the broadcast latency achieved by all possible $S_q \in T_V$ and then choosing the S_q with the least broadcast latency as the CS. The CV of the node is then the latency associated with the chosen CS. If node n uses the transmission sequence S_q, let $D(n)_{S_q}$ denote the resulting latency required to reach all the descendants of n, we can formally define CS and CV of node n as

$$CS(n) = \arg\min_{S_q \in T_V} (D(n)_{S_q}) \tag{8}$$

$$CV(n) = \min_{S_q \in T_V} (D(n)_{S_q}) \tag{9}$$

We will now detail how $D(n)_{S_q}$ can be computed. Let S_q be the r-tuple $(S_{q,1}, ..., S_{q,x}, ..., S_{q,r})$. Since the coverage area of a higher latency transmission is larger, thus with the transmission sequence $S(q)$, some of the child nodes of n will receive the same packet multiple times. In particular, let $N(n)_{S_{q,x}}$ denote the child nodes of n that are reachable by a multicast of latency S_{qx} but are not reachable by $S_{q,x-1}$. In other words, the nodes in $N(n)_{S_{q,x}}$ receives their packets from n for the first time via a multicast of latency $S_{q,x}$ and will receive the same packet a total of $(r - x + 1)$ times. Note also that the sets $N(n)_{S_{q,x}}$ $(x = 1, ..., r)$ effectively partition the child nodes of n into r disjoint subsets. Let $D(n)_{S_{q,x}}$ denote the delay it takes n to reach all the nodes in the set $N(n)_{S_{q,x}}$ and their descendants. Assuming that the transmission of the descendants of $N(n)_{S_{q,x}}$ do not interfere with each other, we have

$$D(n)_{S_q} = \max_{1 \leq x \leq r} D(n)_{S_{q,x}} \tag{10}$$

As mentioned a number of times before, the decisions we need to make are highly coupled. Thus, by ignoring the inter-branch interference, we obtain an approximation which makes the problem tractable. The inter-branch interference will be taken into account in the scheduling stage in Section 5.3.

We propose to compute $D(n)_{S_{q,x}}$ using the following formula:

$$D(n)_{S_{q,x}} = \sum_{i=1}^{x} S_{q,i} + \max_{i \in N(n)_{S_{q,x}}} CV(i) + \sum_{i=1}^{x-1} SCDelay_{(S_{q,i})} \tag{11}$$

This equation is obtained by assuming the following *modus operandi*: Node n first transmits at latency $S_{q,1}$ reaching the nodes in $N(n)_{S_{q,1}}$. If some of the nodes in $N(n)_{S_{q,1}}$ are transmitting nodes, they will then begin their transmission to their respective downstream neighbours in parallel. (Note that we are again ignoring inter-branch interference). Note that node n does not begin transmitting at latency $S_{q,2}$ immediately after finishing transmitting at $S_{q,1}$. We assume that node n waits until all the transmissions from $N(n)_{S_{q,1}}$ and their descendants have proceeded sufficiently so that the $S_{q,2}$-transmission of node n does not interfere with those of $N(n)_{S_{q,1}}$ and their descendants. This operation then repeats itself until all transmissions in S_q have been made.

With this *modus operandi* in mind, we can now explain how Equation (11) comes about. We begin with the case for $x = 1$ where we have $D(n)_{S_{q,1}} = S_{q,1} + \max_{i \in N(n)_{S_{q,1}}} CV(i)$. Recall that $D(n)_{S_{q,1}}$ is the delay it takes to reach all the nodes in $N(n)_{S_{q,1}}$ and their descendants. The first term $S_{q,1}$ is simply the time it takes to reach the nodes in $N(n)_{S_{q,1}}$. After the packets have been received by the nodes in $N(n)_{S_{q,1}}$, we assume that the transmissions by the nodes in $N(n)_{S_{q,1}}$ will proceed in parallel, so the maximum time it takes all these transmissions to reach the end of their branches is given by the second term. Note that this follows from our definition of CV.

We now explain the derivation of Equation (11) for $x > 1$. The first two terms of the equation bear similar meaning to what is explained in the last paragraph, so we will focus on the third term only. Recall from our description of the *modus operandi* that the $S_{q,x}$-transmission of node n will only begin after the downstream transmissions caused by the $S_{q,x-1}$-transmission have proceeded sufficiently. The time gap between these two transmissions by node n is $SCDelay_{(S_{q,x-1})}$. Here the prefix "SC" stands for single-channel as this delay is caused by the fact that we have only one single-channel in the system.

Recall that the time separation $SCDelay_{(S_{q,x-1})}$ is needed so that the $S_{q,x}$-transmission is not interfered by the transmissions by the nodes in $N(n)_{S_{q,x-1}}$ and their descendants. In order to compute $SCDelay_{(S_{q,x-1})}$, we will first need to identify those transmissions which may interfere with the reception of the nodes in $N(n)_{S_{q,x}}$. Let $\mathcal{T}_{S_{q,x-1}}$ be all transmitting node in $N(n)_{S_{q,x-1}}$. Let $\tilde{t} \in \mathcal{T}_{S_{q,x-1}}$, the set $\mathcal{N}(\tilde{t})$ consists of all nodes $\tilde{n}$ with the following properties (1) $\tilde{n}$ is a descendant of $\tilde{t}$. (2) The transmission of the parent of $\tilde{n}$ interferes with the reception of nodes in $S_{q,x}$. (3) Either $\tilde{n}$ is a leaf node or the transmission of $\tilde{n}$ and its descendants do not interfere with the reception of nodes $S_{q,x}$. In other words, the transmissions in $\mathcal{N}(\tilde{t})$ are the first ones that do not interfere with the $S_{q,x}$-reception. Thus, we have

$$SCDelay_{(S_{q,x-1})} = \max_{\tilde{t} \in \mathcal{T}_{S_{q,x-1}}} (CV(\tilde{t}) - \min_{\tilde{n} \in \mathcal{N}(\tilde{t})} CV(\tilde{n})) \tag{12}$$

The term in parenthesis in the above equation essentially estimates the time it takes the transmissions due to $\tilde{t}$ and its descendants to clear the interference range of the nodes in $N(n)_{S_{q,x}}$.

Having looked at how $D(n)_{S_{q,x}}$ (Equation (11)) was obtained, we can see how equations (8), (9), (10), and (11) can be used together to obtain the CV at a transmitting node. This process can be performed recursively starting from the leaf nodes back to the root of the broadcast tree.

In addition to deciding on the transmission sequence at each transmitting node, the results of the above computation will also be helpful in deciding the timing of the transmissions in the scheduling stage. Recall that the CV of a transmitting node n can be interpreted as the time required to reach all the descendants of node n. Thus, when it comes to scheduling all the multicast transmissions that are to be made, we can use the analogous concept of the CV of a transmission as a measure of the urgency of the transmission. If S_{q_x} is a transmission within the CS of node n (i.e. S_{q_x} is a chosen transmission), then the CV of S_{q_x} is in fact given by Equation (11).

Example

Following on the example in Section 5.1 where we apply the BIB and WCDST topology construction algorithms to the sample network in Figure 4. If we apply BIB to the sample network, we arrive at the broadcast tree showed in Figure 5. By applying the above multicast grouping algorithm to the BIB broadcast tree, we arrive at the result showed in Figure 7 where the number on each link showed the multicast rate used on that link. Note that Node 1 uses multiple broadcast rates to reach its child nodes in the BIB broadcast tree; after multicast grouping, Node 1 uses only one transmission rate with latency 8 units to reach all its child nodes.

The result of applying the multicast grouping algorithm to the WCDST broadcast tree is showed in Figure 8. It shows that Node 1 uses multiple transmission rates to reach its child nodes. By using branch and bound, we can compute the optimal multicast grouping. Figures 9 and 10 show the optimal multicast grouping for the cases where each node can transmit the same packet at most, respectively, one and two times. By comparing Figures 8 and 10, we see that WCDST multicast grouping is optimal for this example.

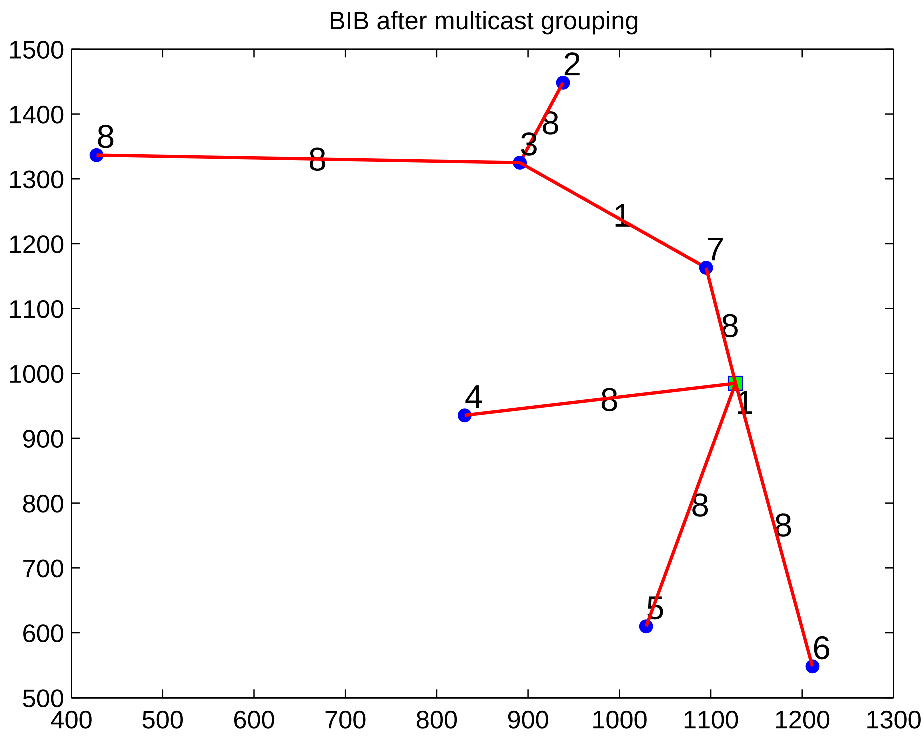

Figure 7: BIB after multicast grouping

5.3 The Scheduling of Transmissions

After both topology construction and multicast grouping have been done, we know all the multicasts transmissions that have to be performed except their timing. We approach the scheduling problem by formulating it with precedence constraints (which enforces that a

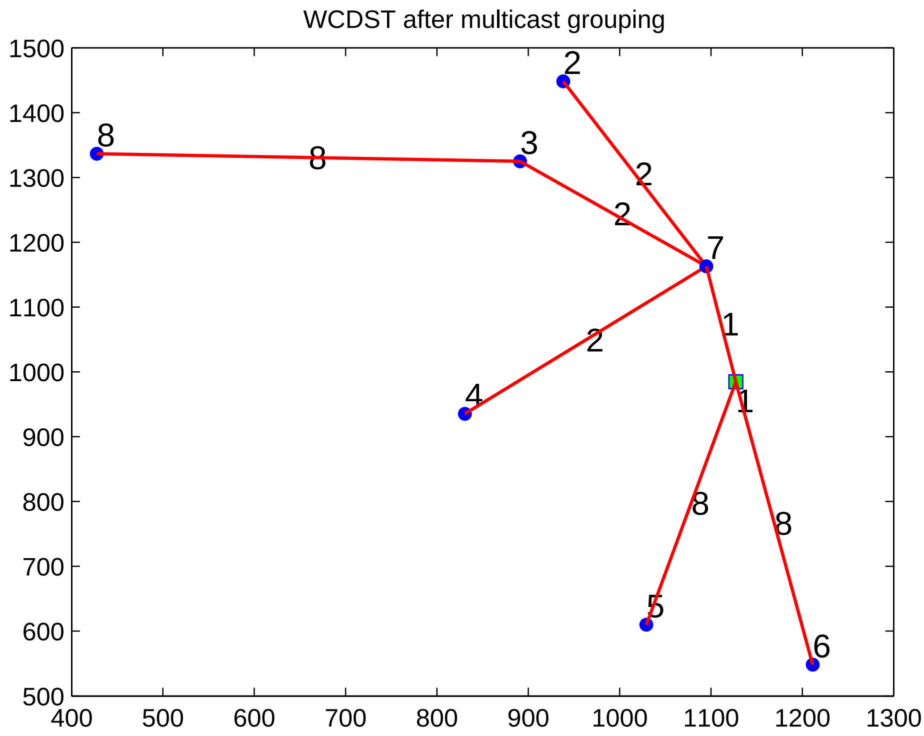

Figure 8: WCDST after multicast grouping

node can only multicast after it has received the packet) and conflict graph (which models the interference between different transmissions). Let $V_b = \{b_1, b_2, ..., b_k\}$ be the set of all the multicast transmissions decided by the multicast grouping algorithm in Section 5.2. Each multicast transmission b_i have four attributes: (1) A sender (which is a non-leaf node of the broadcast tree). (2) A group of recipients (which is a subset of the child nodes of the sender). (3) The latency required by the transmission, denoted by $t(b_i)$, which is the minimum latency it takes the sender to reach all its designated recipients. (4) The CV value of a transmission as defined at the end of Section 5.2. Since the CV value of transmission measures the time it takes a packet to reach the end of the tree, it is viewed as an urgency measure by the scheduling algorithm.

In addition, we define an undirected conflict graph $G_c = (V_c, E_c)$ such that $V_c = V_b$ and $(b_i, b_j) \in E_c$ if and only if (1) The multicast of b_i interferes with the reception of the recipients in b_j or vice versa; or, (2) Both multicasts b_i and b_j have the same sender.

Formally, a schedule can be defined as a mapping $\tau : V_b \rightarrow \mathbb{R}$ which gives the transmission starting time of $b_i \in V_b$. A valid schedule is one which meets the following constraints:

1. The source node multicasts at time zero.
2. A node can only multicast after it has received the packet: if the sender of b_j is a recipient of b_i, then $\tau(b_j) \geq \tau(b_i) + t(b_i)$
3. For any edge $(b_i, b_j) \in G_c$, we have $(\tau(b_i), \tau(b_i)+t(b_i)) \cap (\tau(b_j), \tau(b_j)+t(b_j)) = \phi$. Note that $(\cdot, \cdot)$ here also denotes an open interval in $\mathbb{R}$. Although the same notation is used

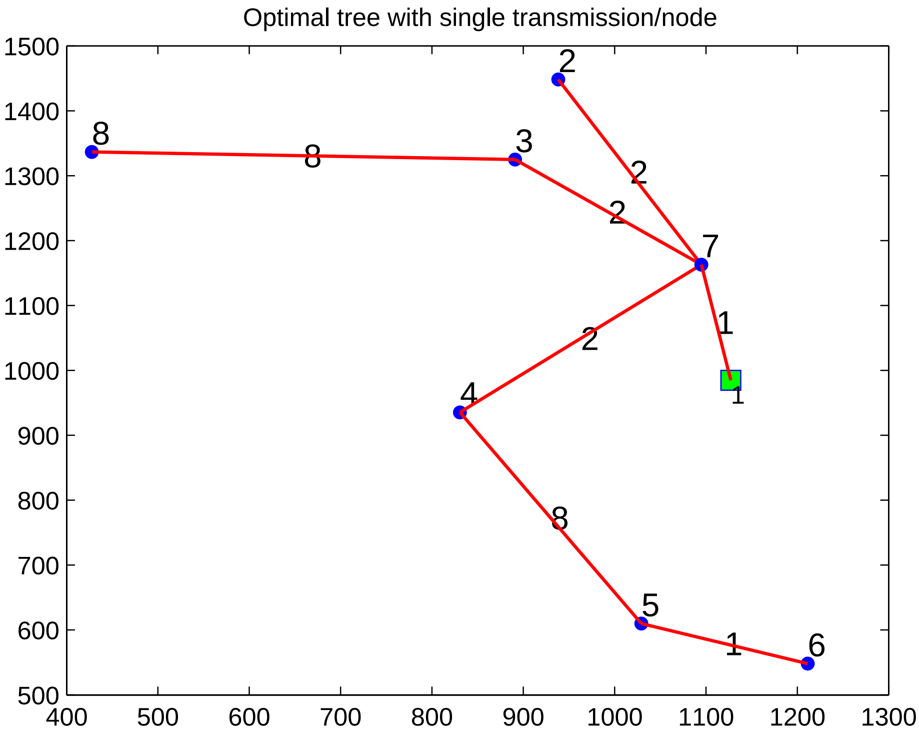

Figure 9: Optimal tree with single transmission/node

to denote both an open interval and an edge of a graph, the usage should be clear from the context.

The scheduling algorithm is depicted in Algorithm 3. The input to the algorithm is transmissions information (TX) which contains the attributes discussed earlier. The aim of the scheduling algorithm is to find out the starting time (τ) and ending time (δ) of all transmissions at each transmitting node.

Initially, *time* depicting current running time is initialized to zero and E depicting *eligible transmissions* is initialized with all transmissions of the source node. A transmission is said to be eligible when the node performing this transmission receives the multicast from its parent, all transmissions of the source node are eligible at time 0. The scheduling process starts by scheduling the lowest latency transmission of the source node at time 0. This transmission is added to the set T which contains all transmissions currently being performed. The starting time (τ) and ending time (δ) of transmissions are decided as they are added to T or in other words as they start transmitting. The minimum of $\delta(\forall t \in T)$ is the earliest any transmission in T will finish and also the earliest a waiting eligible transmission can be scheduled and is called the next-stop time.

At the next-stop time, since the channel becomes available again due to completion of some transmission, a new transmission must be slotted for transmission. The transmission $t \in E$ having the maximum transmission CV is determined, and is assumed to be more *'critical'* as it connects to sub-trees of higher broadcast delay. Thereafter, it is checked that t does not interfere with any of the transmissions in T. In case of no interference, t is added

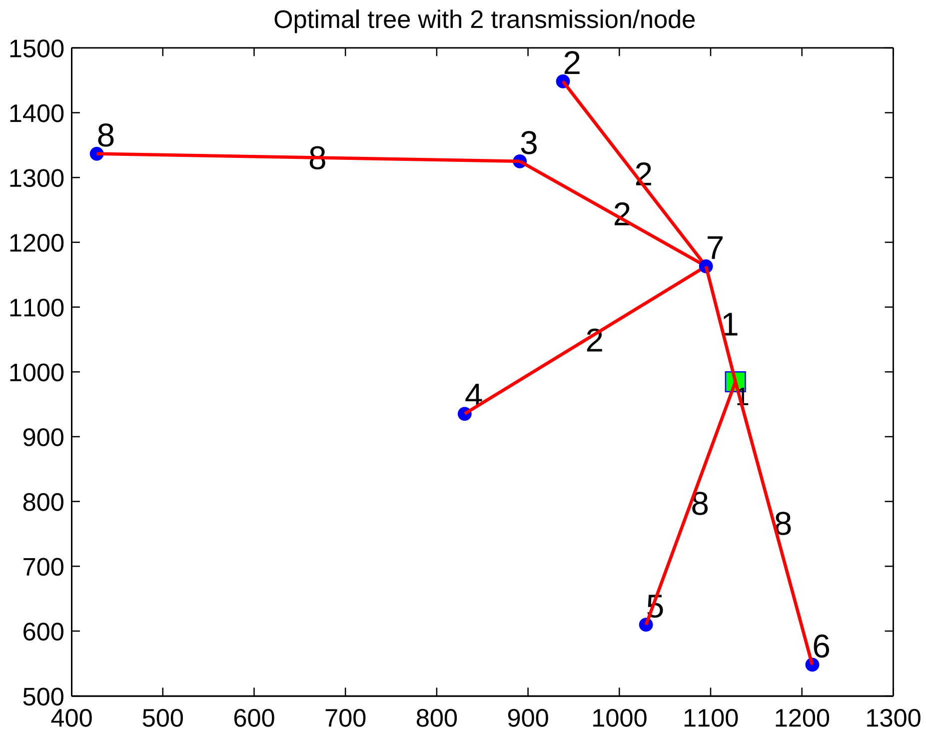

Figure 10: Optimal tree with multiple transmission/node

onto T and deleted from E. The starting time $\tau(t)$ and ending time $\delta(t)$ for the transmission t is also decided at this time. However in case t interferes with any existing transmissions in T, it is held back until next-stop time. It is also ensured that a high-rate transmission does not follow a low-rate transmission at the same node.

After we have iterated through all eligible transmissions i.e. all t belonging to E, the next-stop time is found out by determining which transmission is going to finish the earliest. At the next-stop interval, the children nodes of the transmission finishing at next-stop interval receive the message and thus are eligible for transmitting. Thus at next-stop interval, the transmissions of these recently eligible nodes are added to the eligible transmissions E alongside those transmissions which were held back in the last round. We abide by the precedence constraint in this manner i.e. by allowing a transmission to be added to E only after the transmission has been enabled. A transmission is considered to be enabled when the node making the transmission has received from its parent. At the next-stop interval, all transmissions which are finishing are deleted from T. The algorithm runs in rounds and finishes when the starting time for all transmissions $\tau(\forall t \in V_b)$ and ending time for all transmissions $\delta(\forall t \in V_b)$ have been decided.

5.4 Maximum end-to-end throughput

The above discussion of the tree construction and scheduling algorithms focused on the case of a *single* packet, attempting to minimize the broadcast latency for a single packet. This ap-

Algorithm 3 Scheduling

Input: TX
Set $time = 0$
Initialize $E \leftarrow \cup_{TX.Node=n_s}\{TX\}$
Initialize $T = \emptyset$
while ($E \neq \emptyset$ or $T \neq \emptyset$) **do**
  while $E \neq \emptyset$ **do**
    $t = \arg\max_{TX \in E} TX.CV$
    $E = \{E \backslash t\}$
    if $|T| >= 1$ **then**
      if $TX(t).node$ and $TX(\cup_{t' \in T \backslash t}\ \{t'\}).node$ do not interfere **then**
        $T \leftarrow \{T \cup t\}$;
        Set $\tau(t) = time$
        Set $\delta((t) = time + TX(t).latency$
      else
        $E_{Next} \leftarrow t$
      end if
    else if $|T| < 1$ **then**
      $T \leftarrow \{T \cup t\}$;
      Set $\tau(t) = time$
      Set $\delta(t) = time + TX(t).latency$
    end if
  end while
  $NextStop$= min $(\delta(\cup_{t \in T}\{t\}))$
  $NextTrans$= $\{t\}$: $(\forall t)$ $(\delta(t)$= $NextStop)$
  $E \leftarrow E \cup t_{children}$ of $NextTrans$
  $T = T - NextTrans$
  $E = E - NextTrans$
  $E = E \cup EN$
  $time \leftarrow NextStop$
end while
Output: $\tau(t), \delta(t)\ \forall_{(1 \leq t \leq |TX|)}$

proach is clearly directly applicable when the data rate of the broadcast stream is low enough (e.g., for control traffic), where one can safely assume the absence of interference/scheduling conflicts among successive packets of the same flow. For higher rate data flows, it is important to compute the maximum achievable throughput of a broadcast tree, defined as the maximum data rate that can be sustained without their being any scheduling-related conflicts between packets of the same flow.

The maximum achievable throughput can be computed from the packet transmission schedule computed in Section 5.3. Using the same notation as in Section 5.3, the set of all multicast transmissions are $V_b = \{b_1, b_2, ..., b_k\}$ and the schedule says that transmission b_i will take place during the time interval $[\tau(b_i), \tau(b_i) + t(b_i)]$. Assuming that packets are generated by the source node at regular time at $(m-1)\Delta$ (for $m = 1, 2, ...$). Our goal is to maintain the same schedule computed earlier so node b_i is expected to multicast the m-th packet during $[(m-1)\Delta + \tau(b_i), (m-1)\Delta + \tau(b_i) + t(b_i)]$. The maximum throughput is achieved by the smallest possible Δ such that there is no conflict between the scheduling of all the packets. By defining

$$\mathcal{I}(m, b_i) \quad = \quad ((m-1)\Delta + \tau(b_i), (m-1)\Delta + \tau(b_i) + t(b_i)), \tag{13}$$

we can formally express the above problem as:

$$\min \Delta \ \ s.t. \ \ \mathcal{I}(m_1, b_i) \cap \mathcal{I}(m_2, b_j) = \phi \ \ \forall m_1, m_2 = 1, 2, ... \text{ if } (b_i, b_j) \in G_c \tag{14}$$

Note that G_c is the conflict graph defined in Section 5.3. Since the schedules repeat themselves periodically, it is sufficient to examine possible conflicts in $[0, T_{\max}]$ where $T_{\max}$ is the broadcast latency. Thus, we can simplify the problem to

$$\min_{\Delta \in [0, T_{\max}]} \Delta \ \ s.t. \ \ \mathcal{I}(1, b_i) \cap \mathcal{I}(m, b_j) = \phi \ \ \forall m = 1, 2, ... \text{ if } (b_i, b_j) \in G_c \tag{15}$$

Assuming two transmissions b_i and b_j do interfere with each other, the constraints in equation (15) can alternatively be expressed as:

$$(m-1)\Delta + \tau(b_j) \notin (\tau(b_i) - t(b_j), \tau(b_i) + t(b_i)). \tag{16}$$

The left-hand-side of the above expression is the start transmission time of the m-th packet by transmission b_j and it must not lie in the time interval given on the right-hand-side in order to avoid conflict. This expression also means that Δ cannot take certain values. Thus by identifying all the values that Δ cannot take within $[0, T_{\max}]$, we can easily find the optimal value of Δ. This computation method is similar to domain reduction in constraint logic programming [2]. This algorithm can find the optimal Δ in polynomial time.

6 Simulated Performance Studies

In this section we study the performance of the algorithms proposed in Section 5 to solve the low latency network-wide broadcast problem in a multi-rate wireless mesh network. For the purpose of comparison, we will study altogether 4 heuristics. All these four heuristics have the same structure, computing a broadcast tree and then followed by the multicast grouping (Section 5.2) scheduling algorithm (Section 5.3). In other words, these algorithms only differ in how the broadcast tree are computed. The algorithms to be considered are:

1. Algorithm BIB: Uses BIB in Section 5.1.1 to compute the broadcast tree.

2. Algorithm WCDS: Uses WCDS in Section 5.1.2 to compute the broadcast tree.

3. Algorithm SPT: The broadcast tree is the shortest path tree (SPT) computed by Dijkstra's algorithm. (This algorithm does not exploit the broadcast advantage while computing the tree; however, during transmission, each node transmits to its child nodes in a single transmission).

4. Algorithm CDS: This heuristic assumes that all broadcasts are done at the lowest transmission rate. The broadcast tree can be computed by using WCDS in Section 5.1.2 with only the lowest rate allowed.

The simulations in this section are based on the rate-range relationship in Table 1.

6.1 Small, Regular grid topology

We consider a regular 2-by-4 planar grid network whose physical topology is given in Figure 11. (This topology is deliberately chosen to be a simple and small thus allowing us to compute the optimal broadcasting solution via the integer programming formulation of Section 4.3). The horizontal and vertical separations between the nodes are denoted by, respectively, L_x and L_y. We use 9 different set of values for L_x and L_y, which can be found in Table 2. Each set of values is designed to give different connectivity pattern and transmission link rates. For small values of L_x and L_y, each node can have 6 or 7 neighbours, however, for large values of L_x and L_y, each node may have only 2 or 3 neighbours.

For each given physical topology (i.e. given values of L_x and L_y) and each possible choice of broadcast source node s, we compute the worst case delay given by the heuristics (denoted as $d_{\mathrm{BIB}}(L_x, L_y, s)$, $d_{\mathrm{WCDS}}(L_x, L_y, s)$, $d_{\mathrm{SPT}}(L_x, L_y, s)$ and $d_{\mathrm{CDS}}(L_x, L_y, s)$) and the optimal solution given the integer programming formulation (denoted as $d_{\mathrm{OPT}}(L_x, L_y, s)$). As a measure of performance of each heuristic, we compute, for each given physical topology, the following indices:

$$r_{\mathrm{METHOD}}(L_x, L_y) = \left(\prod_{s=1,\ldots,8} \frac{d_{\mathrm{METHOD}}(L_x, L_y, s)}{d_{\mathrm{OPT}}(Lx, Ly, s)} \right)^{\frac{1}{8}} \tag{17}$$

where METHOD = BIB, WCDS, SPT or CDS. The results are tabulated in Table 2. It can be seen that BIB performs best out of the four heuristics for all the 9 different topologies used. This is due to the fact that BIB is able to exploit both the differential increment in link rates as well as the wireless broadcast advantage. As an illustration, let us consider the case where $L_x = 120$, $L_y = 360$ and source node is 6. In Figure 12, we show the connectivity of Node 4 for this topology. The connectivity of the other nodes can be readily deduced from this. The number next to the arrow shows the relative cost in packet transmission delay. The minimum cost is 1 (when transmitted at 11Mbps) and the maximum is 11 (when transmitted at 1 Mbps). The latency given by the heuristics BIB, WCDS, SPT and CDS are, respectively, 6.5, 11, 22 and 11 time unit. The optimal is 6.5 which is achievable by BIB.

Figures 13 and 14 show the broadcast tree computed by, respectively, SPT and BIB. Although Node 6 (the source node) multicasts to the same set of neighbours in all cases, there are significant differences subsequently. For SPT, in addition to the source node, 3 other nodes (Nodes 5, 7 and 8) will multicast, making the total number of multicasts four. In addition, these three multicasts interfere with each other so their transmission cannot take place in parallel. This result in poor performance of SPT.

On the other hand, the BIB algorithm exploits the wireless multicast advantage and requires only two multicasts in total. The second multicast in BIB (see Figure 14) is performed by Node 2 and it reaches all the three remaining nodes in one go. This demonstrates that BIB is able to exploit wireless multicast advantage. Another feature of BIB is that it exploits incremental link rates. Consider the SPT tree in Figure 13, note that there is a simple modification of the tree which will result in a better latency. This can be seen by noticing that there are two shortest paths from Nodes 6 to 3: 6-7-3 and 6-2-3. Either one of these may be chosen by the Dijkstra's algorithm. However, if we replace the link (7,3) with cost 5.5 in Figure 13 by link (2,3) with cost 1, we still have a shortest path tree but the broadcast latency will be reduced. This is precisely what BIB does and chooses link (2,3) because it has

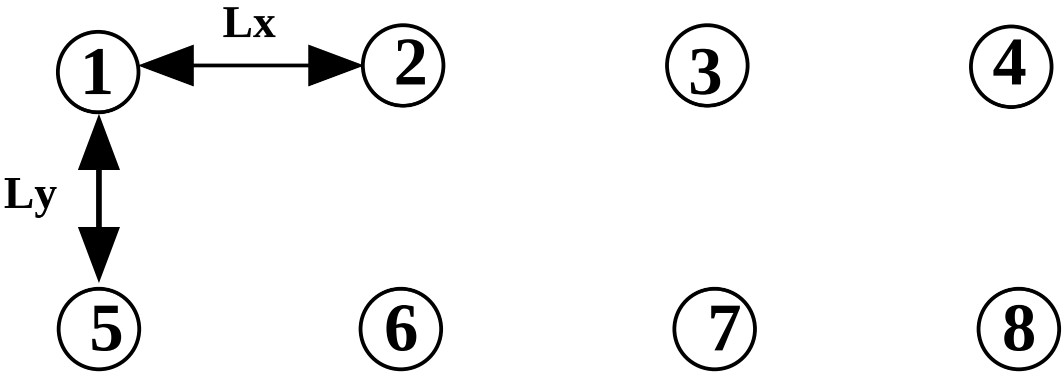

Figure 11: Regular grid topology used.

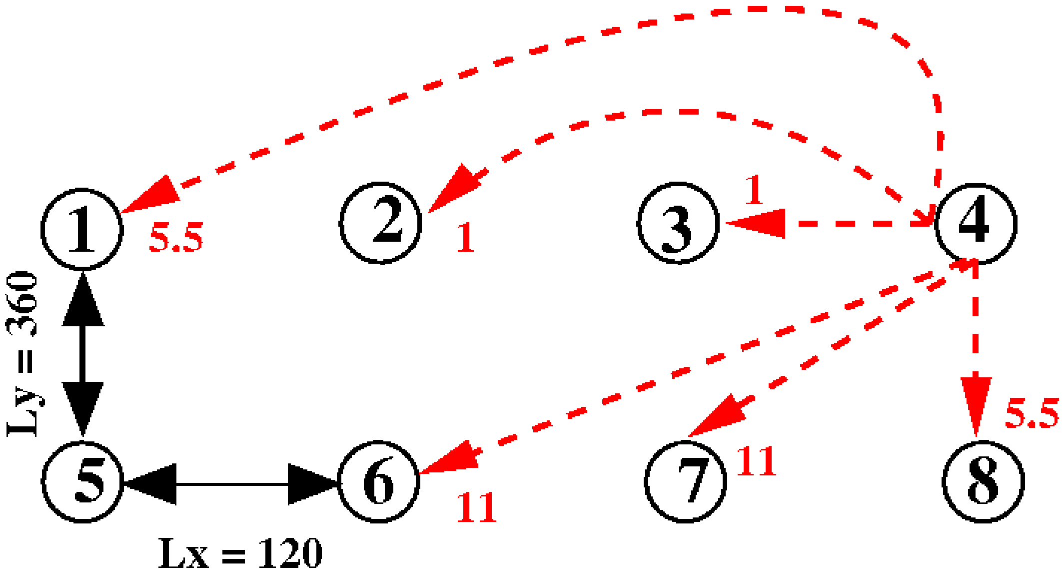

Figure 12: Transmission cost of a regular grid topology.

a smaller incremental cost. It is also important to point out that the BIB tree in Figure 14 is in fact also a shortest path tree though one with better multicast property.

For the network in Figure 12, since all nodes are within the transmission range of Node 6. The CDS algorithm will use one broadcast which takes 11 time units. WCDS also returns the same tree as CDS for this example.

6.2 Heuristic performance in random topology

In this section we compare the performance of the four heuristics using randomly generated topologies of different sizes as measured by the number of nodes in the network. For each network size, we generate 100 topologies whose nodes are uniformly randomly distributed in a square of ℓ^2 km^2. Since the network size that we use is at least 20, integer programming is not able to give us a reference in reasonable time. Instead, we choose to normalise the delay obtained from the heuristics by the delay given by the Dijkstra's algorithm which is the

$L_y = 320$			
heuristic	$L_x = 120$	$L_x = 220$	$L_x = 320$
BIB	1.0773	1.1115	1.1624
WCDS	1.0801	1.0488	1.1547
SPT	1.2123	1.2649	1.1892
CDS	2.1909	2.5298	2.8284
$L_y = 360$			
heuristic	$L_x = 120$	$L_x = 220$	$L_x = 320$
BIB	1.0231	1.0168	1.0595
WCDS	1.6733	1.3093	1.3663
SPT	2.6833	1.8516	1.2649
CDS	1.8974	1.8516	1.7889
$L_y = 400$			
heuristic	$L_x = 120$	$L_x = 220$	$L_x = 320$
BIB	1.0513	1.1642	1.0148
WCDS	1.1547	1.4015	1.2677
SPT	1.3093	1.9640	1.8516
CDS	1.3093	1.6036	1.9897

Table 2: The performance of the heuristics BIB, SPT and CDS compared to the optimal solution for a regular 2-by-4 grid network topology.

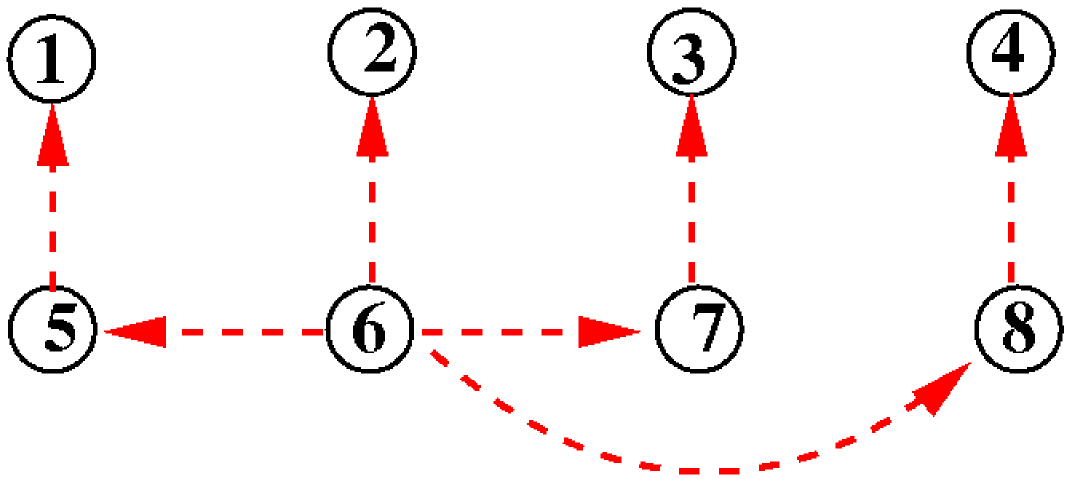

Figure 13: The SPT tree with $L_x = 120$ and $L_y = 360$.

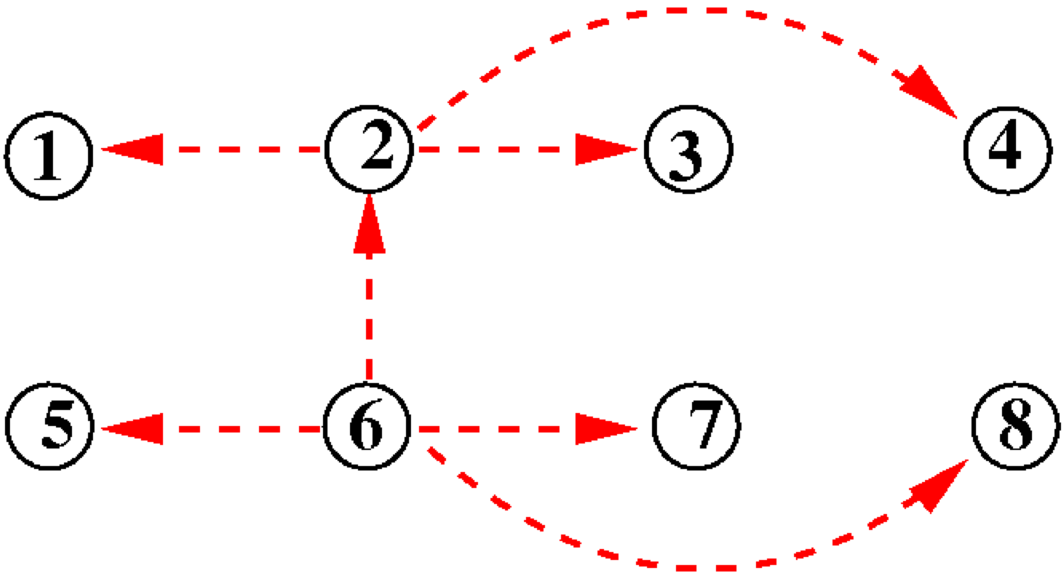

Figure 14: The BIB tree with $L_x = 120$ and $L_y = 360$.

shortest delay possible when there is no limit to the number of radios, channels and times a node can transmit a packet. Thus the minimum value of normalised delay is unity. The result that we will show is the geometric mean, over 100 network instances of a fixed size, of the normalised delay and the throughput.

Our modelling assumption in Section 4.2 states that the interference range is κ times of the transmission range of the lowest transmission rate. We will refer κ to as the normalised interference range. Unless otherwise specified, κ is 1.7 which is identical to that used in [25].

6.2.1 Single transmission case

We first consider the case when each node can transmit a packet at most once. For the simulation, we set $\ell = 1$ and vary the number of nodes from 30 to 100. The results are given in Figures 15 (for delay) and 16 (for throughput). It turns out that good performance for delay also means good performance for throughput and vice versa. WCDS performs best in these experiments and then followed by BIB, SPT and CDS. It shows that both BIB and WCDS are able to exploit the multiple transmission rates available. The SPT algorithm does not exploit the wireless multicast advantage as well and results in higher latency and lower throughput. The failure of SPT to exploit wireless broadcast can also be seen from Figure 17 which shows that SPT on average uses the most number of multicasts per tree out of the four heuristics.

Although CDS uses the least number of multicasts per tree, it fails to exploit the higher transmission rates thus resulting in the worst latency and the lowest throughput.

We also study the sensitivity of the results to the value of interference range. We vary the normalised interference range κ from 1 to 3 in steps of 0.2. Note that since the nodes are distributed within a square of 1 km^2 and the maximum transmission range is 483m (see Table 1), a *kappa* value of 3 corresponds to infinite intererence range. The results for 30-node networks are showed in Figures 18 and 19. Similarly, the results for 100-node networks

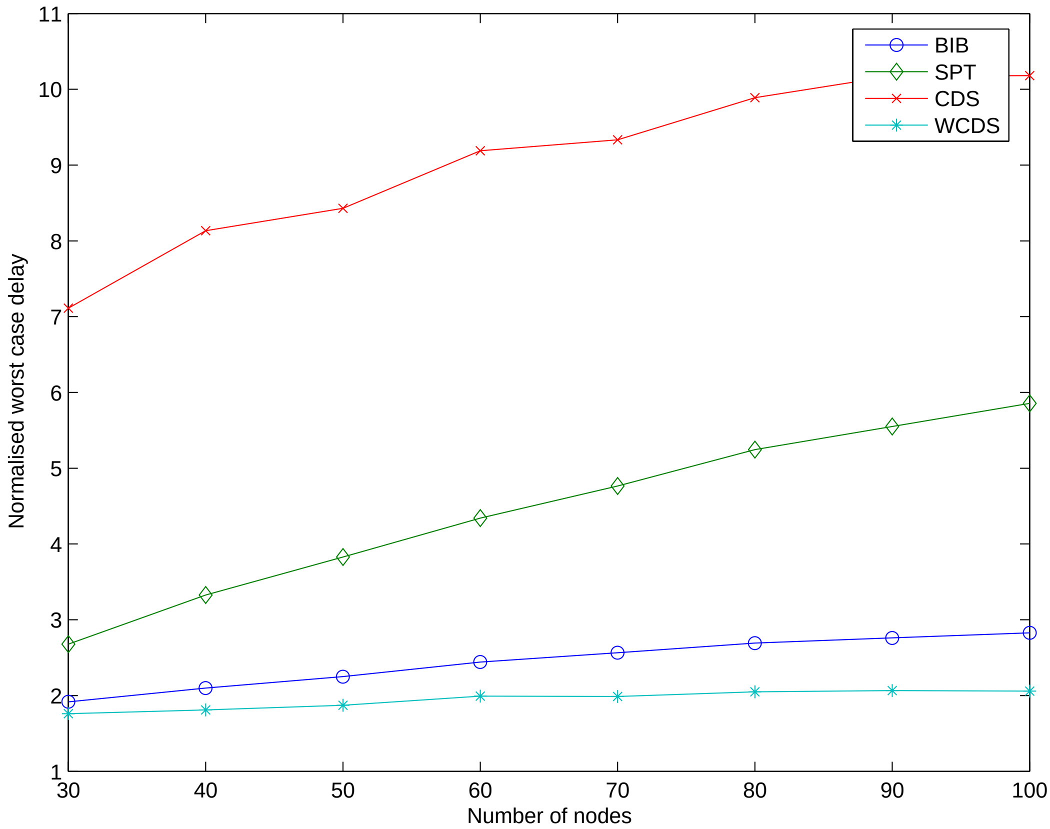

Figure 15: This graph shows the geometric mean of the normalised broadcast latency of BIB, WCDS, SPT and CDS.

are given in Figures 20 and 21. With increasing interference range, these figures show that broadcast latency increases while throughout decreases, which is not surprising. These figures also show that the interference range does not effect the relative performance of the algorithms.

6.2.2 Multiple transmission case

We now consider the case when the nodes are allowed to transmit the same packet multiple times but at different rates. We run our simulation in the same manner as in Section 6.2.1 but we do not limit the number of times a node can transmit the same packet. We find that, over 100 random topologies of fixed number of nodes in a fixed area, multiple transmission do not significantly reduce the broadcast latency. In Figure 22 we plot the percentage of topologies that require multiple transmissions and for those topologies that result in multiple transmissions, the percentage of reduction in delay. It appears that multiple transmission is only required by a fairly small number of topologies, e.g. consider the WCDS algorithm, only 2 out of 100 topologies for a network area of 1 km^2 require multiple transmissions and for those 2 topologies, multiple transmissions result in a 10% reduction in broadcast latency. It appears that multiple transmission may not be required in the single-radio single-channel scenario. However, multiple transmissions is likely to be more useful in the multi-radio multi-channel environment since we know that for the infinite-radio infinite-channel case, the best broadcast

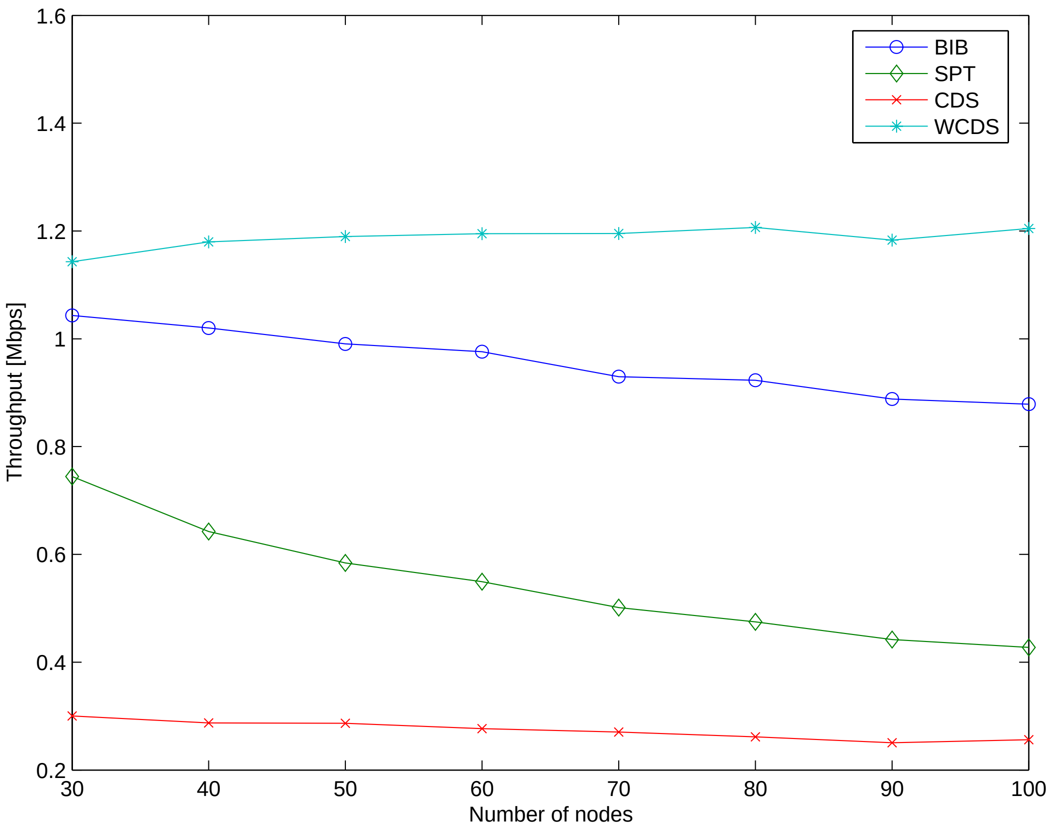

Figure 16: This graph shows the mean throughput of BIB, WCDS, SPT and CDS.

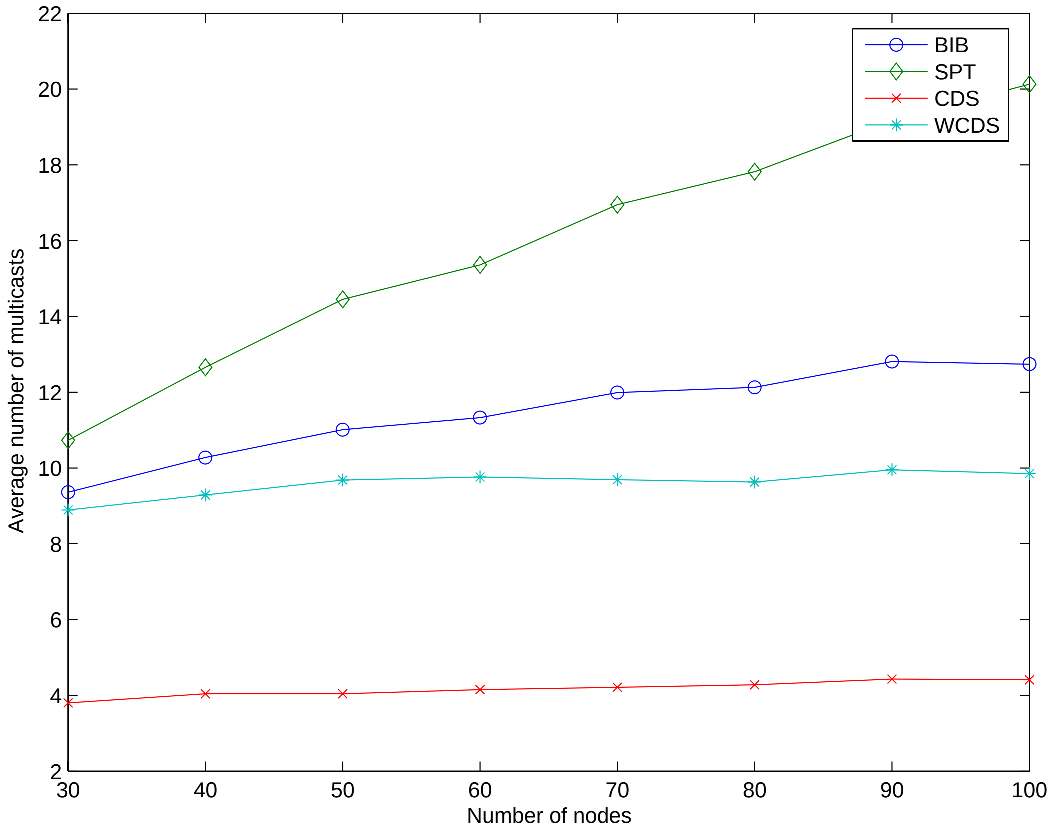

Figure 17: This graph shows the mean number of multicasts per tree for BIB, WCDS, SPT and CDS.

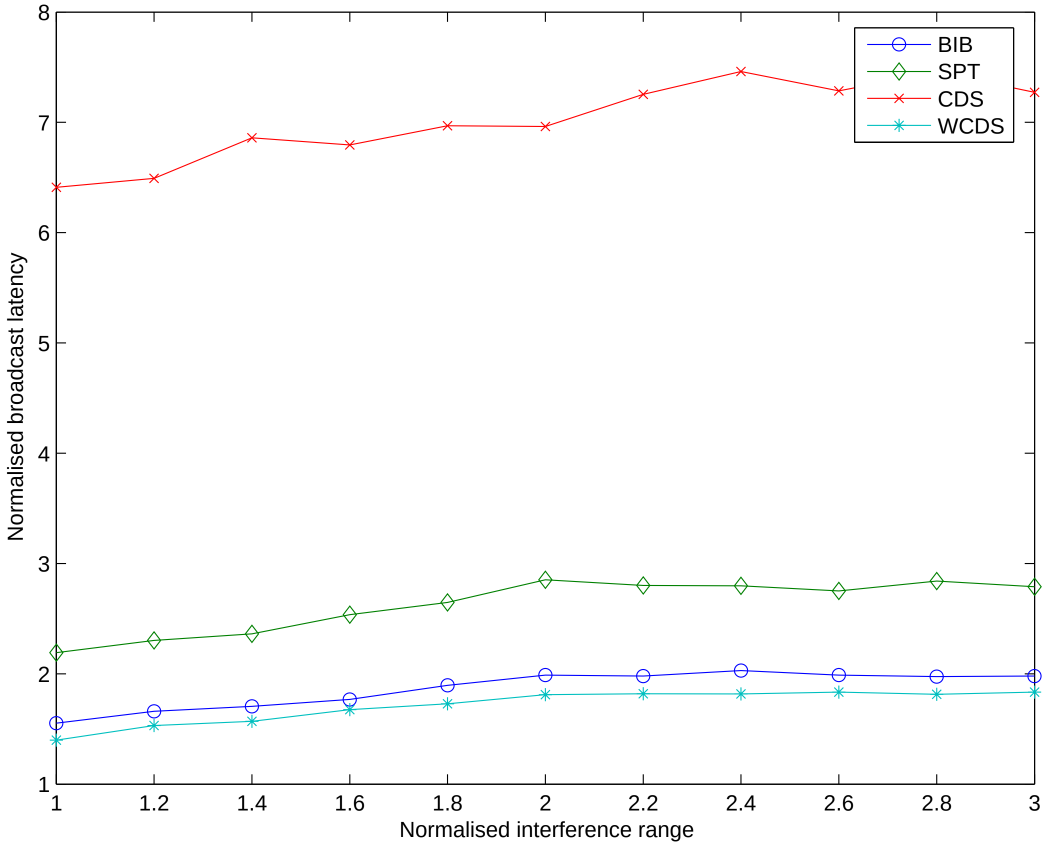

Figure 18: Impact of interference range on the normalised broadcast latency for BIB, WCDS, SPT and CDS. Number of nodes = 30.

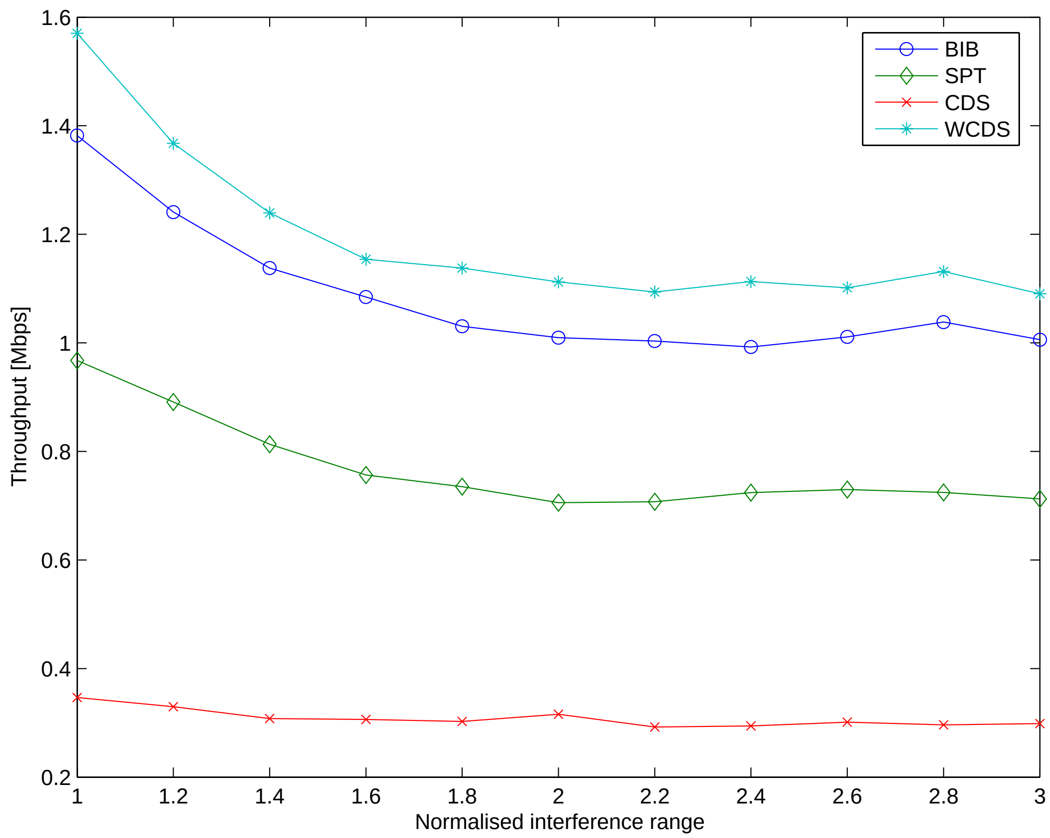

Figure 19: Impact of interference range on the throughput for BIB, WCDS, SPT and CDS. Number of nodes = 30.

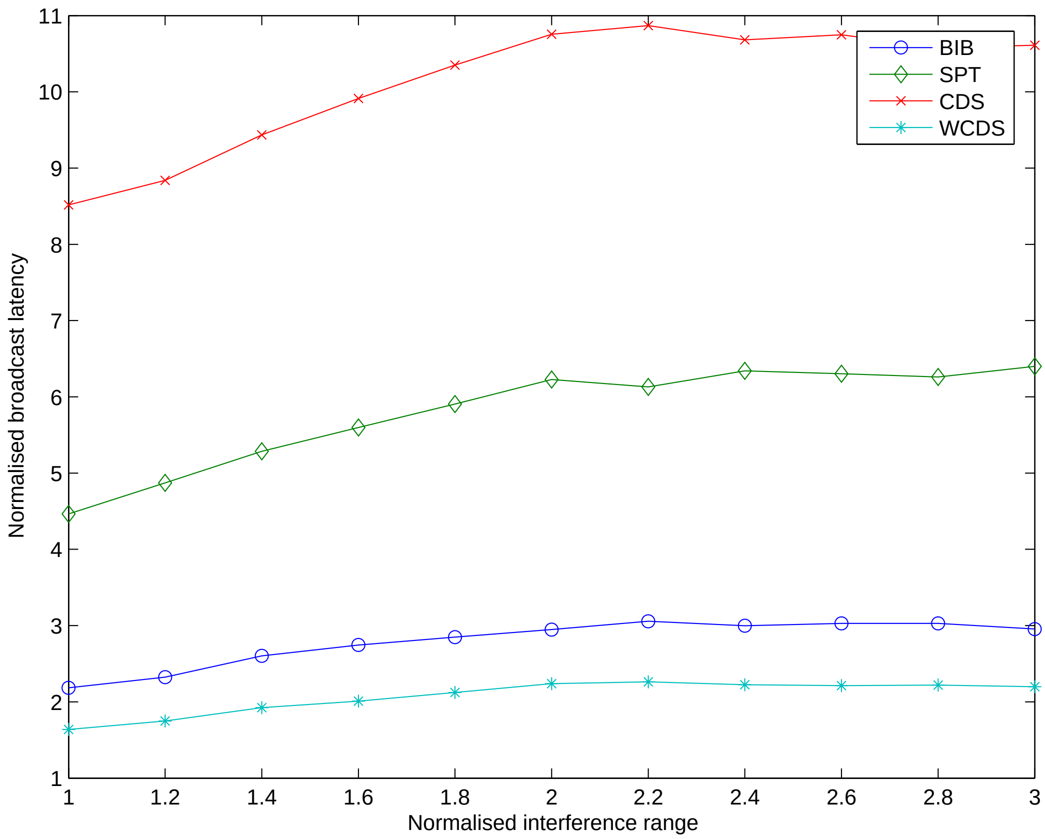

Figure 20: Impact of interference range on the normalised broadcast latency for BIB, WCDS, SPT and CDS. Number of nodes = 100.

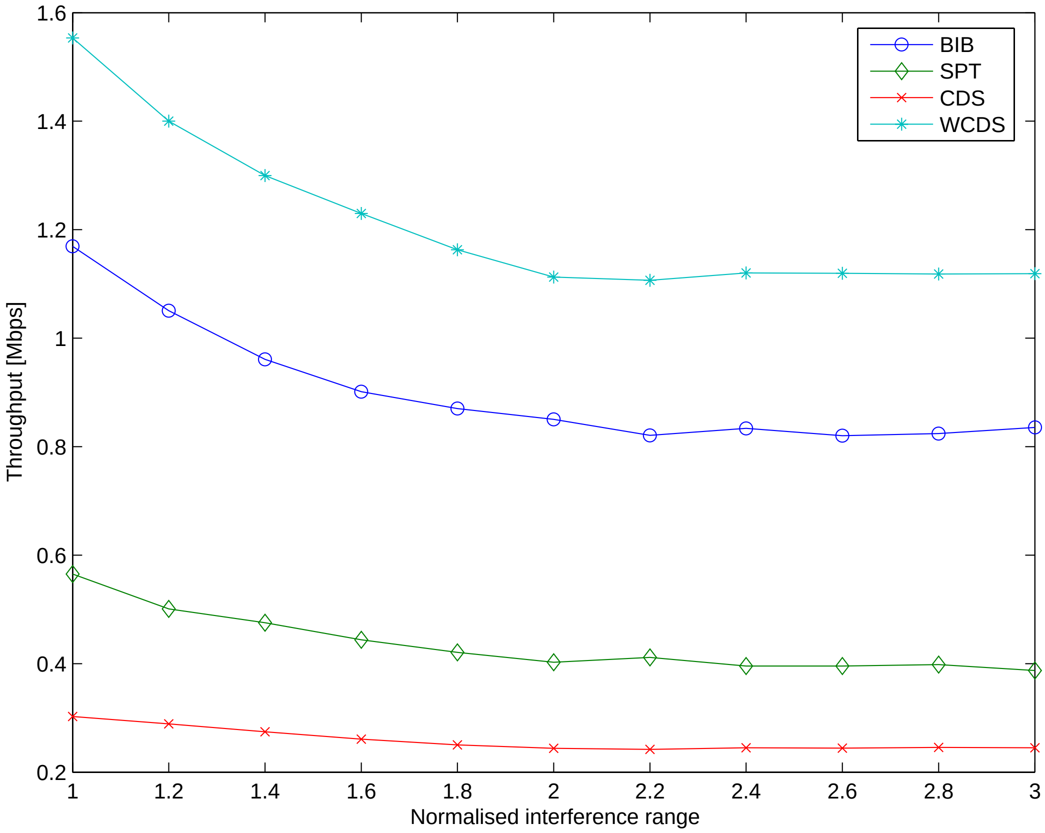

Figure 21: Impact of interference range on the throughput for BIB, WCDS, SPT and CDS. Number of nodes = 100.

latency is achieved by using the shortest path tree which requires multiple transmissions. From Figure 15, we see that single transmission can result in a normalised broadcast latency of about 2. *This means that the potential improvement offered by multi-radio multi-channel for latency reduction is still large, and should be investigated further.*

7 Fundamental Design Principles of Broadcast in Multi-rate Meshes

In Section 6, we study the performance of the heuristics using the transmission rate-transmission range characteristics (or rate-range curve for short) given in Table 1. The results show multi-rate multicast using BIB and WCDS give a lower worst case latency compared with the multicasting using the lowest rate only as given by CDS. In this section, we will study how sensitive this result is to the choice of rate-range curves. The result of this investigation can help us to answer a number of fundamental design questions for multi-rate systems, such as: (1) Given a multi-rate given with n different rates, is it necessary to use all the n different rates? (2) If not, which of the n different rates should we use and what is an efficient method to decide that?

7.1 The transmission rate-transmission coverage area product

In order to study the effect of rate-range curves on the broadcast performance, we use a family of hypothetical rate-range curves as given in Table 3. Our hypothetical system has a minimum transmission rate of r_0 Mbps whose transmission range is d_0 m. Each subsequent transmission rate is a factor of $\rho(> 1)$ greater but whose transmission range is a factor of $\gamma(< 1)$ smaller.

Let us assume for the time being $\gamma = \frac{1}{2}$. Consider the transmission of a frame of size p bits. If the lowest rate is used, this packet will reach all nodes in the area of πd_0^2 in a time of $\frac{p}{r_0}$. However, if this is to be transmitted using the second lowest rate $r_1 = \rho r_0$, then each transmission will only cover an area of $\frac{1}{4}\pi d_0^2$ requiring a shorter time of $\frac{p}{\rho r_0}$ for each transmission. Therefore, four transmissions at rate r_1 can cover the same area as one transmission at rate r_0. Furthermore, in the worst case where these four transmissions at rate r_1 are within the interference range of each other, then they can only take place one after the other and this will take a total time of $4\frac{p}{\rho r_0}$ to complete. Thus, if $\rho > 4$, it will be always be more efficient to transmit at rate r_1.

Generalising the argument used in the last paragraph, we propose to use the product of transmission rate and transmission coverage area (or rate-area product or RAP for short) as a measure of efficiency of a certain transmission rate. Thus, with the hypothetical system given in Table 3, it will be more efficient to use the higher rate if $\gamma^2\rho > 1$, otherwise the lowest rate should be used instead. Alternatively, a transmission rate with a higher RAP is more efficient for broadcast. In order to verify this conjecture, we perform a number of simulations using the same method as in Section 6.2 except that the rate-range curve in Table 3 is used.

In the first set of simulations, we use $\rho = 1.5$, $r_0 = 1$ and $d = 500$. Thus, we expect that, if the above hypothesis holds, then it will be more efficient to use the higher transmission rates if $\gamma > \frac{1}{\sqrt{\rho}} = 0.82$. Five different values of $\gamma = 0.7, 0.75, 0.8, 0.85$ and 0.9 are used. Only heuristics BIB, WCDS and CDS are used. We normalised the delay and throughput by using those of CDS. The normalised delay and throughput of WCDS are given in Figures 23 and

Transmission rate (Mbps)	Maximum transmission range (m)
r_0	d_0
ρr_0	γd_0
$\rho^2 r_0$	$\gamma^2 d_0$
$\rho^3 r_0$	$\gamma^3 d_0$

Table 3: This table shows the relationship between the transmission rates and transmission range for a hypothetical multi-rate system. The range-rate curves are parameterized by $\gamma \in (0, 1)$.

24. It shows that WCDS gives a better latency and throughput than CDS for all values of γ. For $\gamma \geq 0.8$, WCDS exploits multi-rate and gives far better delay and throughput than CDS; for $\gamma < 0.8$, WCDS still performs better than CDS but the results are comparable. These observations therefore confirm our earlier conjecture.

The normalised delay and throughput of BIB are given in Figures 25 and 26. It shows that if $\gamma \geq 0.8$ then BIB has better latency and throughput compared with CDS, but if $\gamma \leq 0.8$, CDS performs better. This suggests that BIB is only better at exploiting multi-rate when the rate-range curve is more favourable. Also, by comparing these Figures, we find that WCDS performs better than BIB for all network sizes and rate-range curves.

We repeat the above experiment but this time we choose $\rho = 2$ which means that it is more efficient to use the higher rate for $\gamma > \frac{1}{\sqrt{\rho}} = 0.72$. The results for WCDS are plotted in Figures 27 and 28. These results again show that WCDS performs better than CDS for all values of γ though the performance gap diminishes for $\gamma \leq 0.7$. We can understand this by looking at the average percentage of times that each transmission rate is used for each value of γ. The results are shown in Table 4. It shows that if the rate-range curve is favourable, then the higher transmission rates are used most of the time. However, even when the rate-range curve is less favourable, the higher rate transmissions are also used but less often.

The results for BIB are given in Figures 29 and 30. They again show that BIB only performs well when the rate-range curves are favourable.

We also study the sensitivity of the above results to interference range. All the above simulations are conducted with a normalised interference range κ, which is defined as the inference range to the maximum transmission range, of 1.7. The experimental settings for this one are identical to the last one except that we use only network with 100 nodes and we use six different normalised interference ranges: 1,1.5,2,2.5, 3 and 5. Since the maximum transmission range is 500m, a normalised interference range of 3 or 5 means an infinite interference range. The effect of interference range on the normalised delay and throughput of WCDS for different range-rate curves is showed in Figures 31 and 32. It shows that the interference range has little effects on the results.

The results in this section show that whether one should try to exploit multi-rate for network wide-broadcast depends on rate-area product of the transmission rates. If the higher transmission rates have a higher rate-area product compare with the lowest rate, then using multi-rate link layer broadcasts can result in significant reduction in broadcast latency. Applying this rule-of-thumb to the rate-range characteristics in Table 1, it can easily verified that the rate-area product is higher for higher transmission rates and this agrees with the results we have in Section 6.2.

	% of times each transmission rate is used			
γ	Rate $\rho^3 r_0$	Rate $\rho^2 r_0$	Rate ρr_0	Rate r_0
0.9	100	0	0	0
0.8	96	4	0	0
0.7	70	18	6	6
0.6	41	27	12	20
0.5	14	28	21	37

Table 4: The table shows the average percentage of times each transmission rate is used for different value of γ. The parameter $\rho = 2$ thus the rates decreases from left to right,

7.2 Channel capacity and multi-rate networks

In Section 7.1, we demonstrate that transmission rates with large RAP are good for achieving low broadcast latency. With improvement in coding, wireless signal processing etc., the achievable wireless transmission rate is pushing closer to the Shannon capacity. An interesting question is to study the RAP if the transmission rate at a distance is given by the Shannon capacity. We consider a system where the bandwidth $B = 10MHz$, the SNR at distance $d_0 = 50m$ is $30dB$. (We will see later that these parameter values will not affect the general discussion here). Assuming that the rate R at distance d is given by the Shannon capacity formula, as follows:

$$R = B\log_2(1 + SNR(\frac{d_0}{d})^\theta) \quad (18)$$

where θ is the path loss exponent. Assuming that $\theta = 4$, Figure 33 shows R and RAP as a function of d. It shows that the RAP increases for small values of d and decreases for large d. This is understandable since for small d, $R \sim log_2(\frac{1}{d})$ and for large d, $R \sim \frac{1}{d^\theta}$. It can be shown, via differentiating $R\pi d^2$, that the transmission rate (whose corresponding spectral efficiency is ψ) that maximises the RAP is the solution to the equation $\psi - \frac{\theta \log_2 e}{2}(1 - 2^{-\psi})$ which says that the optimal ψ is a function of the path loss exponent θ only and not of other parameters. For $\theta = 4$, the maximum RAP (indicated by the dashed lines in Figure 33) occurs around a spectral efficiency of 2.3 bps/Hz. The lowest transmission rates for both 802.11a/b has a spectral efficiency far lower than this and therefore have poor RAP. By adding higher transmission rates with better RAP to 802.11b (see Table 1), the broadcast latency of 802.11b is improved as seen in Section 6. However, the Shannon RAP predicts that RAP will eventually fall for higher transmission rates. From the technical specifications of a commercial 802.11b/g product in [1], we find that the outdoor transmission ranges for rates 1, 6, 11, 18 and 54 Mbps are respectively 610, 396, 304, 183 and 76m, giving RAP of 1.2, 3.0, 3.2, 1.9 and 1.0 Mbps-km^2 which eventually falls for high transmission rates.

We assume a hypothetical multi-rate system by selecting five points from the Shannon rate-range curve indicated by the diamonds in Figures 33. Since it is likely that future wireless systems will have rates with efficiency above and below 2.3bps/Hz, the rate that gives the maximum RAP is selected as well as two points on each side of it. (Note also that the Shannon transmission rate can only be used if no other nodes are transmitting, or in other words, the interference range is infinity. Since we find that in the last section that the interference range has little impact on the result, we keep the normalised interference range as 1.7 as before). We use the same simulation set up as in Section 7.1 except that we use the following five algorithms:

1. WCDS with all the five transmission rates
2. WCDS with only the lowest four transmission rates
3. WCDS with only the lowest three transmission rates
4. WCDS with only the lowest two transmission rates
5. CDS with the lowest transmission rate only

We normalise the results for the various WCDS algorithms using those from CDS. The results are showed in Figure 34. It can be seen that the best results are given by WCDS using all the five rates, thus again confirming that multi-rate is useful for reducing broadcast latency. Since the third rate has the highest RAP, note that there is sizeable performance gap between using the lowest 2 rates and the lowest 3 rates.

8 Conclusions

We propose the novel concept of multi-rate link layer multicast as a way to introduce low-latency network layer multimedia broadcast (or multicast) in a wireless mesh networks. We show that by exploiting both multiple transmission rates and wireless multicast advantage, we can *get* significant reduction in broadcast latency compared with using the lowest rate alone. For example, based on simulations using typical 802.11-based values, the use of our rate-aware WCDS heuristic results in a 3-6 fold reduction in the broadcast latency compared to the CDS algorithm that always performs link broadcasts at the lowest rate. Moreover, at least for a single-channel, single-radio environment, it is more important to exploit the rate diversity than allow each individual node to engage in multiple transmissions–we conjecture that this will change when multiple radios are present on a mesh node.

In addition, we find that the efficiency of a particular transmission rate for broadcast can be predicted by the product of the transmission rate and its transmission coverage area. This provides a rule-of-thumb that the designer of a multi-rate system can use to determine which transmission rates should be included in a multi-rate system. Investigation of theoretical Shannon limits suggest that the case for using at least a small subset of the available choice of rates for link-layer multicasts will become even more compelling, as better modulation and coding techniques are introduced.

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A Integer programming formulation

Under the modelling assumptions stated in 4.2, we can formulate the minimum latency network-wide broadcast problem as a mixed-integer programming problem whose optimal solution can be used as a benchmark for heuristic solution to the problem. The basic decision variables of the optimisation problem are:

1. Whether a node should multicast and to which neighbours this multicast should be made.
2. The timing sequence of these multicasts.

We assume that each node is represented by a point in a metric Euclidean space with V being the set of nodes. The distance between two nodes is given by their Euclidean distance. We first define a number of notation:

1. Each node can broadcast at b different rates $r_1, r_2, ..., r_b$ where $r_1 > r_2 > ... > r_b$. The transmission range of these b broadcast rates are $s_1, ..., s_b$ respectively with $s_1 < s_2 < ... < s_b$. The maximum transmission range $s_{\max} = s_b$.
2. The network is modelled as a directed graph $G = (V, E)$ where $V = \{1, 2, ..., n\}$ is the set of nodes and (i, j) is an element of E (the set of edges) if the distance between nodes i and j is less than $s_{\max}$.
3. We assume a fixed size packet will be transmitted. The time it takes to transmit this packet from node i to node j using the maximum possible rate between these two nodes is denoted by d_{ij}.
4. We assume the source node is node 1.
5. Each node is permitted to multicast the same packet up to $m_{\max}$ times. We will use the term "multicast opportunities" to refer to the multiple times that a node can multicast. These opportunities will be indexed by k in the formulation.
6. Define the following indicator function

$$\delta_{i,j} = \begin{cases} 1 & \text{the reception at node } i \text{ is interfered by transmission by node } j \text{ or vice versa} \\ 0 & \text{otherwise} \end{cases} \quad (19)$$

The decision variables for this optimisation problem are:

$$x_{ik} = \begin{cases} 1 & \text{node } i \text{ uses its } k\text{-the opportunity to multicast the packet} \\ 0 & \text{otherwise} \end{cases} \quad (20)$$

$$y_{ijk} = \begin{cases} 1 & \text{node } j \text{ is a recipient in node } i\text{'s } k\text{-th multicast opportunity} \\ 0 & \text{otherwise} \end{cases} \quad (21)$$

$$\tau_{ik} = \text{the time at which node } i \text{ performs its } k\text{-th multicast opportunity} \quad (22)$$

$$t_{ik} = \text{the earliest possible time at which node } i \text{ performs its } k\text{-th multicast opportunity} \quad (23)$$

$$p_{ik} = \text{the packet transmission time for node } i \text{ at its } k\text{-th multicast opportunity} \quad (24)$$

$$d = \text{the time to reach all the nodes in the network} \quad (25)$$

Based on the above description, we can formulate the above optimal broadcast problem as a mixed-integer programming problem. In order to improve the readability following formulation, we have left some constraints as logical constraints, note that it is straightforward to convert these logical constraints to a set of linear constraints [23].

The optimisation problem can be formulated as follows:

$$\min d \quad (26)$$

$$d \geq t_{i1} \quad \forall i \in V \quad (27)$$

$$\tau_{11} = 0 \quad (28)$$

$$t_{11} = 0 \quad (29)$$

$$x_{11} = 1 \quad (30)$$

$$\sum_{(i,j)\in E} y_{ijk} \geq x_{ik} \forall i \in V; k = 1, ..., m_{\max} \quad (31)$$

$$y_{ijk} \leq x_{ik} \forall i \in V; k = 1, ..., m_{\max} \quad (32)$$

$$x_{i,k+1} \leq x_{i,k} \forall i \in V; k = 1, ..., m_{\max} - 1 \quad (33)$$

$$t_{i,k+1} \geq \tau_{i,k} + p_{ik} \forall i \in V; k = 1, ..., m_{\max} - 1 \quad (34)$$

$$\tau_{ik} \geq t_{i,k} \forall i \in V; k = 1, ..., m_{\max} \quad (35)$$

$$\text{if } y_{ijk} = 1 \text{ then } t_{j1} \geq \tau_{ik} + p_{ik} \forall (i,j) \in E, k = 1, ..., m_{\max}, j \neq 1 \quad (36)$$

$$p_{ik} \geq y_{ijk} d_{ij} \forall i \in V, (i,j) \in E, k = 1, ..., m_{\max} \quad (37)$$

$$\sum_k \sum_{(i,j)\in E} y_{ijk} = 1 \forall j \in N \setminus \{1\} \quad (38)$$

$$\text{if } \sum_{(i,q)\in E} y_{i,q,k_1} \delta_{q,j} + \sum_{(j,q)\in E} y_{j,q,k_2} \delta_{q,i} > 0$$
$$\text{then either } \tau_{i,k_1} \geq \tau_{j,k_2} + p_{j,k_2}$$
$$\text{or } \tau_{j,k_2} \geq \tau_{i,k_1} + p_{i,k_1} \forall i,j \in V \text{s.t.} i \neq j; k_1, k_2 = 1, ..., m_{\max} \quad (39)$$

The time t_{i1} in (27) is the the earliest time at which node i can multicast, it is also exactly the same as the time when a node first receives the packet. The constraints (28) to (30) means that the source must multicast at time zero. Constraints (31) ensures that if a node performs a multicast, it must reach some neighbouring nodes. Conversely, constraint (32) ensures that no node is reach if node i does not multicast. Constraints (33), (34) and (35) ensure that if a node uses multiple multicast opportunities, they are used in order and a later multicast opportunity can only take place after an earlier one has finished. Constraint (36) ensures that a node cannot multicast the first time unless it has received the packet. Constraint (37) ensures that transmission time required in a multicast is the minimum possible time to reach all the intended recipients of that multicast. Constraint (38) makes sure that each node is only the intended recipient of a multicast once. Finally, constraint (39) says that if two multicasts — node i's k_1-multicast and node j's k_2-multicast — interferes with each other (which happens when the antecedent is true), then these two multicasts cannot take place at the same time. The consequent part of constraint (39) says that one multicast can only begin after the other has finished.

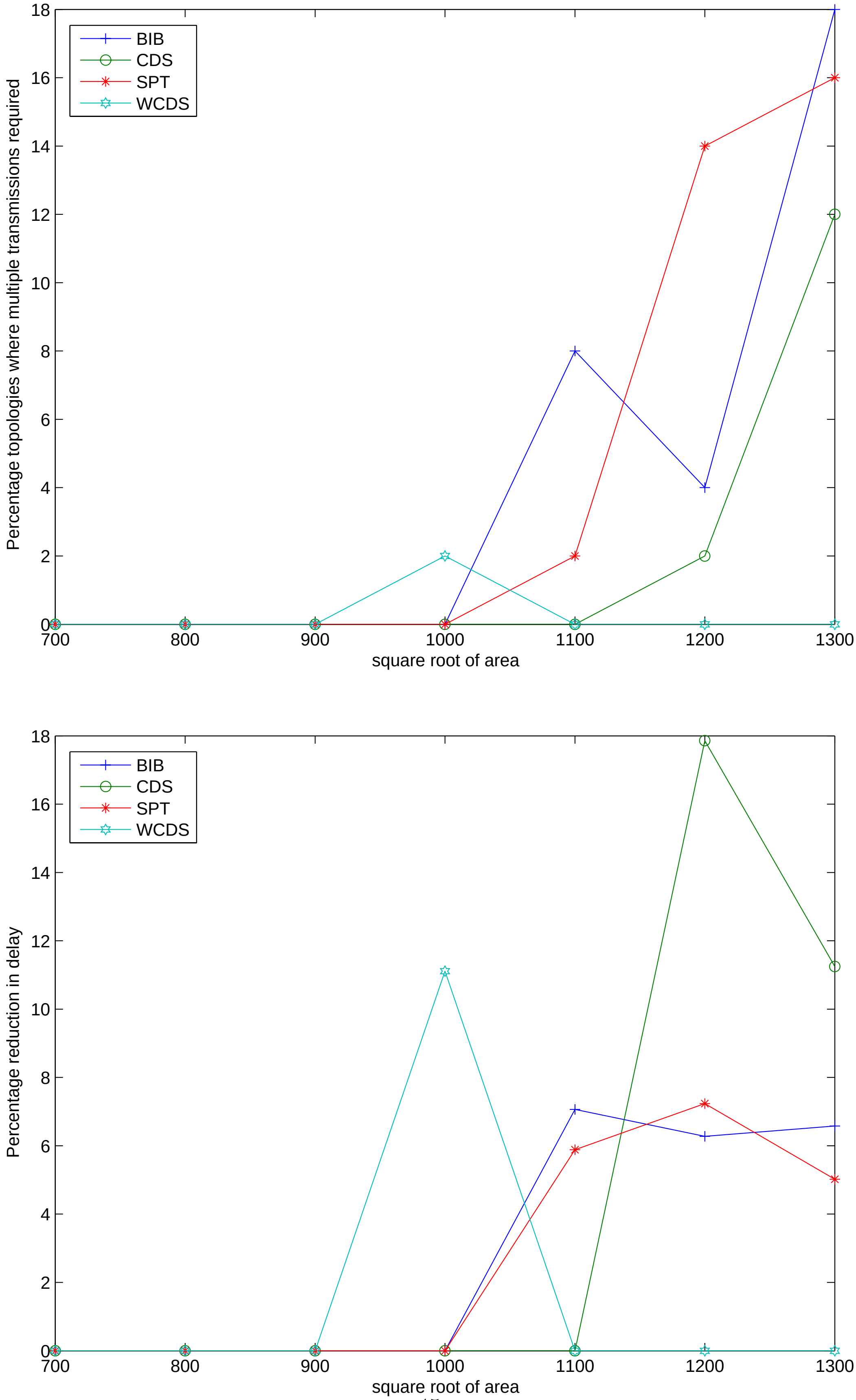


Figure 22: The top graph shows the percentage of topology that requires multiple transmission for a given network area and algorithm. The bottom graph shows the corresponding reduction in delay by using multiple transmission for those topologies that require multiple transmissions.

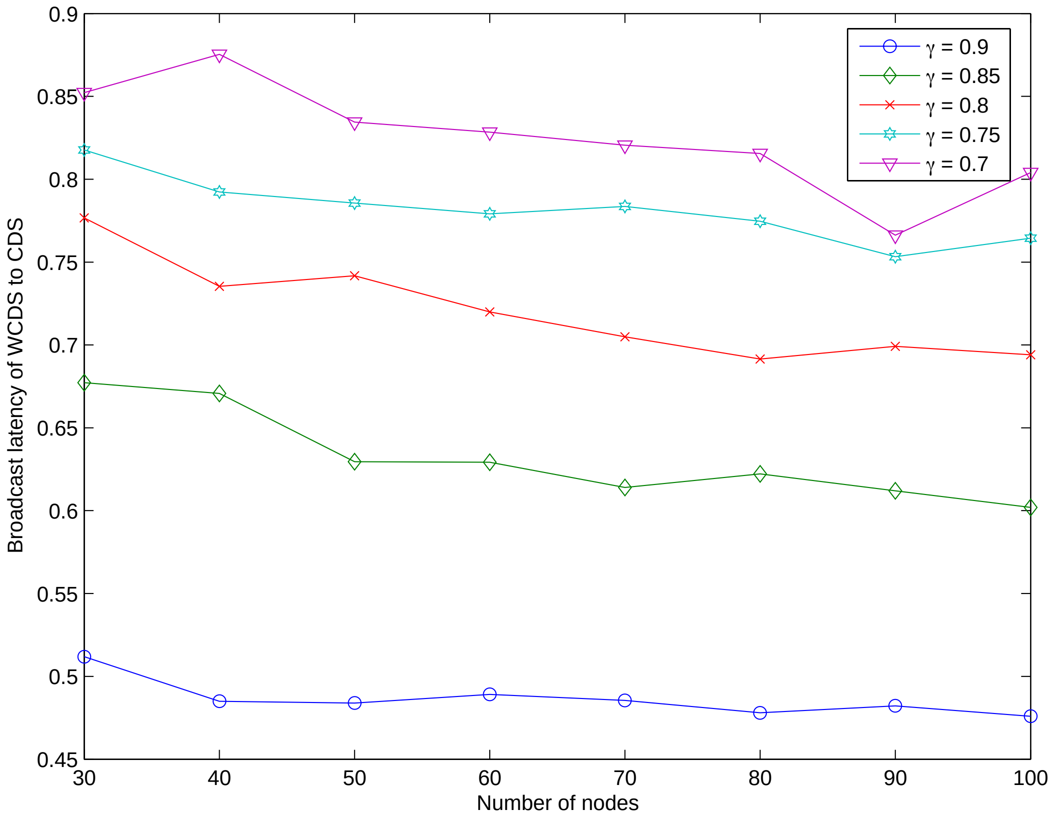


Figure 23: This graph shows the geometric mean of the latency of WCDS to CDS over 100 randomly generated topologies of each network size. The parameter $\rho = 1.5$ and $\gamma = 0.7, 0.75, 0.8, 0.85$ and 0.9.

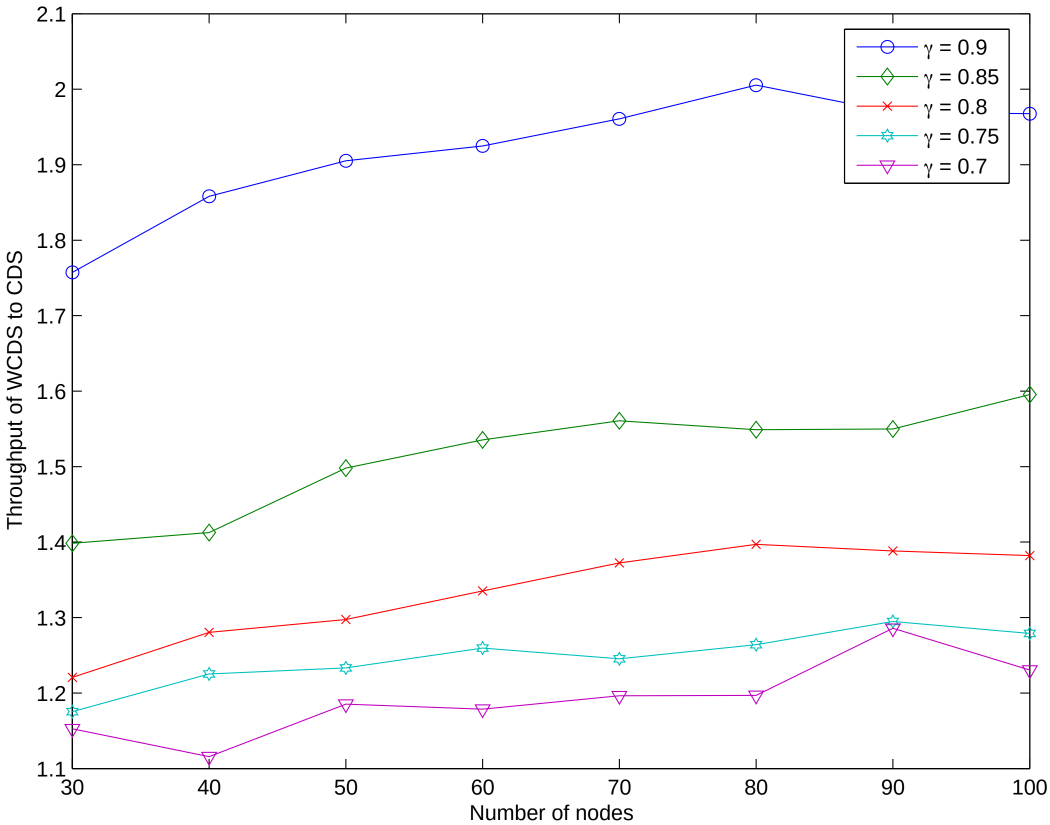


Figure 24: This graph shows the geometric mean of the throughput of WCDS to CDS over 100 randomly generated topologies of each network size. The parameter $\rho = 1.5$ and $\gamma = 0.7, 0.75, 0.8, 0.85$ and 0.9.

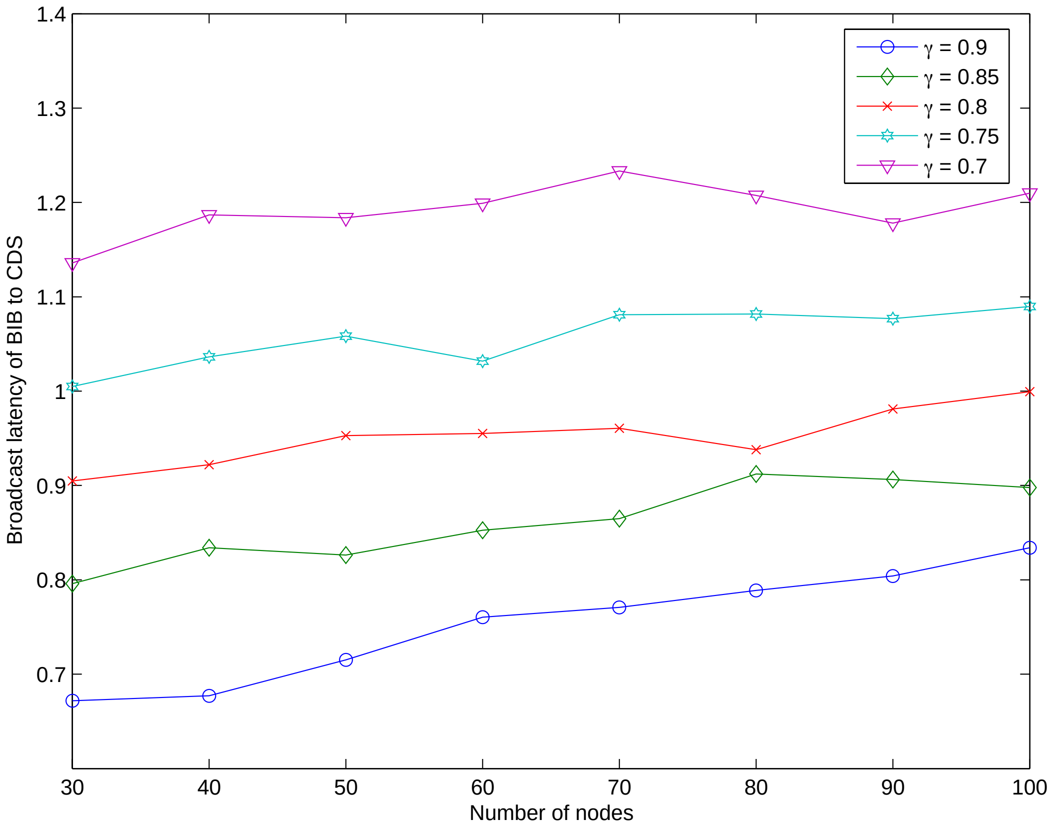


Figure 25: This graph shows the geometric mean of the latency of BIB to CDS over 100 randomly generated topologies of each network size. The parameter $\rho = 1.5$ and $\gamma = 0.7, 0.75, 0.8, 0.85$ and 0.9.

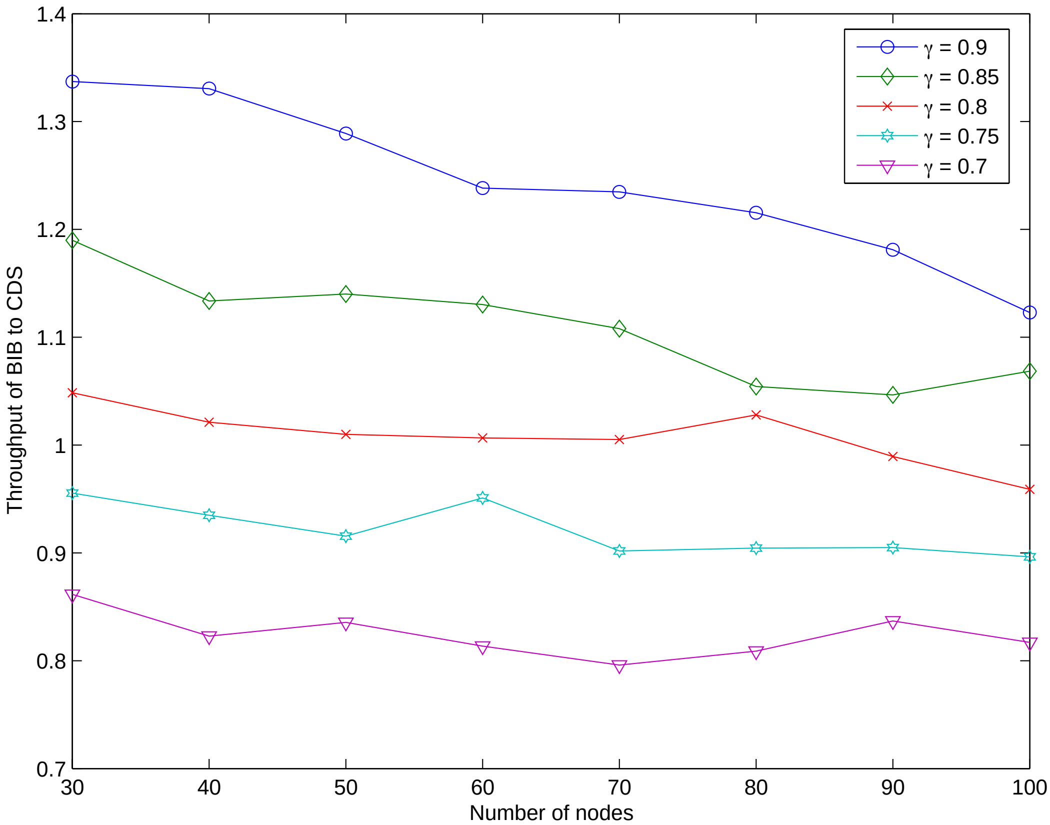


Figure 26: This graph shows the geometric mean of the throughput of BIB to CDS over 100 randomly generated topologies of each network size. The parameter $\rho = 1.5$ and $\gamma = 0.7, 0.75, 0.8, 0.85$ and 0.9.

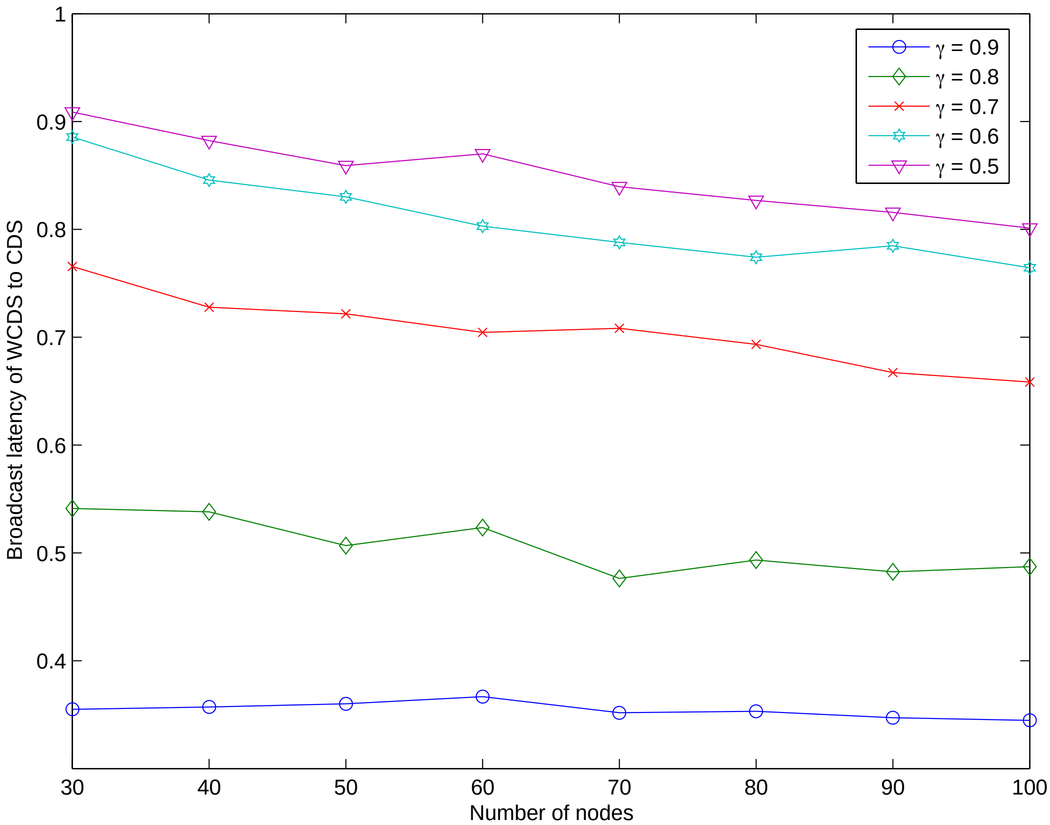


Figure 27: This graph shows the geometric mean of the latency of WCDS to CDS over 100 randomly generated topologies of each network size. The parameter $\rho = 2$ and $\gamma = 0.5, 0.6, 0.7, 0.8$ and 0.9.

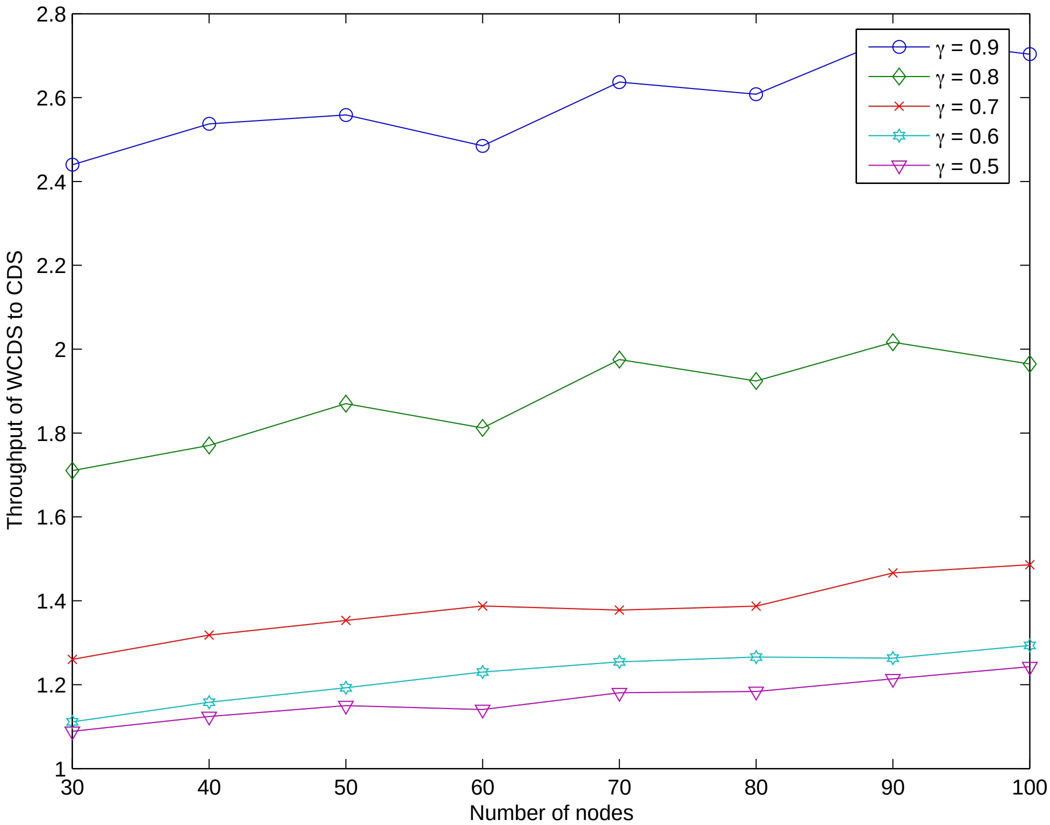


Figure 28: This graph shows the geometric mean of the throughput of WCDS to CDS over 100 randomly generated topologies of each network size. The parameter $\rho = 2$ and $\gamma = 0.5, 0.6, 0.7, 0.8$ and 0.9.

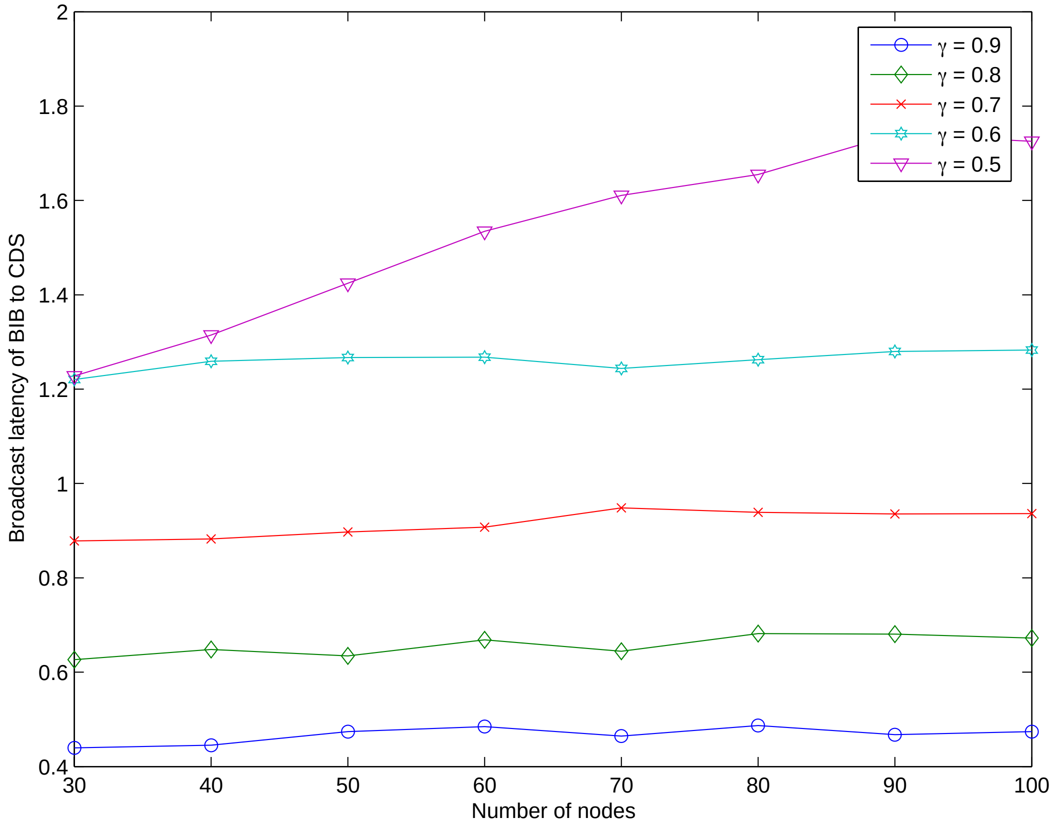


Figure 29: This graph shows the geometric mean of the latency of BIB to CDS over 100 randomly generated topologies of each network size. The parameter $\rho = 2$ and $\gamma = 0.5, 0.6, 0.7, 0.8$ and 0.9.

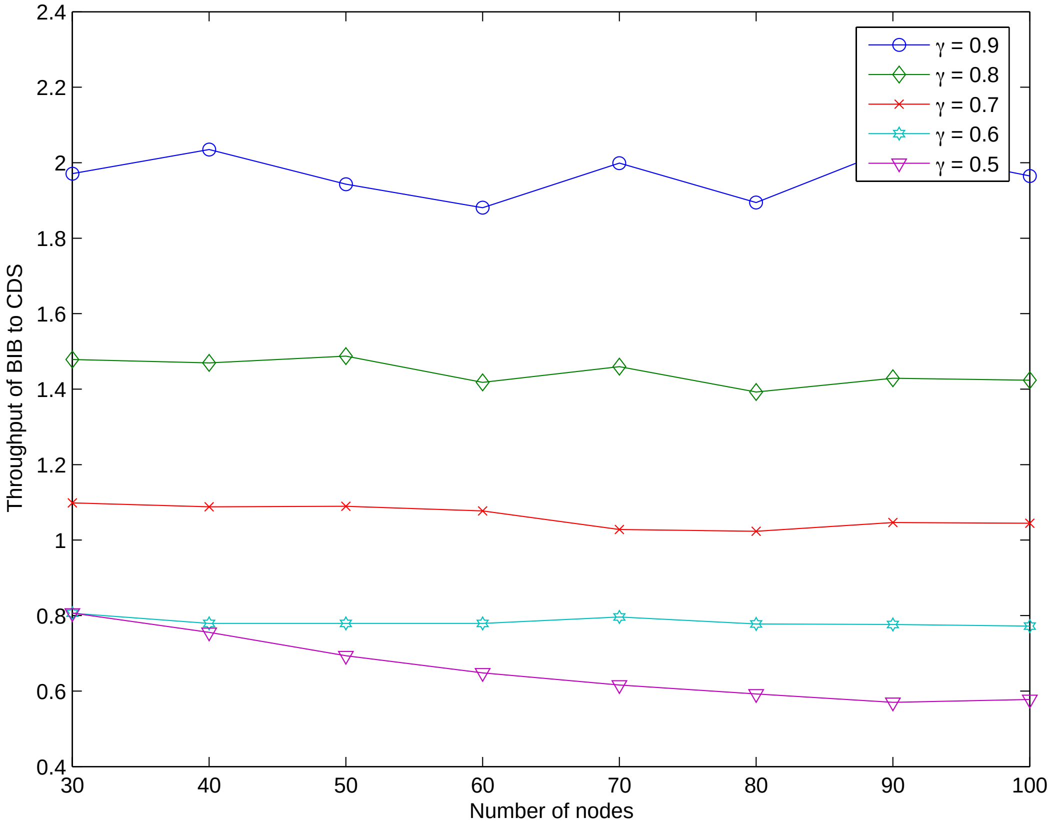


Figure 30: This graph shows the geometric mean of the throughput of BIB to CDS over 100 randomly generated topologies of each network size. The parameter $\rho = 2$ and $\gamma = 0.5, 0.6, 0.7, 0.8$ and 0.9.

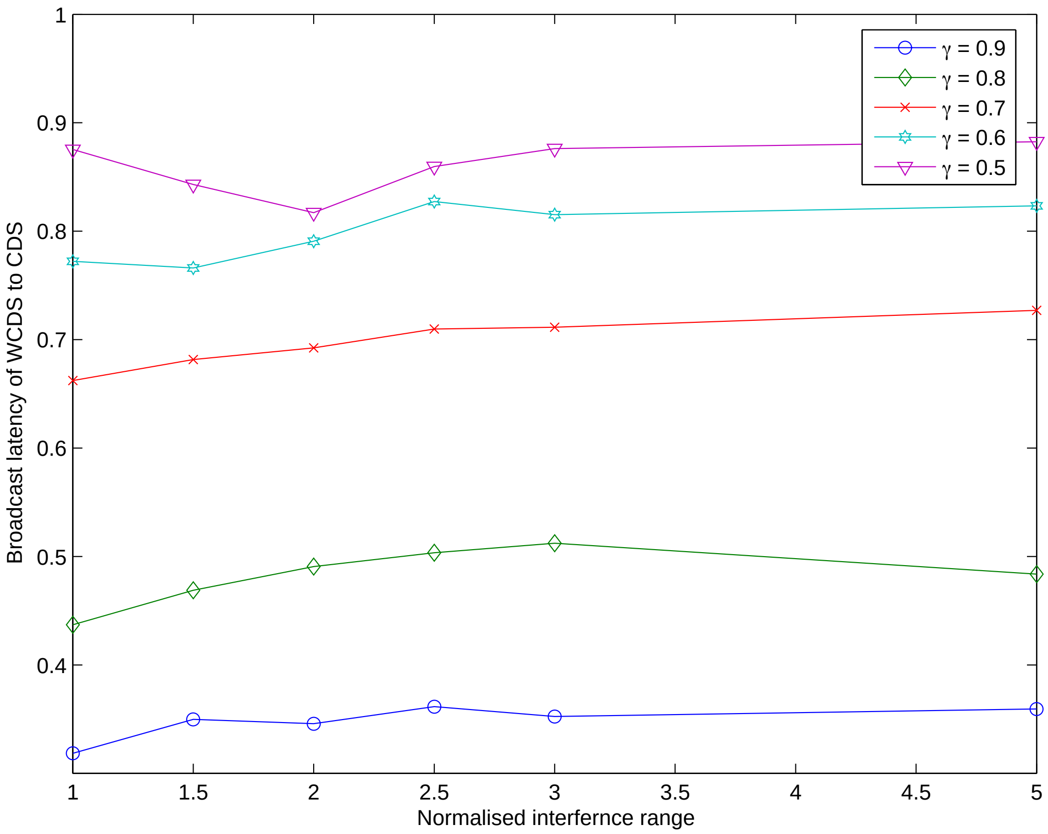


Figure 31: This graph shows the effect of interference range on the normalised latency of WCDS. The network size is 100. The parameter $\rho = 2$ and $\gamma = 0.5, 0.6, 0.7, 0.8$ and 0.9.

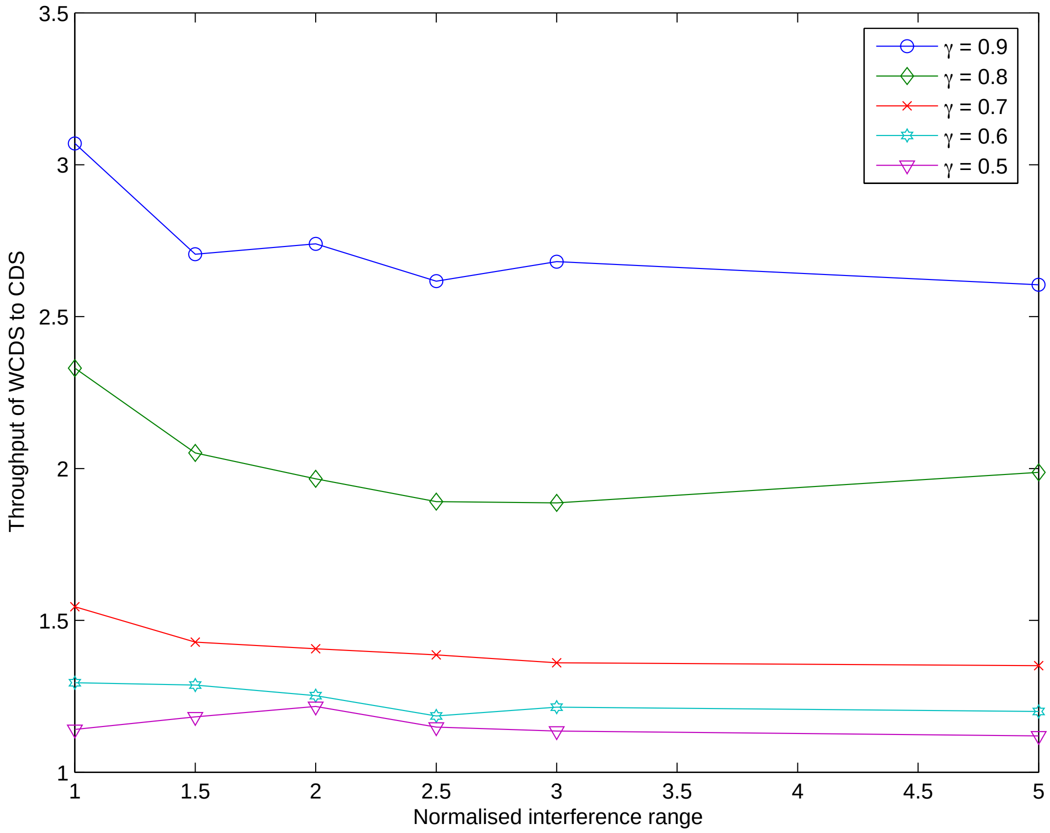


Figure 32: This graph shows the effect of interference range on the normalised throughput of WCDS. The network size is 100. The parameter $\rho = 2$ and $\gamma = 0.5, 0.6, 0.7, 0.8$ and 0.9.

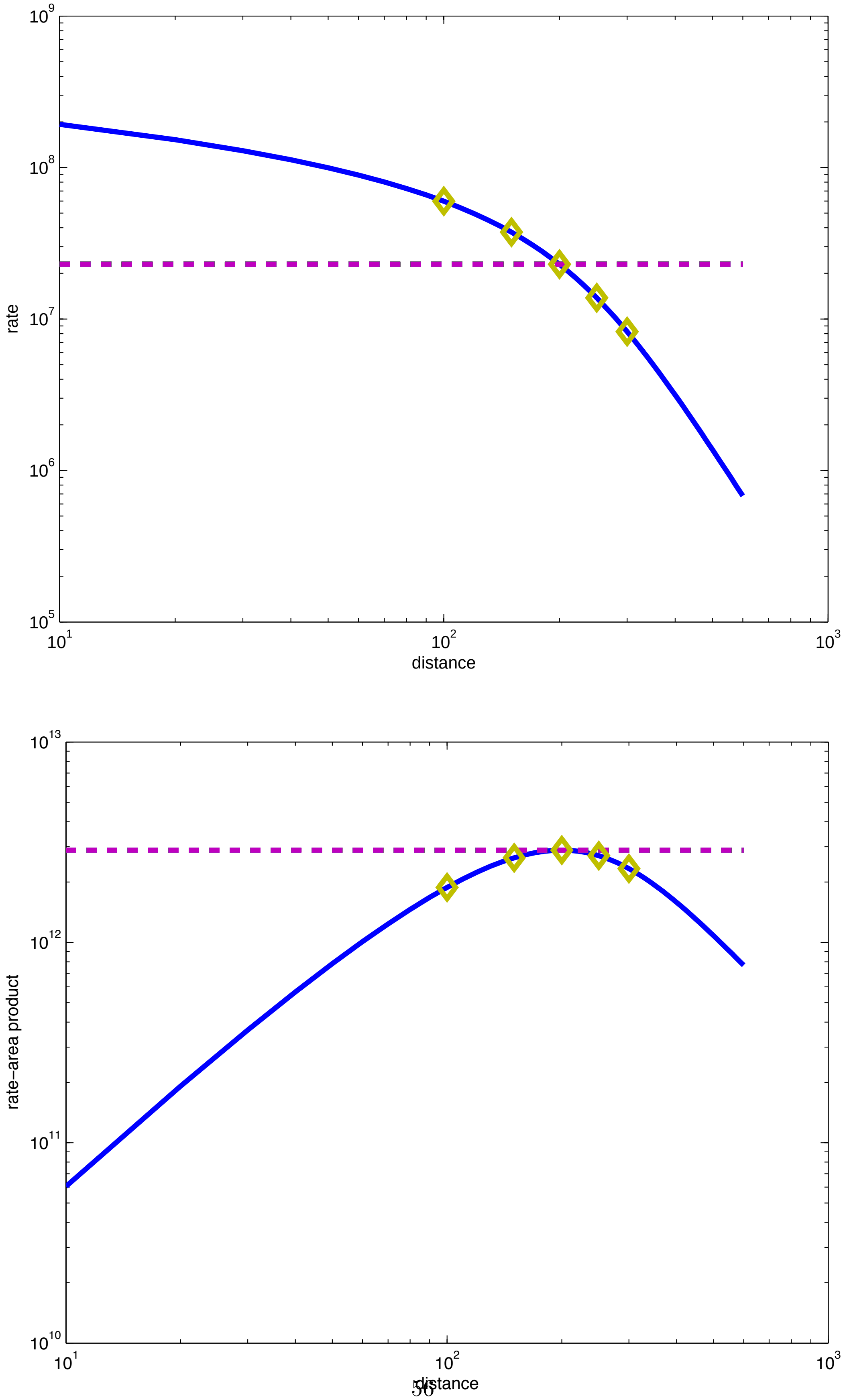
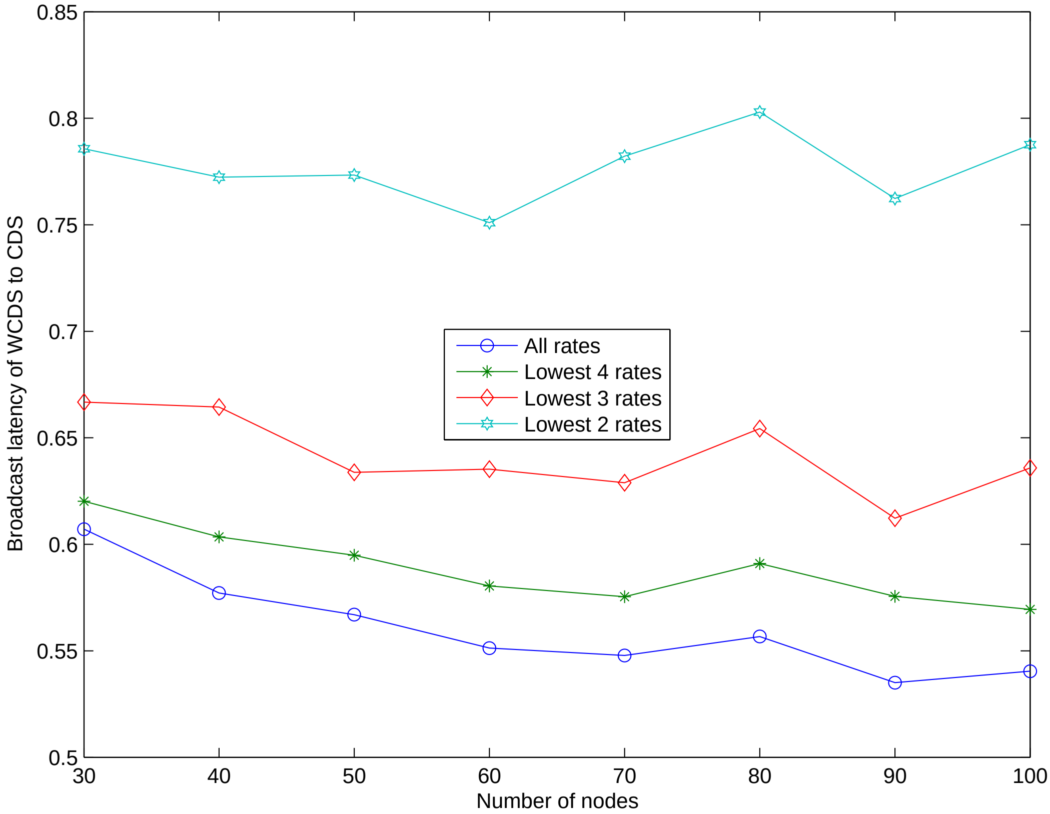


Figure 33: The top graphs shows relation between distance and Shannon capacity. The bottom graph shows the correspond rate-area product.



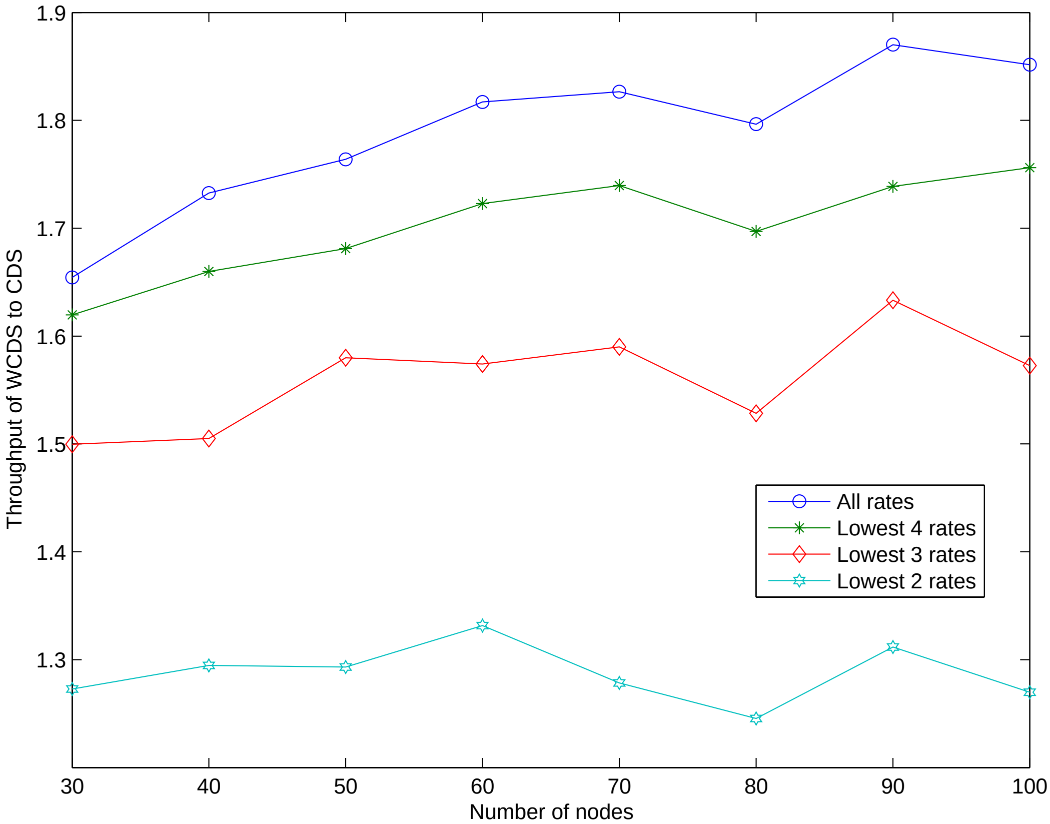


Figure 34: Performance of WCDS algorithm (relative to CDS) using different number of transmission rates. The top graph shows latency while the bottom one shows throughput.