A Theory of Proximity Relations

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Abstract

Next to orientation and connectivity, proximity is one of the key topological properties of many spatial relations. The aim of the work presented in this report is to provide a formalism that can qualitatively account for absolute binary proximity relations, taking into consideration common-sense spatial knowledge. The theory of nearness presented here is based on the concepts of influence areas of spatial objects and distances between these objects abstracted into a pseudometric space.

This theory goes beyond existing models and influence area approaches, by generalising them and providing a formalisation of nearness notions. The most general nearness notions are justified against a set of experimental results obtained from studies conducted by Worboys [30] in the domain of environmental spaces.

The symmetric notion of nearness, which we found to be an adequate representation for most cases, is elaborated on in more detail. Its implications are investigated in the context of a navigational model.

There are however cases where nearness is not symmetric. Therefore a brief discussion on the asymmetric aspect of nearness is given and its implications are investigated in the context of a natural language model.

1 Introduction

The aim of the work presented in this report is to provide a formalism that can qualitatively account for absolute binary proximity relations, taking into consideration common-sense spatial knowledge. This formalism lays the foundations for practical applications in fields such as Geographic Information Systems or Robotics. Next to orientation and connectivity, proximity is one of the key topological properties of many spatial relations. Many of the existing spatial reasoning formalisms do not account for proximity while others stipulate the notion of nearness by using natural language expressions, such as *close to* or *near*, as symbolic values. Guesgen and Albrecht [12] suggest to associate spatial binary relations such as *far from* or *close to*; or unary relations such as *downtown* with fuzzy membership grades, which could be calculated for locations in a map using Euclidian distance.

The use of ternary relative nearness relations has also been a common approach with van Benthem's [29] theory of space introducing nearness relations such as "A is closer to B than C." A very interesting approach to ternary nearness relations is Edwards and Moulin's [8] suggestion of a Voronoi model to represent spatial information. In terms of spatial proximity, the preposition *near*, for example, is interpreted as an n-ary relation (with $n \geq 3$) meaning "nearer than the surrounding points". Guesgen [13] proposed to define proximity without any distance measure by using the notion of fuzzy sets previously defined in Guesgen and Albrecht [12]. These fuzzy sets where used to reason about the relationship between proximities by the means of transitive induction on ternary proximity relations such as "If B is closer to A than C is to A and if C is closer to A than D is to A, then B is closer to A than D is to A." The approach of Edwards and Moulin, and the approach of Guesgen and Albrecht are both very innovative and interesting, however they are computationally quite expensive.

Some models, such as the navigational model by Kettani and Moulin [15], assume that objects are near to a reference object if they are situated within the reference object's area of influence. The work presented in this report also acknowledges the importance of areas of influence to nearness relations. In addition to that however, we assume that some objects can be near when only their influence areas intersect, without any of the objects themselves being positioned within the influence area of the other object. This makes it a much more general approach than any of the previous ones, thus providing for a greater range of spatial relations.

We assume that objects are abstracted as points and positioned into a pseudo-metric space by human beings or more generally cognitive agents. During this abstraction process, the agent's common-sense knowledge about the object is reduced to a weight that codes various properties such as the size, danger impact or desirability of reaching the object. These weights can account for different interpretations of the same real world situation.

The weight associated with an object is used to define the area of influence of this object. Influence areas are combined with notions from the theory of proximity spaces, whose fundamental definition is that two sets are near if they share at least one closure point. This also forms the basic assumption of the most general notion of our nearness formalism. These assumptions are justified against a set of experimental results obtained from studies conducted by Worboys [30] in the domain of environmental spaces.

Our abstraction process is assumed to conform to the figure-ground separation of visual scenes. Originating from Gestalt Psychology ([16], [20]), this perceptual organisation assumes that there are two main visual components necessary for a person to see an object properly. One component is the figure i.e., the object and the other component is the ground i.e., the background or surroundings in which the object occurs. When one looks at a complex scene or listens to a noisy environment, some stimuli are emphasized and stand out clearly and others are perceived to be less relevant background; these are figure and ground respectively.

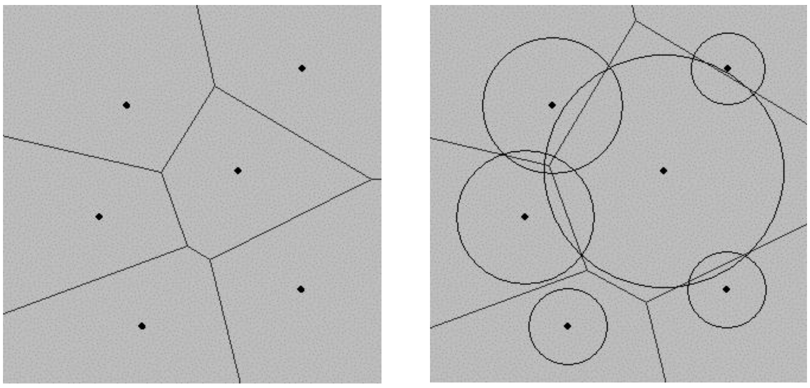
For example, picture a scene with a bicycle being near a house. While there are other objects and backgrounds in the scene that the observer senses, only some of the objects are actually perceived. In our context, only the house and the bicycle are perceived, thus constituting the figure, whereas everything else in the scene is ground. It has also been found that this figure-ground separation is applied to map reading. For example, point symbols are detected on the background of a complex highway map or one region is identified as figure on another region that is functioning as ground [19].

Considering this kind of figure-ground separation, it becomes quite clear that, depending on the focus of attention of the cognitive agent, different objects are chosen as figure or ground at different points in time. For the following expositions, it is always assumed that the universe we are dealing with is the result of an abstraction at one particular point in time.

Objects are only abstracted into weighted points if the agent gives them figure status during his/her perceptual abstraction. All the other objects in the scene have ground status and their abstractions are therefore non-distinct points i.e., points that have no weight associated with them.

More formally, the universe considered has a unary predicate applied to it that decides which points are important within the space and have certain properties that distinguish them from the other points. These distinguished points are very much the same as the previously discussed figure and we call them *sites* or *generating points*. When we talk about nearness, we are always referring to nearness between sites, not between points that are not sites.

The formalism proposed in this report draws on an idea that originated from Computational Geometry. Voronoi Diagrams are used to divide a plane of points into Voronoi regions representing all the points closer to the generating point of a particular region than to the generating point of any other region. This principle is often illustrated with what is termed as the post office problem. Given all the post offices in a particular city, a Voronoi Diagram can be generated using the post offices as generating points. If one is now looking for the closest post office to a given query point, it can easily be identified by verifying which post office is the generating point of the region in which the query point is situated. Another example could be a map in which the cities are used as generating points (or sites) while everything else is considered to be the background. The Voronoi region for each city would include all map objects such as roads or smaller settlements other than cities, which are closer to this particular city than to any other city on the map. Objects being as close to one city as to another[1] form the boundary between the Voronoi regions of these cities. Van Benthem's logic of space [29] also very much follows this principle of ternary nearness relations e.g., by stating that one object is as near to a second as it is to a third object. Power Diagrams add another dimension to Voronoi Diagrams by introducing weights to every site, thus defining the area of influence of every site spreading out from the generating point.



(a) Ordinary Voronoi Diagram (b) Power Diagram

Figure 1: Voronoi Example of a Map of Schools in a District

This can be illustrated by an example. Given several schools in a particular district, an Ordinary Voronoi Diagram can be used to indicate which residential areas are closer to which school. This is shown in Figure 1(a). A school can also have a certain catchment area which might not coincide with its (Ordinary) Voronoi region e.g., because the school might have a large capacity. Power Diagrams do therefore not only add the areas of influence i.e., catchment areas in the case of the schools, but they also adjust the Voronoi

[1] This represents a particular ternary nearness relation.

Edges according to the weight of every site. Therefore, a school with a larger catchment area will also have a larger Voronoi region. Figure 1(b) shows the previous Voronoi Diagram after the weights, represented by circles in the figure, have been added.[2] Aurenhammer [2] discusses Power Diagrams or Generalised Voronoi Diagrams in more detail.

Influence areas spreading out from a centre combined with notions from the theory of proximity spaces form the basic assumption of our nearness formalism. The formalism requires only two parameters, the distance between points and the weight associated with sites.

This provides the means for describing both concepts of proximity valid for any value of the parameters and the scope of each concept for specific values of the parameters. The focus in the work presented here is on describing concepts of proximity valid for any value of the parameters. A full exposition on the scope of the concepts i.e, the coverage of a particular concept for specific values of the parameters, is left subject for future work. It should be noted that proximity of space is also often referred to as *nearness* [25] and both terminologies will be used interchangeably in this report.

In Section 3, a "family" of notions of nearness is discussed, formalised and justified on the basis of case studies, each of which provides a consistent interpretation of two axioms representing the generic notion of nearness.

In Section 4, the "sub-family" of the symmetric notions of nearness introduced in Section 3 is revisited and its properties are discussed. Then its implications are investigated in the context of two models: navigation and natural language. Examples of cross-country navigation are used for the former and a set of spatial expressions for the latter model.

In most cases, the symmetric notion of nearness is adequate. There are however cases where nearness is not symmetric. The asymmetric aspect of nearness is discussed in more detail in Section 5.

One of the classic examples in the linguistic literature, used to show that nearness relations in natural language can be asymmetric, is the nearness relation between a house and a bicycle. Symmetry means that if the bicycle is near the house, then the house would also be regarded as near the bicycle. But, according to Talmy [27], this is not necessarily the case. Crangle and Suppes [4] did also investigate nearness in spatial language, however they did not consider the asymmetry of nearness at all.

The basic assumption of the formalism is justified by validating it against a set of experimental results in environmental spaces obtained by Worboys [30].

[2]Both diagrams were generated using Brendan Lane's Computational Geometry applet http://pages.cpsc.ucalgary.ca/~laneb/Power/index.html.

2 Foundations for a Formalism of Nearness

Connectivity-based topological relations are a very efficient way of representing many spatial relations qualitatively, as demonstrated in calculi such as the Region Connection Calculus [5]. However, as Kuipers [17] points out, a metric is needed to represent certain aspects of spatial knowledge. Nearness, for example, cannot be accounted for in these connectivity-based topological relations. Grading of discreteness[3] is needed and therefore some distance measure is essential.

The formalism proposed here is based on pseudo-distance, which is a generalisation of distance. Pseudo-distance allows for points to be distinct even when the distance between them is zero. A space having a pseudo-distance as its metric is called a pseudo-metric space. In many cases, pseudo-distance will actually be distance[4]. Pseudo-distance and the weight of every site are the only two parameters needed, keeping the formalism as simple as possible.

Pseudo-distance has the advantage that points or regions can be considered to be pseudo-connected without being the same or sharing at least one common point respectively and therefore can be used to describe the cognitive notion of points or regions being next to each other. For a detailed discussion on the mathematical implications of applying proximity space properties to region-based theories of space see Varkarelov et al. [28].

This formalism draws upon and adapts features of the theory of proximity spaces which is discussed in detail by Smirnov [26]. This theory is based on sets and their proximity relationships to each other. The abstraction process creates a point-based universe representing physical space and the objects it contains as points. This universe will also be referred to as the universe of discourse.

Notation 1 (Universe) *We denote by $\mathcal{U}$ a pseudo-metric space.*

As previously mentioned, we assume that some cognitive agent has abstracted real world entities into points, retaining distance information, and some information about important properties of all those original material entities that are perceived as relevant to the agent's reasoning goals. These properties are retained in the form of a weight assigned to the points. Points that have weights associated with them are referred to as sites.

Notation 2 (Sites) *We denote by $sites(\mathcal{U})$ the collection of sites in the universe $\mathcal{U}$ with $sites(\mathcal{U} \subset \mathcal{U}$.*

[3]Several objects that are discrete i.e., not connected, can have very different spatial relations to each other in addition to their discreteness. Some objects might be considered near to each other while others might be considered far from each other. This distinction cannot be achieved by discreteness alone. Grading discreteness therefore refers to the different nearness relations between objects that are spatially discrete.

[4]i.e., where distance=0 implies points are identical

Drawing upon information about the real entities that the points abstract, the agent is assigning an area of influence to each of them. Real entities that were not perceived as relevant to the agent's reasoning goals do abstract into points without weights associated with them.

The areas of influence include all the points in the universe that fall within them. This gives us sets further used within a proximity space setting, which is a pseudo-metric space.

2.1 Pseudo-distance and Pseudo-equality

Recall the definition of pseudo-distance: A function $D : \mathcal{U}^2 \rightarrow \mathbb{R}_+$ is a pseudo-distance iff:
(P1) $\forall x \in \mathcal{U},\ D(x,x) = 0$
(P2) $\forall x,y \in \mathcal{U},\ D(x,y) = D(y,x)$
(P3) $\forall x,y,z \in \mathcal{U},\ D(x,z) \leq D(x,y) + D(y,z)$
Note that the above definition also allows for $x \neq y$ if $D(x,y) = 0$, which differentiates pseudo-distance from distance.

Notation 3 (Pseudo-equality) *We denote by $=_D$ pseudo-equality defined by D.*

The relation $=_D$ is the equivalence relation over $\mathcal{U}$ accounting for both the metric space equivalence relation $x = y$ and the fact of x being as close as possible to y i.e., being intuitively next to each other, so close that they are almost indistinguishable however not equal in the traditional sense. $x =_D y \leftrightarrow D(x,y) = 0$.

2.2 Dense pseudo-metric Spaces

Physical spaces are always dense and even though the notion of dense spaces is not explicitly used in this report, we include the definition of dense pseudo-metric spaces for future reference.

We assume that there exist infinitely many individuals representing material spatial entities. Through an abstraction process, assumed to be performed by the cognitive agent, the universe of discourse $\mathcal{U}$ is a *dense* and possibly infinite pseudo-metric space. Density is defined as follows.

Definition 4 *$(\mathcal{U}, D)$ is said to be dense just in case for all $p,q \in \mathcal{U}$ and $\Gamma > 0$, if $D(p,q) < \Gamma$ then $\{x \in \mathcal{U} \mid D(p,x) < \Gamma \wedge D(q,x) < \Gamma \wedge x \neq p \wedge x \neq q\} \neq \emptyset$.*

2.3 Weights and Influence Areas

Every site p has a weight $\omega(p) \geq 0$ associated with it, retained from the abstracted "real world" entity. The weight is therefore an abstraction to code quantitative and qualitative properties of sites, considered important by a cognitive agent, such factors are discussed in more detail by Gahegan [11].

The interpretation of the weights is restricted to physical constraints such as an agent's vision ability or the danger impact of a falling tree. Therefore the weight of every site is a combination of the object's physical properties and what the agent perceives of it in the context of the current situation or the task to be performed by the agent. The mental or cognitive biases of the agent itself in terms of spatial reasoning is incorporated into newly defined nearness concepts.

Notation 5 (Weight) *We denote by ω a function from sites$(\mathcal{U})$ into $\mathbb{R}_+$.*

The area of influence of a site p is a function of the weight $\omega(p)$; for the nearness formalism it is described as the set of points in the area of influence, including p.

Definition 6 (Influence Area) $IA^{\omega}_{D}(p) = \{x \in \mathcal{U} \mid D(x,p) \leq \omega(p)\}$

When the context permits, we will write $IA(p)$ instead of $IA^{\omega}_{D}(p)$. Note that although the square root of the weight is used in the original Voronoi definition of Power Diagrams (see [2]), only the weight itself is used in the above influence area definition. Every formalisation requires a particular choice of ω and it is therefore left to the user to define the weight appropriately if the square root or in fact any other function of the weight is required.

Let us now consider nearness for some specific distances and weights. We will examine the effects of some distance and weight on the influence area itself. They are simple examples, but are worth mentioning here.

Equality of two points: For any points x and y, let $D_0(x,y) = 0$ if $x = y$ and $D_0(x,y) = 1$ otherwise. We can then conclude by the definition of influence:
For all sites p, if $\omega(p) < 1$ then $IA^{\omega}_{D_0}(p) = \{p\}$
For all sites p, if $\omega(p) \geq 1$ then $IA^{\omega}_{D_0}(p) = \mathcal{U}$.

Pseudo-Equality: Let a pseudo-distance D be given. For any points x and y, let $D_1(x,y) = 0$ if $x =_D y$ and $D_1(x,y) = 1$ otherwise. We can then conclude by the definition of influence:
For all sites p, if $\omega(p) < 1$ then $IA^{\omega}_{D_1}(p) = \{x \in \mathcal{U} \mid x =_D p\}$.
For all sites p, if $\omega(p) \geq 1$ then $IA^{\omega}_{D_1}(p) = \mathcal{U}$.

3 A Theory of Nearness

This theory of nearness is based on the definitions for influence area and distance as stated in the previous foundations section. In the following case studies, several notions of nearness are identified. These different notions of nearness belong to the same "family" of nearness relations for specific (D, ω).

We formalise nearness as satisfied by two axioms. Axiom (A1) is straightforward. We assume that every site is near itself. Linguistically, there might be cases where an object is not considered near itself, but formally this is a natural assumption that does not need to be justified. From the case studies we have done, we could conclude that any two sites whose influence areas do not intersect have no nearness relation. Axiom (A2) expresses this property.

$(A1)$ *For all sites* p $Near(p,p)$
$(A2)$ *For all sites* p, q $IA(p) \cap IA(q) = \emptyset \rightarrow \neg Near(p,q)$

Based on these axioms, which constrain all nearness notions, we will now study various interpretations of one particular abstracted space of four sites, which is shown in Figure 2. These different interpretations each follow a particular case study. For each of the interpretations all possible models in terms of nearness are considered in order to define and justify further notions of nearness.

The case studies are divided into small-scale and large-scale space settings, as these two kinds of space exhibit quite different properties with respect to nearness. The line drawn between these two spaces is not always clear and therefore not always consistent across the literature, which also employs numerous terms in similar contexts. We will try to overcome these discrepancies where possible and aim to provide a clear distinction within our settings. As a rule of thumb, in the English language, we would usually describe the orientation of objects in small-scale spaces by using relative orientation relations such as *left of*, *right of*, *in front of* or *to the back of*.

It should also be noted that every possible model (i.e every possible case) for each interpretation is considered, even if particular cases might be quite unlikely. That means that all possible models, that are not explicitly excluded by a certain interpretation, will be included in the analysis for that interpretation.

3.1 Small-Scale Spaces

Small-scale spaces are characterised as spaces whose structure is within the sensory horizon of the agent. Manipulable objects in a small-scale space are essentially three-dimensional with all three dimensions being about equally important. When an object is moved, its spatial and non-spatial properties, apart from its positioning, remain practically unchanged [9].

We will first consider a magnetic field example. Contrary to other approaches to representing nearness that consider only the influence area of one of the objects, such as Kettani and Moulin's [15] model, nearness is de-

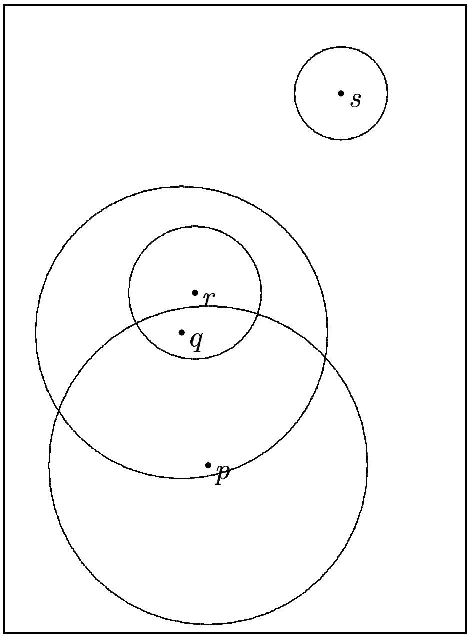


Figure 2: Example Space with four Sites, showing influence areas

termined by the relationship between the influence areas of both objects in our theory. This magnetic field example very clearly shows the significance of our approach. Due to the more general nature of our nearness theory, a model such as Kettani and Moulin's can easily be incorporated as one of its interpretations.

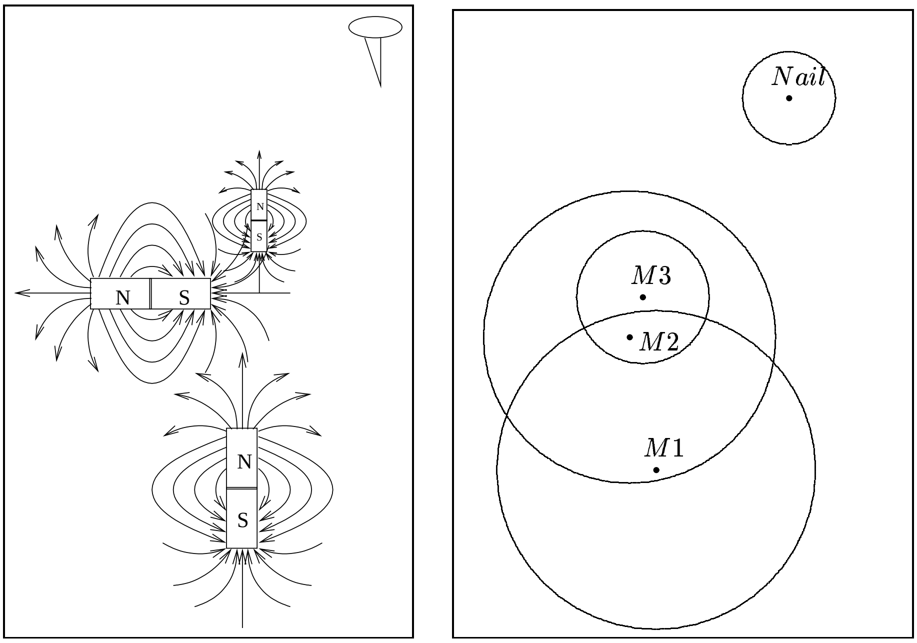
We will then move on to the example of a *table-top* scene, meaning by that the manipulable space on top of a table. It should be noted that we do refrain from calling this scene a *table-top* space example to avoid confusion, because the term table-top space is often used synonymously with small-scale spaces.

The example following the table-top setting is that of, what we will call, egocentric space. In this space, the agent doing the abstraction is part of the scene itself and he or she considers everything that he or she can reach as near.

Finally, an environmental space setting is explored, which is a substantially larger space than the previous one. However, because it can be observed from a single point of view, we still consider it small-scale space.

3.1.1 Magnetic Fields

This particular interpretation shows a space of three permanent two-bar magnets and a nail (i.e. an unmagnetised iron object). The scene is shown in Figure 3.



(a) Magnets and Nail

(b) Abstracted Space

Figure 3: The Influence of Magnetic Fields

It is well known that magnets attract unmagnetised iron objects and attract or repel other magnets depending on their polarity. A bar magnet sets up a magnetic field in the space around it, and a second body responds to it [24]. For example, if a nail happens to be within a magnetic field, it will be drawn towards the magnet. For magnets however, the second magnet does not have to be within the first magnet's field to either be drawn to or repelled from it, it is sufficient when the second magnet's field gets into contact with the first magnet's field.

In this interpretation, the term magnetic field describes the region within which the friction with the table top is not enough to stop an object i.e., the magnets or the nail, from moving. This means that the influence areas depend on the frictional characteristics of the object being attracted or the frictional characteristics in addition to the magnetic properties of a magnet being attracted or repelled. In order to consider the spatial setting of the magnets, we assume that the observing agent is moving the magnets into the positions shown and then holding on to them. This allows their fields to intersect without them being physically moved at first encounter. The nail in the scene is not affected by any of the magnets. The nearness relations between these magnetic objects and even between unmagnetised iron objects and magnets are truly symmetric. This results in exactly one model of the scene shown in Table 1, where T stands for a *True* and F stands for a *False* nearness relation.

	M1	M2	M3	Nail
M1	T	T	T	F
M2	T	T	T	F
M3	T	T	T	F
Nail	F	F	F	T

Table 1: Model of Nearness in the Scene in Figure 3

The model clearly satisfies axiom (A1) with every object being near itself. Axiom (A2) is satisfied by the nail, whose influence area does not intersect with the influence area of any of the magnets, being not near any of the magnets.

It can be observed that the magnets whose influence areas intersect, are also near. The following definition expresses this symmetric nearness notion s-near1.

Definition 7 *For all sites p and q,* s-near1(p,q) *holds iff* $IA(p) \cap IA(q) \neq \emptyset$*.*

This is is the most general nearness notion in our "family" of nearness relations for specific (D,ω). This formulation was derived from the context of proximity spaces, in which two sets are near each other if they share at least one closure point. Adopting this to a universe containing distinctive points i.e., sites and their associated areas of influence, nearness holds for two sites if their influence areas intersect i.e., share at least one point.

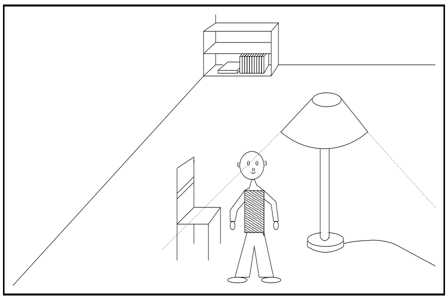
This does not only make sense in the context of proximity spaces, but can also be shown to be a reasonable approach for physical space. We analysed experimental results obtained from studies conducted by [30] in the domain of environmental spaces in the context of influence areas and used these results to validate this general nearness notion s-near1. It was found that s-near1 was satisfied by 99.56% of all empirical cases i.e., 230 out of 231 cases recorded by Worboys.

In addition to notion s-near1, it can also be noted that in the model shown in Table 1, when sites are included in the influence area of another site, they are always near. For example, magnet M2 is in the influence area of magnet M1 and M1 in the influence of M2. This nearness notion s-near4 is a specialisation of s-near1.

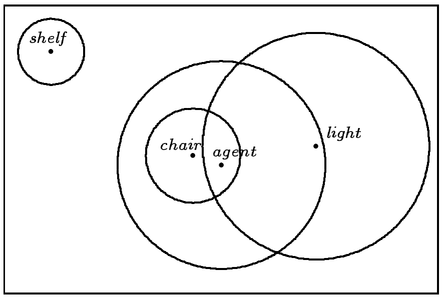
Definition 8 *For all sites p and q,* s-near4(p,q) *holds iff* $p \in IA(q) \wedge q \in IA(p)$*.*

3.1.2 Egocentric Space

In this particular interpretation, the agent is part of the scene and his or her area of influence is determined by how far he or she can reach. This reachability influence area translates into everything within the agent's reach including objects reached by reasonably small forward-backwards or sideways movements such as bending forwards or taking a small step. Everything that is within the reach of the agent is also near the agent, but the reverse is not necessarily the case. For a model of human-arm-reachable workspace see [18].



(a) Egocentric Space



(b) Abstracted Space

Figure 4: Egocentric Space Example

In addition to the agent, the scene in Figure 4 contains a chair, a shining standing light and a book shelf. The influence of the light is the area of its direct light beam. Chair and book shelf exhibit only their own spatial extent.

Model 1

	light	agent	chair	shelf
light	T	T	T	F
agent	T	T	F	F
chair	T	T	T	F
shelf	F	F	F	T

Model 2

	light	agent	chair	shelf
light	T	T	T	F
agent	T	T	T	F
chair	T	T	T	F
shelf	F	F	F	T

Table 2: Models 1 and 2 of Nearness in Egocentric Space Scene

There are four possible models of this interpretation shown in Tables 2 and 3. The chair is always near the light, even though only part of the chair is actually within the light's beam and the agent is between light and chair. This is signified by the nearness relation between chair and light being true in all four models shown in Tables 2 and 3. The light however is not necessarily near the chair, because the chair does not have much influence on the light and even less so with the agent being situated between light and chair.

Models 1 and 2 in Table 2 reflect the symmetric nearness notion, where the light is also considered to be near the chair and the nearness notion s-near1 is sufficient to describe these relations.

Model 3

	light	agent	chair	shelf
light	T	T	F	F
agent	T	T	T	F
chair	T	T	T	F
shelf	F	F	F	T

Model 4

	light	agent	chair	shelf
light	T	T	F	F
agent	T	T	F	F
chair	T	T	T	F
shelf	F	F	F	T

Table 3: Models of Nearness in Egocentric Space Scene

Models 3 and 4 in Table 3 reflect the cases when light and chair are not considered near. While s-near1 is still valid for this case as well, it is certainly not restrictive enough and needs to be refined, resulting in the asymmetric nearness notion a-near2.

Definition 9 *For all sites p and q,* a-near2(p, q) *holds iff* $(IA(p) \cap IA(q) \neq \emptyset) \wedge \omega(p) \leq \omega(q)$.

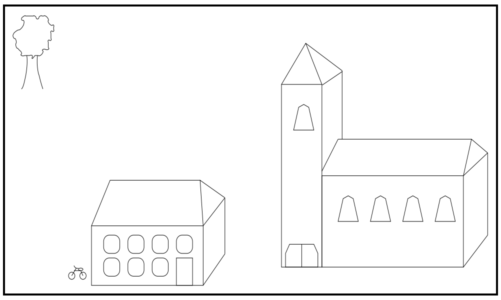
As can be seen in the previous discussion, the "in-betweenness" of the agent with respect to the light and the chair has only played a small role in the model definition of this interpretation. If this kind of "in-betweenness" of the agent needs to be considered for the nearness relation between light and chair, another weight that does not include the influence area of the chair would have to be assigned to the light resulting in a different interpretation. It is not of concern to this interpretation of egocentric space.

The chair is definitely near the agent due to the reachability, but the agent is not necessarily near the chair, as shown in Model 1 in Table 2 and Model 4 in Table 3. This is a quite unlikely model, but valid within the framework by Axiom (A2) and this interpretation.

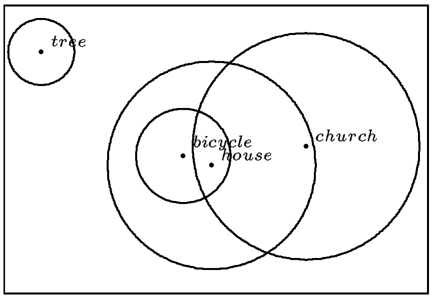
The other relations in Models 1 to 4 in Tables 2 and 3 can all be described by nearness notion s-near1 given in Definition 7.

3.1.3 Environmental Space - A Natural Language Interpretation

Montello [21] described the environmental space as the large-scale space in which we live. We however include this particular interpretation in the small-scale space section, because the scene shown in Figure 5(a) can be viewed from a single view point by the observing agent. In addition to that, this scene is also interpreted as a natural language representation, thus having natural language restrictions imposed on its possible models. The English language is used in this interpretation.



(a) Environmental Small-scale Space



(b) Abstracted Space

Figure 5: Environmental Small-scale Space Example

The scene contains a bicycle next to a house which is in proximity to a church. A tree is shown in the distance. According to Talmy [27], natural language expressions representing spatial relations are quite commonly asymmetric. Among other criteria, this is the case when the objects vary considerably in size. While a small object might be near a large object, the large object is usually not correctly described as being near the small object, in natural language terms. For the scene depicted, the bicycle is definitely near the house, however not vice versa as can be seen in all four models shown in Tables 4 and 5.

Model 1

	church	house	bicycle	tree
church	T	T	F	F
house	T	T	F	F
bicycle	T	T	T	T
tree	F	F	F	T

Model 2

	church	house	bicycle	tree
church	T	F	F	F
house	T	T	F	F
bicycle	T	T	T	T
tree	F	F	F	T

Table 4: Models 1 and 2 of Nearness in the Environmental Small-Scale Space of Figure 5

The whole influence area of the bicycle is enclosed in the influence area of the house, but not conversely. This example and similar ones justify the introduction of an asymmetric nearness notion, namely a-near4, defined as follows.

Definition 10 *For all sites p and q,* a-near4(p, q) *holds iff* $IA(p) \subseteq IA(q)$.

The bicycle can also be considered near the church in certain contexts as shown in Models 1 and 2 in Table 4. For example, if the emphasis is on the bicycle being parked next to the house and near the church, and not, for example, near the station where it is usually parked. The bicycle is therefore considered to be near the church, while in this context, it would not be said that the church is near the bicycle. The asymmetric nearness notion a-near2 given in Definition 9 accounts for these kind of situations.

Model 3

	church	house	bicycle	tree
church	T	T	F	F
house	T	T	F	F
bicycle	F	T	T	T
tree	F	F	F	T

Model 4

	church	house	bicycle	tree
church	T	F	F	F
house	T	T	F	F
bicycle	F	T	T	T
tree	F	F	F	T

Table 5: Models 3 and 4 of Nearness in the Environmental Small-Scale Space of Figure 5

The relationship between house and church is symmetric in some contexts, abstracted in Models 1 and 3, but asymmetric in other contexts, abstracted in Models 2 and 4 in Table 5. While it would always be considered that the house is near the church, it would not always be considered that the church is near the house, due to the church's greater size and also higher priority as a landmark. The relation a-near2 accounts for the asymmetric case.

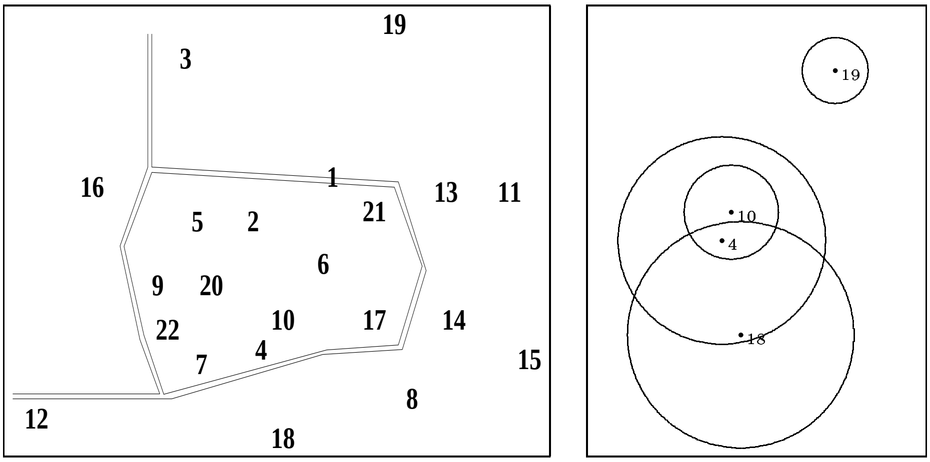
3.2 Large-Scale Spaces

Large-scale spaces are defined as spaces whose structure is at a much larger scale than the sensory horizon of the agent [17].

3.2.1 Environmental Space

Worboys [30] investigated possible regularities in the perception of nearness across a variety of individuals and conducted experiments in the environment of Keele University Campus. A list of 22 significant campus places was presented to the subjects who had to evaluate the nearness of one given reference place, also chosen from the list, to each of these 22 places. The results of the experiments were analysed by Worboys in several ways, here we are concerned only with the results of his three-valued logic analysis. The nearness relations between each two places are thus either *True*, *False* or *Indeterminate*. The experiments and their three-valued logic analysis are described in more detail in Section 3.4.

Influence areas can be assigned to all the places included in the experimental setting. The influence area of each place i.e., of each reference, is established by assigning to it every other place that is near the reference or whose nearness relation to the reference is indeterminate.



(a) Environmental Large-scale Space

(b) Abstracted Space

Figure 6: Environmental Large-Scale Space

	4	10	18	19
4	T	T	T	F
10	T	T	I	F
18	I	I	T	F
19	F	F	F	T

Table 6: Nearness Relations for Scene in Figure 6(a)

From the scene in Figure 6 and the nearness information available from Worboys' experiments shown in Table 6, eight different possible models of the scene can be derived from the different interpretations of the three indeterminate values. Even though Worboys suggested that nearness relations in environmental spaces are weakly symmetric[5], we will still consider asymmetric interpretations. For example, while the nearness relation between 4 and 18 is true, the nearness relation between 18 and 4 is either false or true in different models.

If in Table 6 we consider only the pair (p, q) such that $Near(p, q)$ and $Near(q, p)$[6], then s-near1 given in Definition 7 adequately describes the *true* nearness relations for these pairs. The *false* nearness relations satisfy axiom

[5] *Weakly symmetric* is not a formal notion, but is used by Worboys to refer to the fact that his test data exhibit no two sites a and b for which $Near(a, b)$ is true and $Near(b, a)$ is false or vice versa.

[6] Hence consider the following pairs (4,4), (4,10), (4,19), (10,4), (10,10), (10,19), (18,18), (18,19), (19,4), (19,10), (19,18) and (19,19) as symmetric nearness relations.

(A2).

The following models will be discussed in detail to determine what implications all possible interpretations of the indeterminate values in Table 6 have.

Model 1

	4	10	18	19
4	T	T	T	F
10	T	T	F	F
18	F	F	T	F
19	F	F	F	T

Model 2

	4	10	18	19
4	T	T	T	F
10	T	T	T	F
18	F	F	T	F
19	F	F	F	T

Model 3

	4	10	18	19
4	T	T	T	F
10	T	T	F	F
18	F	T	T	F
19	F	F	F	T

Table 7: Models 1-3 of Environmental Large-scale Space

Models 1 to 3 in Table 7 are quite unlikely models, because sites 18 and 4 are not near in this model, even though the sites are included in each other's influence areas, which should indicate a quite strong nearness relation. In Model 1, site 18 is not near site 10, nor is site 10 near site 18. These models do, however, satisfy axioms (A1) and (A2) and are therefore valid models.

Site 18 is not near site 10 in Model 2, however site 10 is near site 18. As the weight of site 18 is much greater than the weight of site 10, this situation can be described by nearness notion a-near2 given in Definition 9. In Model 3, site 18 is considered near site 10, but site 10 not near site 18. This is again a very unlikely situation, but it does satisfy axiom (A2).

Model 4

	4	10	18	19
4	T	T	T	F
10	T	T	T	F
18	F	T	T	F
19	F	F	F	T

Model 5

	4	10	18	19
4	T	T	T	F
10	T	T	F	F
18	T	F	T	F
19	F	F	F	T

Model 6

	4	10	18	19
4	T	T	T	F
10	T	T	T	F
18	T	F	T	F
19	F	F	F	T

Table 8: Models 4-6 of Environmental Large-scale Space

As in Models 1 to 3, Model 4 in Table 8 assumes that site 18 and site 4 are not near, this being a very unlikely, but valid interpretation. Site 18 and 10 are near each other, describable by nearness notion s-near1. In Model 5 and 6 in Table 8, sites 18 and 4 are near, which can be described by nearness notion s-near4 given in Definition 8 assuming that the sites are located within each other's influence area. Sites 18 and 10 are not near each other in Model 5. This situation does satisfy axiom (A2) and also nearness notion s-near4, because the condition of this notion for being near i.e., that the sites are located within each other's influence area, is not given for sites 18 and 10. Model 6 exhibits the situation where site 18 is not near site 10, but site 10 is near site 18. The influence area of site 10 is much smaller than the one of site 18 and their asymmetric nearness relation can therefore be described by nearness notion a-near2 given in Definition 9.

Model 7

	4	10	18	19
4	T	T	T	F
10	T	T	F	F
18	T	T	T	F
19	F	F	F	T

Model 8

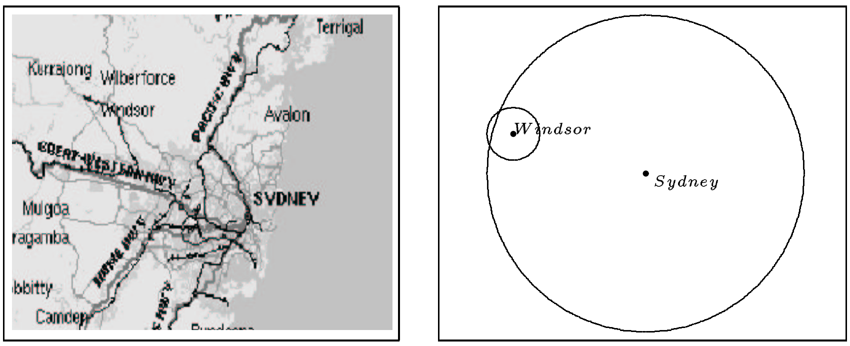
	4	10	18	19
4	T	T	T	F
10	T	T	F	F
18	F	F	T	F
19	F	F	F	T

Table 9: Models 7 and 8 of Environmental Large-scale Space

In Model 7 in Table 9, sites 18 and 4 are near, which can be described by nearness notion s-near4 given in Definition 8, assuming that the sites are located within each other's influence area. Model 7 also displays the situation where sites 18 and 10 are near each other, describable by nearness notion s-near1 given in Definition 7. This nearness notion obviously can not describe the situation in Model 7, where site 18 is near site 10, but site 10 is not near site 18. While it does satisfy axiom (A2), it is a very unlikely interpretation. As in Models 1 to 4, Model 8 in Table 9 assumes that site 18 and site 4 are not near, this being a very unlikely, but valid interpretation.

3.2.2 Geographic Space

Geographic space[7] is, in contrast to small-scale space, essentially two-dimensional. The following geographic space interpretation is the map shown in Figure 7. For this interpretation, the influence area of Windsor does not need to be completely included in the influence area of Sydney.



(a) Geographic Space (b) Abstracted Space

Figure 7: Geographic Space Example

[7] according to Naive Geography [9]

The perceiving agent focuses on two of the places shown in the map, one being Sydney, a large city in South-Eastern Australia, and the other one being Windsor, a town North-West of Sydney. There are two possible models of this interpreation as shown in Table 10. In each of the two models, the town of Windsor is near the city of Sydney. Sydney is also considered near Windsor, when one for example considers the train trip between Sydney and Windsor and vice versa. An agent who considers that this train trip between Sydney and Windsor is short and therefore considers Sydney to be near Windsor, will also conclude that Windsor is near Sydney. Model 1 in Table 10 reflects this situation. As the abstracted space is quite different to both previous interpretation examples, a more specific nearness notion is necessary to acurately describe this model of the scene. The site representing Windsor is situated within the influence area of Sydney, however Sydney is not situated within Windsor's influence area. The nearness notion s-near2 captures this situation.

Definition 11 *For all sites p and q,* s-near2(p, q) *holds iff* $p \in IA(q) \vee q \in IA(p)$.

In contrast, Model 2 represents contexts where Sydney is not considered to be near Windsor. For an example of such a context, imagine someone describing where Windsor is to people who only know Sydney but not Windsor. One would say that Windsor is near Sydney, however not that Sydney is near Windsor, because Sydney is a far more important landmark.

Model 1

	Sydney	Windsor
Sydney	T	T
Windsor	T	T

Model 2

	Sydney	Windsor
Sydney	T	F
Windsor	T	T

Table 10: Models of Nearness in the Scene in Figure 7

Influence areas of cities can also be determined by the catchment area of their infrastructures. For example, there are many specialist hospitals and government offices located in the Sydney area that also service Windsor. In that respect, Windsor is near Sydney. Sydney however is not near Windsor, because none of Windsor's infrastructures service Sydney; thus exhibiting the same asymmetric nearness relation as in the previous example. We therefore need a more specific nearness notion, an asymmetric refinement of s-near2. The relation a-near3 provides for this.

Definition 12 *For all sites p and q,* a-near3(p, q) *holds iff* $p \in IA(q)$.

3.3 Synopsis

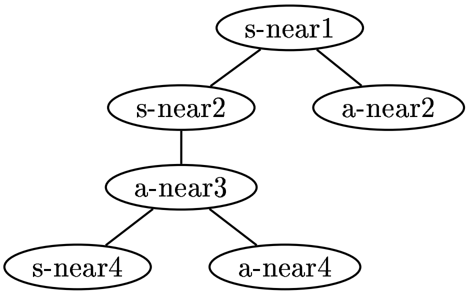


Figure 8: A "Family" of Nearness Relations for Specific (D, ω)

The theory of nearness developed in this section is based on the generic notion of nearness satisfying axioms $(A1)$ and $(A2)$. A "family" of nearness relations for specific distance and weight satisfying $(A1)$ and $(A2)$ were defined resulting in a relational tree shown in Figure 8, starting with s-near1 as the most general nearness notion of the "family." This nearness notion s-near1 will be justified against the previously used set of experimental results obtained from studies conducted by [30] in the domain of environmental spaces.

s-near1(p,q)$=_{Def}$ $IA(p) \cap IA(q) \neq \emptyset$
s-near2(p,q)$=_{Def}$ $p \in IA(q) \vee q \in IA(p)$
a-near2(p,q)$=_{Def}$ $(IA(p) \cap IA(q) \neq \emptyset) \wedge \omega(p) \leq \omega(q)$
a-near3(p,q)$=_{Def}$ $p \in IA(q)$
s-near4(p,q)$=_{Def}$ $p \in IA(q) \wedge q \in IA(p)$
a-near4(p,q)$=_{Def}$ $IA(p) \subseteq IA(q)$

3.4 Evaluation of the Generic Nearness Notion

As stated before, all our notions of nearness are constrained by axioms (A1) and (A2). Recall that (A1) requires $Near(p, p)$ and (A2) requires $IA(p) \cap IA(q) = \emptyset \rightarrow \neg Near(p, q)$. Axiom (A1) can be assumed to be true, because it is generally accepted that everything is near itself. However, the assumption of axiom (A2), that objects are not near when their influence areas do not intersect, needs to be evaluated. We will use experimental results obtained by Worboys [30] to justify the assumption of axiom (A2).

Within the frame of spatial knowledge representation, Worboys [30] investigates nearness relations in environmental spaces. He considers nearness as one of the "conceptual" distance relations and conducted experiments in the environment of Keele University Campus.

Worboys investigated possible regularities in the perception of nearness across a variety of individuals. The goal was to find out whether formal theories can be applied to reasoning with vague spatial notions such as nearness.

Figure 9 shows a sketch of Keele Campus as used by Worboys, the same sketch used in the case study of Environmental Large Scale Spaces in the previous section. It is included here to describe the experimental setting and show the relevance of its results to our formalism.

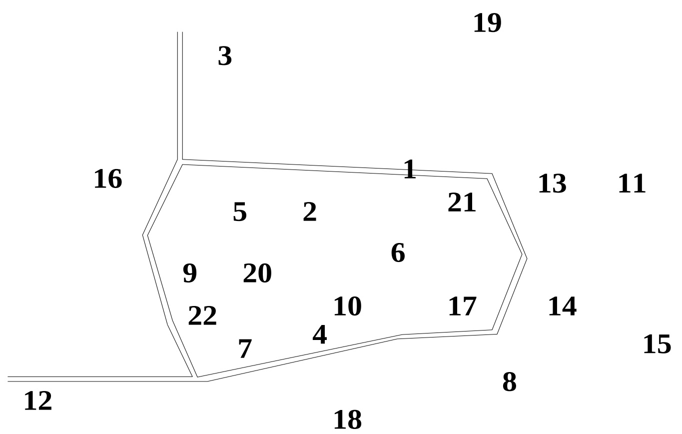


Figure 9: Sketch of Keele Campus

Worboys conducted the experiment in the following way. A list of 22 significant campus places was compiled by the means of questioning the twenty-two subjects each of whom was familiar with the campus setting. The chosen places can be seen in Table 11. The subjects were then equally divided into a truth and a falsity group. They were asked to fill in 22 questionnaires each concerning the location of all significant places with regard to one reference place. The reference place would obviously be one out of the list of the significant places and would not be included in the list of places, as it was assumed that every place is near itself. At least one day would be allowed between filling in each questionnaire.

The truth group was asked to indicate for which of the significant places it was true that they were near the reference place. For example, taking the library (number 17 in the list) as a reference place, the subjects of the truth group were asked to indicate for which of the other places it was true to say that it was near the library.

The falsity group was asked to indicate for which of the significant places it was false that they were near the reference place. For example, taking the library again as a reference place, the subjects of the falsity group were asked to indicate for which of the other places it was false to say that it was near the library. Worboys' results for the library as a reference place are reproduced in Table 11.

Place	Truth Group	Falsity Group
1 *24 Hour Reception*	4	4
2 *Academic Affairs*	5	2
3 *Barnes Hall*	0	11
4 *Biological Sciences*	5	4
5 *Chancellor's Building*	4	6
6 *Chapel*	10	0
7 *Chemistry*	4	6
8 *Clock House*	4	6
9 *Computer Science*	1	10
10 *Earth Science*	7	0
11 *Health Centre*	1	11
12 *Holly Cross*	1	11
13 *Horwood Hall*	4	10
14 *Keele Hall*	8	2
15 *Lakes*	1	11
16 *Leisure Centre*	0	11
17 *Library*	11	0
18 *Lindsay Hall*	2	8
19 *Observatory*	0	11
20 *Physics*	5	5
21 *Student Union*	10	0
22 *Visual Arts*	1	10

Table 11: Truth and Falsity Group responses for Nearness of places to the library reproduced from [30]

Worboys analysed these test results in the context of three-valued logics, fuzzy nearness neighbourhoods and higher-valued logics. Additional details on these analyses can be found in [30]. We will be concerned with the three-valued logics analysis, as it allows for the determination of influence areas for all the significant places. Worboys himself also claims that an analysis of the data in terms of three-valued logics "provides a set of 'nearness neigbourhoods' of particular places" ([30], p638). The results shown in Table 11 were then evaluated using a χ^2 test with a significance level of 0.001 (0.1% level) to determine if the nearness relation between two places was true or false, or in cases where the data did not allow for any conclusion either way, the relation was termed indeterminate. This significance level of 0.001 is a very conservative choice, with generally accepted significance level of 0.05 (5% level) or 0.01 (1% level) commonly used in statistical analysis of experimental results. It can therefore be assumed that the nearness relations termed *true* i.e, T, and *false* i.e., F, result from this analysis are reliable values for the particular nearness relations.

Place	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1 *24 Hour Reception*	T	I	I	F	I	I	F	F	F	F	I	F	I	F	F	I	I	F	I	F	T	F
2 *Academic Affairs*	I	T	I	I	T	I	I	F	I	I	F	F	F	F	F	I	I	I	F	I	I	I
3 *Barnes Hall*	I	F	T	F	I	F	F	F	F	F	F	F	F	F	F	I	F	F	I	F	F	F
4 *Biological Sciences*	I	I	F	T	I	I	T	I	I	T	F	I	I	I	F	F	I	T	F	T	I	I
5 *Chancellor's Building*	I	T	I	I	T	I	I	F	I	I	F	F	F	F	F	I	I	F	F	I	I	I
6 *Chapel*	I	T	I	I	I	T	I	I	I	I	F	F	I	I	F	I	T	I	F	I	T	F
7 *Chemistry*	F	I	F	T	I	I	T	I	I	I	F	I	F	F	F	F	I	I	F	T	F	T
8 *Clock House*	F	F	F	I	F	F	F	T	F	I	F	I	I	T	I	F	I	I	F	F	I	F
9 *Computer Science*	F	I	I	I	I	I	I	F	T	I	F	I	F	F	F	I	F	I	F	I	F	T
10 *Earth Science*	I	I	F	T	I	I	I	I	I	T	F	I	I	I	F	F	I	I	F	T	I	I
11 *Health Centre*	I	F	F	F	F	F	F	F	F	F	T	F	T	I	I	F	F	F	I	F	I	F
12 *Holly Cross*	F	F	F	I	F	F	I	F	I	F	F	T	F	F	F	F	F	I	F	F	F	I
13 *Horwood Hall*	I	F	F	F	F	I	F	I	F	F	T	F	T	T	T	F	I	F	I	F	T	F
14 *Keele Hall*	I	F	F	F	F	I	F	T	F	I	I	F	I	T	T	F	I	I	F	F	I	F
15 *Lakes*	F	F	F	F	F	F	F	I	F	F	I	F	I	T	T	F	F	I	F	F	I	F
16 *Leisure Centre*	F	F	I	F	I	F	F	F	I	F	F	I	F	F	F	T	F	F	F	F	F	I
17 *Library*	I	I	F	I	I	T	I	I	I	T	I	F	I	I	I	F	T	I	F	I	T	F
18 *Lindsay Hall*	F	F	F	I	F	F	I	T	I	I	F	T	F	I	F	F	I	T	F	I	F	I
19 *Observatory*	I	F	I	F	F	F	F	F	F	F	I	F	I	F	F	F	F	F	T	F	I	F
20 *Physics*	F	I	F	T	I	I	T	I	I	T	F	I	I	F	F	I	I	I	F	T	I	I
21 *Student Union*	T	I	I	I	I	T	I	I	I	I	I	F	T	I	I	F	T	F	I	I	T	F
22 *Visual Arts*	F	F	F	I	I	F	I	F	T	I	F	I	F	F	F	I	F	I	F	I	F	T

Table 12: Worboys' three-valued analysis of the test results reproduced from [6]

The experimental results represented in a three-valued logic were summarised and discussed in terms of its implications for computational efficiency in Duckham and Worboys [6]. Table 12 reproduces the summary of the experimental results in three-valued logic format.

While the concept of influence areas has not been explicitly considered by Worboys, it is a reasonable assumption that all places that are indicated as *near* a particular place p are within the influence area of p.

The influence area of the library (i.e. 17 in sketch and table) for example, could be drawn from the test results as follows. Considering that the indeterminate values could be assigned either far or near in different contexts, it is possible to indicate the area of influence of the library by including all the "near places" thereby setting a certain radius of the influence area also including indeterminate places within this radius as seen in Figure 10.

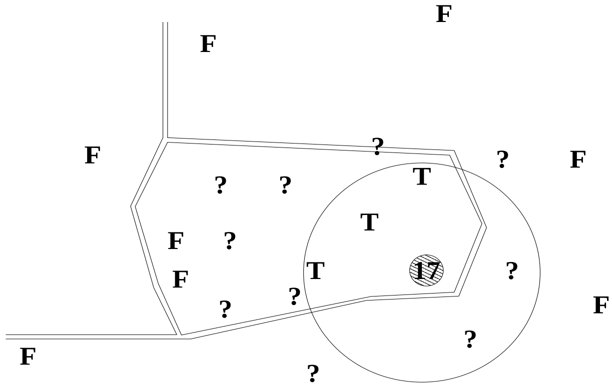


Figure 10: Influence Area of Library

We will now use the data in Table 12 for the evaluation of axiom (A2). To avoid confusion with the nearness notions of our theory, we will use the symbol ν when referring to Worboys' three-valued nearness relations in the following. In many ways, Worboys' three-valued analysis provides the preamble for the derivation of influence areas for the significant places across Keele Campus. Duckham and Worboys [7] also found that for significance levels less than 0.169, if the nearness relation between places p and q is true, the nearness relation between q and p would not be false. Based on this observation, we will, for the purpose of defining influence areas for all the experimentally defined significant places on Keele campus, assume the following:

- $near(p,q) = \nu(p,q)$ if $\nu(p,q) \neq I$
- $near(p,q) = I$ if $\nu(p,q) = \nu(q,p) = I$
- $near(p,q) = T$ if $\nu(p,q) = I \wedge \nu(q,p) = T$
- $near(p,q) = F$ if $\nu(p,q) = I \wedge \nu(q,p) = F$

It should be noted that $near(p,q)$ is different to $Near(p,q)$ in Section 3, which can be either *true* or *false*. Here we now define a predicate, which can be either *true*, *false* or *indeterminate*. Based upon these assumptions, we can now generate a new nearness relation from the original one in Table 12, which can be used to determine the influence areas of all places.

In general, the influence area of a reference place in this particular environmental space can be defined as containing all the reference places that are definitely near (i.e. where the nearness relation between place 1 and place 2 is true in Worboys' three-valued analysis of his test results) or are indeterminate. For the following evaluation of axiom (A2) against Worboys' experimental results, *SWI Prolog* was used in order to derive the influence areas and test our hypothesis i.e., axiom (A2), against the database generated in this way.

The nearness relations from Table 12 were represented as Prolog facts. For example, the fact that reference place 2 is near reference place 5 is expressed as `v(2,5,t).` From these facts following the assumptions above, symmetry can be introduced into the dataset. This is not suggesting that nearness is a symmetric relation per se[8], but interpreting Worboys' test data symmetrically is a reasonable approach. This symmetric relation is expressed as Prolog facts as well. For example, `near(2,5,t)` or `near(9,4,i)`.

We know that axiom (A2) states that two sites are not near if their influence areas do not intersect. Thus, in order to justify (A2), we must show that the nearness relation of any two reference places whose influence areas do not intersect is false. Two tests were conducted, each of them assuming a different interpretation for indeterminate nearness relations.

3.4.1 Test 1 - I interpreted as T

In this test, the indeterminate values in Table 12 were interpreted as *true* and the following Prolog logic is used to assign places to the influence area of a reference place Y:

```
iamember(X,Y) :- near(X,Y,t).
iamember(X,Y) :- near(X,Y,i).
```

Table 13 shows the influence areas generated for each reference by including places that have true and indeterminate nearness relations with it.

It needs to be verified whether any of the reference places whose influence areas do not have a place in common, do exhibit a true or indeterminate nearness relation, which would contradict axiom (A2). Using Prolog, this verification was done for all 22 significant places by checking for each of them if their influence area has a place in common with the influence area of the other place. If they have a place in common, no further verification was necessary as it is of no concern for axiom (A2). If their influence areas, however, do not share any place, there is a need to verify that these references do not have a true or indeterminate nearness relationship i.e., no occurance of `near(X,Y,t)` or `near(X,Y,i)`. Appendix A shows the Prolog code used

[8] as we did already discuss in our previous case study and in the definition of the nearness notions

Reference	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1	1	2	3	-	5	6	-	-	-	-	11	-	13	-	-	-	17	-	19	-	21	-
2	1	2	-	4	5	6	7	-	9	10	-	-	-	-	-	-	17	-	-	20	21	-
3	1	-	3	-	5	-	-	-	-	-	-	-	-	-	-	16	-	-	19	-	-	-
4	-	2	-	4	5	6	7	8	9	10	-	12	-	-	-	-	17	18	-	20	21	22
5	1	2	3	4	5	6	7	-	9	10	-	-	-	-	-	16	17	-	-	20	21	22
6	1	-	-	4	5	6	7	-	9	10	-	-	13	14	-	-	17	-	-	20	21	-
7	-	2	-	4	5	6	7	-	9	10	-	12	-	-	-	-	17	18	-	20	-	22
8	-	-	-	4	-	-	-	8	-	10	-	-	13	14	15	-	17	18	-	-	21	-
9	-	2	-	4	5	6	7	-	9	10	-	12	-	-	-	16	-	18	-	20	-	22
10	-	2	-	4	5	6	7	8	9	10	-	-	-	14	-	-	17	18	-	20	21	22
11	1	-	-	-	-	-	-	-	-	-	11	-	13	14	15	-	-	-	19	-	21	-
12	-	-	-	4	-	-	7	-	9	-	-	12	-	-	-	-	-	18	-	-	-	22
13	1	-	-	-	-	6	-	8	-	-	11	-	13	14	15	-	17	-	19	-	21	-
14	-	-	-	-	-	6	-	8	-	10	11	-	13	14	15	-	17	18	-	-	21	-
15	-	-	-	-	-	-	-	8	-	-	11	-	13	14	15	-	-	-	-	-	21	-
16	-	-	3	-	5	-	-	-	9	-	-	-	-	-	-	16	-	-	-	-	-	22
17	1	2	-	4	5	6	7	8	-	10	-	-	13	14	-	-	17	18	-	20	21	-
18	-	-	-	4	-	-	7	8	9	10	-	12	-	14	-	-	17	18	-	20	-	22
19	1	-	3	-	-	-	-	-	-	-	11	-	13	-	-	-	-	-	19	-	21	-
20	-	2	-	4	5	6	7	=	9	10	-	-	-	-	-	-	17	18	-	20	21	22
21	1	2	-	4	5	6	-	8	-	10	11	-	13	14	15	-	17	-	19	20	21	-
22	-	-	-	4	5	-	7	-	9	10	-	12	-	-	-	16	-	18	-	20	-	22

Table 13: Places contained in the influence areas of the reference considering true and indeterminate nearness relations

for this verification. Note that the complete list of the nearness relation facts is not included in the code and is denoted by `[...]`. No example could be found that contradicted axiom (A2).

3.4.2 Test 2 - I interpreted as F

In this test, the indeterminate values in Table 12 were interpreted as *false* and the following Prolog logic is used to assign places to the influence area of a reference place Y:

```
iamember(X,Y) :- near(X,Y,t).
```

The resulting influence areas are indicated in Table 14.

The verification approach is similar to Test 1. In this test, however, if the influence areas of the references do not share any place, there is a need to verify that these references do not have a true nearness relationship i.e., no occurance of `near(X,Y,t)`. Appendix B shows the Prolog code used for this verification. Note that the complete list of the nearness relation facts is not included in the code and is denoted by `[...]`. No example could be found that contradicted axiom (A2) in Test 2 either.

We can therefore conclude that there are no two sites in the test data set whose nearness relation is true, if their influence areas do not intersect. Thus axiom (A2) is consistent with Worboy's empirical data.

Reference	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	21	-
2	-	2	-	-	5	6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	-	-	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	-	-	-	4	-	-	7	-	-	10	-	-	-	-	-	-	-	18	-	20	-	-
5	-	2	-	-	5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6	-	-	-	-	-	6	-	-	-	-	-	-	-	-	-	-	17	-	-	-	21	-
7	-	-	-	4	-	-	7	-	-	-	-	-	-	-	-	-	-	-	-	20	-	22
8	-	-	-	-	-	-	-	8	-	-	-	-	-	14	-	-	-	18	-	-	-	-
9	-	-	-	-	-	-	-	-	9	-	-	-	-	-	-	-	-	-	-	-	-	22
10	-	-	-	4	-	-	-	-	-	10	-	-	-	-	-	-	17	-	-	20	-	
11	-	-	-	-	-	-	-	-	-	-	11	-	13	-	-	-	-	-	-	-	-	-
12	-	-	-	-	-	-	-	-	-	-	-	12	-	-	-	-	-	18	-	-	-	-
13	-	-	-	-	-	-	-	-	-	-	11	-	13	14	15	-	-	-	-	-	21	-
14	-	-	-	-	-	-	-	8	-	-	-	-	13	14	15	-	-	-	-	-	-	-
15	-	-	-	-	-	-	-	-	-	-	-	-	13	14	15	-	-	-	-	-	-	-
16	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	16	-	-	-	-	-	-
17	-	-	-	-	-	6	-	-	-	10	-	-	-	-	-	-	17	-	-	-	21	-
18	-	-	-	4	-	-	-	8	-	-	-	12	-	-	-	-	-	18	-	-	-	-
19	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	19	-	-	-
20	-	-	-	4	-	-	7	-	-	10	-	-	-	-	-	-	-	-	-	20	-	-
21	1	-	-	-	-	6	-	-	-	-	-	-	13	-	-	-	17	-	-	-	21	-
22	-	-	-	-	-	-	7	-	9	-	-	-	-	-	-	-	-	-	-	-	-	22

Table 14: Places contained in the influence areas of the reference considering only true nearness relations

3.5 Evaluation of Nearness Notion s-near1

In the context of proximity spaces, two sets are near each other if they share at least one closure point. Adapting this to a universe containing discrete points i.e., sites and their associated areas of influence, two sites are near if their influence areas share at least one point. The relation s-near1, identified as the most general notion in the "family" of nearness notions for specific distances and weights of our theory of nearness, represents exactly this fact. This representation does not only make sense in the context of proximity spaces, but can also be shown to be a reasonable approach for physical space by verifying s-near1 against a set of experimental data. We will again use Worboys' experimental results.

As mentioned before, we assume that two sites are near if their influence areas intersect.

In the previous subsection, two tests were conducted, one where the indeterminate values were considered to be *true* and in the second test, they were considered to be *false*. As we were looking for cases where the nearness relations were *true* for any two sites that did not have a common place in their influence areas, it gave us a much stronger validation to interpret I as T, in addition to interpreting I as F. Also, axioms $A1$ and $A2$ should be valid for all possible cases and interpretations.

This time, however, we need to verify a more refined notion, albeit the most general nearness notion in our "family" of nearness notions for specific

D and ω, and therefore need to restrict our test data to more certain interpretations. Therefore, we will use the influence areas derived only from *true* nearness notions as we did in Test 2 in the previous section. The validation whether there are any sites that intersect but whose nearness relation is false, will only use the *false* notions ignoring the indeterminate values.

The influence areas are expressed as Prolog facts in the same way as in Test 2 in the previous subsection. These facts, shown in Table 14, can now be used to validate s-near1. If the influence areas of any two reference places p and q share a common place, then the fact `near(p,q,f)` should not appear in the database. The Prolog program used to evaluate the formalism against Worboys' results is stated in Appendix C. Note that the complete list of the nearness relation facts is not included in the code and is denoted by `[...]`.

The evaluation resulted in one case that contradicted s-near1. The nearness relation between 8 and 12 did not follow the prescribed pattern for s-near1. Taking a closer look at these references it can be seen that the influence areas of both 8 and 12 share the place 18, which implies that their influence areas intersect and the references should therefore be near according to s-near1. A glance at Figure 9 clarifies the locations of the references within the environmental space. While `near(8,12,i)` indicates that nearness between 8 and 12 is not determined, nearness between 12 and 8 is clearly not given by `near(12,8,f)`. Relation s-near1 does not account for the nearness relation between 12 and 8. Their relation is however accounted for in our formalism by Axiom (A2). Axiom (A2) states that sites are not near if their influence areas do not intersect, however it can not be inferred from this that they are near if their influence areas do intersect. Therefore, even though the influence areas of 12 and 8 do intersect, they are not required to have a *true* nearness notion by Axiom (A2).

This result is very encouraging, because out of all the possible combinations, the nearness relation between 12 and 8 is the only case that was inconsistent with our most general nearness notion s-near1. As the number of all possible combinations between sites, excluding the nearness relations between equal sites[9] and the reverse cases[10], is: $C_{k=2}^{n=22} = \frac{n!}{k!*(n-k)!} = \frac{22!}{2!*20!} = 231$, one case is only a very small percentage. The correctness of s-near1 validated against the experimental of Worboys is: $Correctness = \frac{230*100\%}{231} = 99.56\%$. This is a strong justification for s-near1.

We will therefore have a closer look at this symmetric nearness notion in the following section, before moving on to a discussion on the asymmetric aspects of nearness.

[9] They are always *true*.

[10] They do not need to be included, because we asumed symmetry.

4 Symmetric Nearness and its Properties

As previously discussed, Worboys concluded from his test results that nearness was a weakly symmetric relation, because there were no pairs (p, q) such that p near q and q not near p or vice versa[11]. Please recall that our formalism is based on a two-valued logic.

The following section discusses the properties of symmetric nearness based on nearness notion s-near1. Recall that s-near1$(p, q) =_{Def} IA(p) \cap IA(q) \neq \emptyset$. In terms of proximity spaces, this means that the distance between the closest closure points of the two set (i.e. influence areas) is zero.

Axiom (A1) and Axiom (A2) are satisfied by s-near1(p, q) and we can add a third Axiom (A3) stating the symmetric nature of this nearness notion, which is valid for all symmetric nearness notions.

$(A3)$ *For all sites* p, q $Near(p, q) \leftrightarrow Near(q, p)$

4.0.1 A brief look at the bidimensionality of *Nearness*

With weight and distance being the parameters of nearness, nearness can be considered in two different dimensions. One is a discrete dimension, in which D and ω can have any value with the constraints imposed on their behaviour resulting in different discrete concepts of *Near*. The second dimension considers D and ω as specific and continuous parameters, which can be applied to any concept, therefore allowing "fine-tuning" of the specific concepts of *Near* in the other dimension, showing the scope of every concept for particular D and ω. The scope of a concept is the set of all sites in the universe for whose weights and distances between each other satisfy this concept. Figure 11 shows this principle with the *Concepts*-axis representing the first dimension and the *Parameters*-axis representing the second dimension. The dotted line shows that for any given specific distance D and weight ω, each concept has a certain restricted scope. Note that this scope might cover the entire universe for certain values of D and ω and certain concepts.

As can be seen in Figure 11, we are also aiming at introducing an ordering amongst the given concepts e.g., s-near4 > s-near1. The ordering requires all the concepts to be placed in the same branch of a conceptual lattice and the more general notion is implied by the more specific notions.

Definition 13 (Partial Ordering) *Let* μ *and* δ *be two nearness relations on* U *then* $\mu > \delta$ *iff* $\forall p, q\ \mu(p, q) \rightarrow \delta(p, q)$.

[11] In addition to that, Duckham and Worboys [7] found that for significance levels less than 0.169, if the nearness relation between places p and q is true, the nearness relation between q and p would not be false.

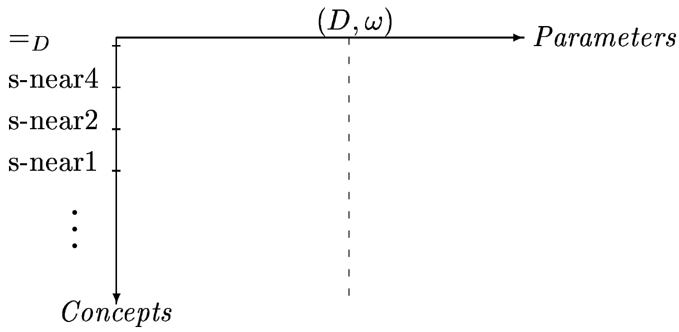


Figure 11: The two dimensions of Nearness where D is distance and ω is weight

Section 4.1 will extend on this idea by supplying such a set of concepts in the context of a navigational setting.

We will now consider properties of symmetric nearness that hold for any distance D and weight ω.

4.0.2 Equality of Sites

Equality of sites i.e., the distance between sites being zero, is not very interesting, but is included for completeness.

Property 14 *The equivalence relation defined by $=$ is a refinement*[12] *of the equivalence relation defined by $=_D$.*

Note that this property of symmetric nearness is also a more specific concept of symmetric nearness.

4.0.3 Inclusion of at least one Site

This property covers the case when only one site is included in the other's influence area.

Proposition 15 *For all $p, q \in \mathcal{U}$, if $D(p,q) \leq sup(\omega(p), \omega(q))$ then* s-near1(p,q).

Proof. If $D(p,q) \leq \omega(p)$ then by Definition 6, $q \in IA(p)$. q is also an element of $IA(q)$. Therefore $IA(p)$ and $IA(q)$ have at least the point q in common. The case of $D(p,q) \leq \omega(q)$ is analogous.□

From this proposition, we will now derive and discuss two corollaries.

[12] i.e., a special case

4.0.4 External Connectedness of Sites

In the Region Connection Calculus (RCC) [5] the relation $EC(x, y)$ is defined as applicable to regions R_1, R_2 and holds if R_1 and R_2 are externally connected i.e., have at least one common point in the RCC terminology. This was extended by Asher and Vieu [1] to assume that the regions have only closure points[13] in common when they are externally connected. That way, the relation can also account for real world objects considered to touch each other without sharing any of their matter.

If regions were singletons i.e., points, the RCC definition would not allow for external connectedness. In this formalism, we resort to pseudo-metric space, because it allows us to define points as *near to each other* when the distance between them is zero. They are as close as possible to each other without being identical.

Corollary 16 *For all* $p, q \in \mathcal{U}$*, if* $p =_D q$ *then* s-near1(p, q).

In this case we can also conclude with the definition of s-near4, but in order to keep this discussion straightforward, we will continue to refer to s-near1.

4.0.5 Inclusion of both Sites

This property covers the case when site p and q are included in each other's influence area, including the previous property.

Corollary 17 *Suppose that* $\mathcal{U}$ *is dense. For all* $p, q \in sites(\mathcal{U})$*, if* $D(p, q) \leq inf(\omega(p), \omega(q))$ *then* s-near1(p, q).

We will now discuss different concepts of symmetric nearness in a navigation setting.

4.1 Concepts of Symmetric Nearness in Navigation

In this subsection, specific concepts of symmetric nearness as refinements and generalisations of s-near1 are discussed.

4.1.1 Generalisations of s-near1

Let us consider the following example in which the distinct points in the space are now a distinct set of trees in a forest; the agent is a bushwalker trying to navigate from one tree to an unknown point in the forest and back to the initial position. In this context, the weight assigned to every tree could be the distance the agent can see while standing beside the tree. Another aspect that needs to be added is the reasoning abilities of this agent during navigation. An agent with rather bad navigating skills might only

[13] See Subsection 2.1 for an explanation of what a closure point denotes.

feel comfortable to walk to the next tree that he or she can see and back to the one he or she started from. For a better navigator, he or she might feel comfortable to navigate along a couple of these "near" trees and back. Thus, given that the total distance is not very far nor hard to cover, by considering the first and the last tree actually being near to each other, s-near1 in Definition 7 can be generalised as seen in the following definition.

Definition 18 *Let $i > 0$ given. For any two sites p and q: $Near_i(p, q)$ holds iff there exist $r_0, ..., r_i$ with $r_0 = p$, $r_i = q$ and for all $j < i$, $IA(r_j) \cap IA(r_{j+1}) \neq \emptyset$.*

A clear ordering occurs among these generalised nearness relations as expressed in the next property. By Definition 18 we can say that $Near_{i+1} \rightarrow Near_i$, therefore by (partial ordering) Definition 13 we can conclude that $Near_{i+1} > Near_i$.

Property 19 *For all $i > 0$, $Near_{i+1} > Near_i$.*

Note that this partial ordering can actually be total.

4.1.2 Refinements of s-near1

Let us now look at a bushwalker, who might consider trees near each other if at least one of them is in sight of the other. This situation could occur if the agent is, for example, at a smaller tree from which he or she can see a tall tree in the distance but not vice versa. The relation s-near2(p, q) given in Definition 11 accounts for this fact that two trees p and q are near to each other if and only if $p \in IA(q)$ or $q \in IA(p)$.

Another agent might only consider trees near each other, if he or she can see either tree positioned at the other. Relation s-near4(p, q) given in Definition 8 accounts for this fact that two trees p and q are near to each other if and only if $p \in IA(q)$ and $q \in IA(p)$.

The concept of pseudo-equality is stated in Corollary 16 and would, in this context, refer to a bushwalker only feeling comfortable to walk to another tree virtually next to the tree he or she is positioned at. Indeed a rather bad navigator.

A clear ordering occurs among these refined nearness relations as expressed in the following property. By Definitions 8 and 11 we can say that s-near4 $\rightarrow$ s-near2 and therefore we can conclude by (partial ordering) Definition 13 that s-near4 $>$ s-near2.

Property 20 s-near4 $>$ s-near2.

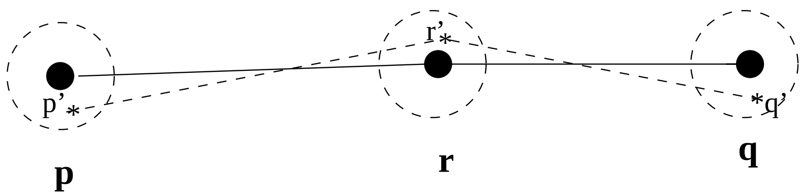


Figure 12: Fuzzy Notion of *Between* where p', q', r' are pseudo-equal points to p, q, r respectively

4.2 Enriching the Formalism

In order to enrich the formalism to gain more expressive power for reasoning, we will now add the notion of one site being in between two others.

Definition 21 ($Between(r, p, q)$) *Given the distinct points p, q, r in $\mathcal{U}$ and $p \neq r$, if $D(p, q) = D(p, r) + D(r, q)$ then r is said to be between p and q.*

There is the limit case that all three points p, q, r are pairwise pseudo-equal and any of them is between the other two. Another, very interesting, aspect of Definition 21 is that all pseudo-equal points to p, q, r also comply to the *Between*-relation, resulting in a fuzzy notion of *Between*. This is shown in Figure 12, where the points p', q', r' denote the pseudo-equal points to p, q, r respectively and $Between(r, p, q) \rightarrow Between(r', p', q')$.

The *Between* relation could be made more expressive if orientation relations were added as suggested in Frank [10] and Hong et al. [14], but this is not in the scope of this discussion as it would add more parameters to the formalism than desired.

The *Between*-relation could also very effectively be represented using Ordinary Voronoi Diagram information, as has been shown by Edwards and Moulin [8] in their Voronoi model approach. However, the potential user has to weigh up the advantages of a more effective representation against this computationally more expensive approach.

5 The Asymmetric Nature of Nearness

In most cases, the symmetric notion of nearness, as discussed in the previous section, is adequate. However, when it comes to the everyday world around us and the way we perceive it, navigate around and talk about it, this symmetric property does not always hold. There are cases, as we have seen in the case study in Section 3, where nearness is not symmetric. Asymmetric nearness does satisfy axioms (A1) and (A2), but obviously not axiom (A3), which only applies to the class of symmetric nearness notions. This asymmetric aspect of nearness is now discussed in more detail in this section.

As pointed out by Gahegan [11], there appear to be several factors that influence the perception of nearness. One such factor is disparity in the size of the spatial entities. This and other causes of perceptual disparity have been extensively researched cross-linguistically and is apparent as the natural language division of objects into reference objects (RO) and objects to be localised (LO).

For example, picture a bicycle near a house. Expressing this spatial situation as "the house is near the bicycle" would be considered as incorrect by most and "the bicycle is near the house" as the only correct description. This is a classic example from the linguistic literature originating with Talmy [27], who conducted cross-linguistic experiments and summarised his findings on spatial language reference in terms of general properties valid across languages. Talmy defines reference objects as objects whose location and sometimes also other properties, referred to as "geometric" properties, are already known. The site, path or orientation of the object to be localised is therefore defined in terms of the distance between the reference objects and the object to be localised or in terms of the relation to geometry of the reference objects.

Talmy identified several properties of an object that might lead to its choice as either the reference object or the object to be localised, with at least one of these properties applying to each of the scene's objects. These properties are shown in Table 15.

Object to be localised	Reference Object
spatial variables need to be determined	acts as a reference object with known spatial characteristics
more movable	more permanently located
smaller	larger
conceived as geometrically simpler (often point-like)	taken to have greater geometric complexity
more salient	more backgrounded
more recently on the scene/in awareness	earlier on the scene/in memory

Table 15: Properties that determine an object's choice as reference object or object to be localised [27]

Some of these criteria are also reflected in the area of influence that is perceived of an object. As we will see in the following subsection, a navigation system has succesfully been implemented by using size and shape of objects by Kettani and Moulin [15]. Generally, when objects with smaller weights are perceived as being near objects with much larger weights, the reverse is not always true. The relation a-near2 in our family of nearness notions does reflect this possibility. In the context of natural languages, this is caused by the likely reference object role of the object with the larger influence area, to which the object with the smaller influence area is related, as Talmy points out, in terms of distance.

5.1 A Natural Language Example

As part of the GRAAD project, Kettani and Moulin [15] developed a knowledge-based navigation system that incorporated orientation and proximity information to generate routes and also provide natural language descriptions. For the representation of proximity, the GRAAD utilises a model that grades closeness around objects by the means of influence areas. Instead of having only one influence area for each object, several influence areas are assigned each representing a different degree of "closeness".

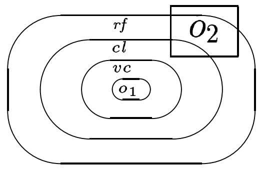


Figure 13: Influence Areas according to Kettani and Moulin [15]

Kettani and Moulin's notion of influence area describes imaginary areas surrounding objects perceived in the environment in order to grade proximity, in the same way that natural language does, reflected in words such as *very close*, *close* and *relative far*. These influence areas are described as portions of space surrounding an object such that each influence area has an interior and an exterior border having the same shape as the object. The length of an imaginary perpendicular line crossing each influence area from one point on the interior border to another point on the exterior border is called the width of this influence area. While this width is a subjective measure of a person's perception, Kettani and Moulin [15] used simple Euclidean geometry to calculate the width of the influence area of objects in their cognitive maps so as to make practical use of their ideas in the navigating system. An object would then be considered to be *very close* to, *close* to or *relative far* from another if it was situated in the appropriate influence area. For example in Figure 13, object o_2 is *close* to object o_1, because part of o_2 is situated within the *close* (i.e. *cl*) influence area of o_1.

The differently graded influence areas for each object were introduced to provide a differentiation between different degrees of proximity. The nearness notions defined in our theory of nearness can also be used to provide different degrees of proximity between objects and it can possibly also account for gradings finer than natural language, similar to the concept definitions in Section 4.1.

Considering the example of Kettani and Moulin's model shown in Figure 13 where *vc* stands for *very close*, *cl* stands for *close* and *rf* stands for *relative far*. According to their model, these relations have the degrees of proximity of $dp - 2$, $dp - 1$ and dp respectively. This means that

$(O_2 \cap IA_{dp}) = \emptyset$, $(O_2 \cap IA_{dp-1}) \neq \emptyset$ and $(O_2 \cap IA_{dp-2}) \neq \emptyset$, where O_n is the region that represents object o_n.

The question now is, if our nearness notions, as previously defined, would be sufficient to account for these different degrees of proximity without having to resort to grading the influence area of an object itself. We will presume that the influence area of the object to be localised is equal to the object's spatial extent as represented by O_2. The influence area of the reference object is the union of the *very close*, *close* and *relative far* influence areas. Given this interpretation, different nearness notions could be used to describe the three different influence areas shown in Figure 13:

- For the *very close* degree of proximity, nearness notion s-near4 could be used, thus assuming that both O_1 and O_2 have to be in each other's influence areas for *very close* to be true. O_2 is within the influence area of O_1, but not vice versa. Therefore O_2 is not *very close* to O_1.

- For the *close* degree of proximity, nearness notion a-near3 could be used, thus assuming that O_2 has to be in O_1's influence area for *close* to be true. O_2 is within the influence area of O_1, therefore O_2 is *close* to O_1.

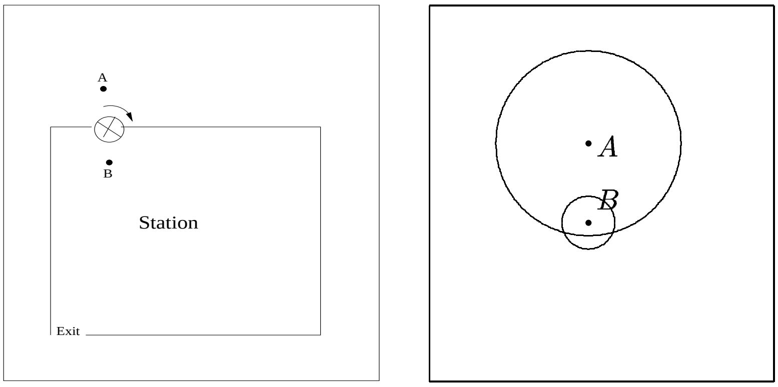
- For the *relative far* degree of proximity, nearness notion s-near1 could be used, thus assuming that the influence areas of O_1 and O_2 have to intersect for *relative far* to be true. We know that the influence areas of O_1 and O_2 intersect, therefore O_1 is *relative far* to O_2.

We know that the more specialised notion does always imply all of the more general notions. If for example, an object O_1 would be *very close* to an object O_2, the degrees of proximity *close* and *relative far* would always hold, even if O_2 would not be situated in Kettani and Moulin's *relative far* influence area. This is, however, not as much of a problem as it might seem at first sight, as generally the stronger nearness notion would be considered. In the above example, we would reason with the relation *close*, rather than with the *relative far*, as it gives us more specific knowledge.

Therefore, we can add to the above descriptions, that by Definition 13, *very close* $>$ *close* $>$ *relative far* and that always the greatest proximity degree will be chosen for reasoning.

6 Distorted Metrics

Asymmetric nearness can also be caused by what we will refer to as distorted metrics. Take the following example in Figure 14 showing a railway station with a turnstile on one of its ends. Given that A is perceived as being near B, we cannot necessarily conclude that B is also near A. In order to get



(a) Turnstile in a Station (b) Abstracted Space

Figure 14: Asymmetric Distance Example

from B to A, we will have to proceed through the exit of the station and cover a far greater distance than was needed to get from A to B.

In the framework of our theory, this situation can be abstracted into the space shown in Figure 14(b). Relation a-near2 can be used to describe this situation, with the weights accounting for the fact that A's influence goes "beyond" the turnstile, while B does not. In that sense, we do not need to worry about asymmetric distances, which will be taken care of by the choice of weight. Axioms (A1) and (A2) still hold and the "family" of nearness notions can also be used to represent this situation. The distance used for determining the correct nearness relation between A and B still follows the pseudo-distance properties (P1) to (P3). However, because asymmetric distances can have an impact on the choice of the weights, we will discuss them briefly.

It is quite clear that there are cases when a non-symmetric distance is considered i.e., the distance of the path needed to be taken from one point to another. Berendt [3] differentiates Euclidian distance from route distance when considering the properties of subjective distance (i.e. a qualitative distance as perceived by humans), representing straight-line distance and distances along the route or path respectively. While the former exhibits the same symmetric properties as pseudo-distance described by (P1) to (P3), the latter would require an additional restriction to property (P2) such that: $\forall x, y \in \mathcal{U},\ D(x,y) = D(y,x)$ iff $path(x,y) = path(y,x)$.

In addition to this path distance, Egenhofer and Mark [9] argue that in the context of naive geographic space, distance is often also seen as a measure of other units such as the time it takes to get from A to B [23], road tolls that have to be paid or petrol consumption. Golledge [22] found that even when the same path is travelled in the opposite direction, the distance perceived might be different for each direction. This difference might result from the terrain of the path (e.g. uphill versus downhill), which can influence the speed of travel or the travel time, for example during rush hour.

These kind of cognitive factors might need to be considered when defining the weight for sites, depending on the application.

7 Conclusions and Outlook

The aim of the work presented in this report is to provide a formalism that can qualitatively account for absolute binary proximity relations, taking into consideration common-sense spatial knowledge. The theory of nearness presented here is based on the concepts of influence areas of spatial objects and distances between these objects abstracted into a pseudo-metric space. The nearness constraints of Axioms (A1) and (A2) were defined, and in a series of case studies we found a whole "family" of nearness notions, all of which conform to the axioms.

This theory goes beyond existing models and influence area approaches, by generalising them and providing a formalisation of nearness notions. Both the generic nearness notion and the most general of the "family" of nearness notions were justified against a set of experimental results obtained from studies conducted by [30] in the domain of environmental spaces. The generic nearness notion was confirmed with 100% and the most general of the "family" of nearness notions was confirmed with 99.56% by the experimental data. These results are a strong justification for the proposed theory.

The symmetric notion of nearness, which was found to be an adequate representation for most cases, was elaborated on in more detail. Its implications were investigated in the context of a navigational model.

There are however cases where nearness is not symmetric. Therefore a brief discussion on the asymmetric aspect of nearness was given and its implications investigated in the context of a natural language model.

Future work will involve the identification of further properties of nearness and an evaluation of the theory in the context of existing spatial reasoning applications such as Geographic Information Systems and Robotics.

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A Prolog Program to verify Axiom (A2) with I interpreted as T

```
%Facts of Nearness Relations between Significant Places
near(1,1,t).
[...]
near(22,22,t).

%Facts of Influence Areas of Significant Places including indet. Relations
iati(1,[1,2,3,5,6,11,13,17,19,21]).
iati(2,[1,2,4,5,6,7,9,10,17,20,21]).
iati(3,[1,3,5,16,19]).
iati(4,[2,4,5,6,7,8,9,10,12,17,18,20,21,22]).
iati(5,[1,2,3,4,5,6,7,9,10,16,17,20,21,22]).
iati(6,[1,4,5,6,7,9,10,13,14,17,20,21]).
iati(7,[2,4,5,6,7,9,10,12,17,18,20,22]).
iati(8,[4,8,10,13,14,15,17,18,21]).
iati(9,[2,4,5,6,7,9,10,12,16,18,20,22]).
iati(10,[2,4,5,6,7,8,9,10,14,17,18,20,21,22]).
iati(11,[1,11,13,14,15,19,21]).
iati(12,[4,7,9,12,18,22]).
iati(13,[1,6,8,11,13,14,15,17,19,21]).
iati(14,[6,8,10,11,13,14,15,17,18,21]).
iati(15,[8,11,13,14,15,21]).
iati(16,[3,5,9,16,22]).
iati(17,[1,2,4,5,6,7,8,10,13,14,17,18,20,21]).
iati(18,[4,7,8,9,10,12,14,17,18,20,22]).
iati(19,[1,3,11,13,19,21]).
iati(20,[2,4,5,6,7,9,10,17,18,20,21,22]).
iati(21,[1,2,4,5,6,8,10,11,13,14,15,17,19,20,21]).
iati(22,[4,5,7,9,10,12,16,18,20,22]).

%-------------------------------------------------------
%Loop through all significant places - outer loop 1 to 22
%-------------------------------------------------------
checko(22) :- checki(22,1).

checko(I)  :- checki(I,1),
              succ(I,INC),
              checko(INC).

%-------------------------------------------------------
%Loop through all significant places - inner loop 1 to 22
%-------------------------------------------------------
checki(I,22) :- share_place(I,22).

checki(I,22) :- near(I,22,t),
                write('Contradiction for: '),write(I),write(' and '),write(22),nl.

checki(I,22) :- near(I,22,i),
                write('Contradiction for: '),write(I),write(' and '),write(22),nl.

checki(I,22) :- near(I,22,f).

checki(I,J)  :- share_place(I,J),
                succ(J,JNC),
                checki(I,JNC).

checki(I,J)  :- near(I,J,t),
                write('Contradiction for: '),write(I),write(' and '),write(J),nl,
                succ(J,JNC),
```

```
                checki(I,JNC).

checki(I,J)  :- near(I,J,i),
write('Contradiction for: '),write(I),write(' and '),write(J),nl,
                succ(J,JNC),
                checki(I,JNC).

checki(I,J)  :- near(I,J,f),
                succ(J,JNC),
                checki(I,JNC).

share_place(X,Y) :- iati(X,L1),
                    iati(Y,L2),
                    have_common_place(L1,L2).

have_common_place([H1|T1],L2) :-  member(H1,L2).

have_common_place([H1|T1],L2) :-  have_common_place(T1,L2).
```

B Prolog Program to verify Axiom (A2) with I interpreted as F

```
%Facts of Nearness Relations between Significant Places
near(1,1,t).
[...]
near(22,22,t).

%Facts of Influence Areas of Significant Places only including true Relations
iat(1,[1,21]).
iat(2,[2,5,6]).
iat(3,[3]).
iat(4,[4,7,10,18,20]).
iat(5,[2,5]).
iat(6,[6,17,21]).
iat(7,[4,7,20,22]).
iat(8,[8,14,18]).
iat(9,[9,22]).
iat(10,[4,10,17,20]).
iat(11,[11,13]).
iat(12,[12,18]).
iat(13,[11,13,14,15,21]).
iat(14,[8,13,14,15]).
iat(15,[13,14,15]).
iat(16,[16]).
iat(17,[6,10,17,21]).
iat(18,[4,8,12,18]).
iat(19,[19]).
iat(20,[4,7,10,20]).
iat(21,[1,6,13,17,21]).
iat(22,[7,9,22]).

%-------------------------------------------------------
%Loop through all significant places - outer loop 1 to 22
%-------------------------------------------------------
checko(22) :- checki(22,1).

checko(I)  :- checki(I,1),
              succ(I,INC),
              checko(INC).

%-------------------------------------------------------
%Loop through all significant places - inner loop 1 to 22
%-------------------------------------------------------
checki(I,22) :- share_place(I,22).

checki(I,22) :- near(I,22,t),
                write('Contradiction for: '),
                write(I),
                write(' and '),
                write(22),
                write(' : true'),
                nl.

checki(I,22) :- near(I,22,i).

checki(I,22) :- near(I,22,f).

checki(I,J)  :- share_place(I,J),
                succ(J,JNC),
                checki(I,JNC).
```

```
checki(I,J)  :- near(I,J,t),
                write('Contradiction for: '),write(I),write(' and '),write(J),
                write(' : true'), nl,
                succ(J,JNC),
                checki(I,JNC).

checki(I,J)  :- near(I,J,i),
                succ(J,JNC),
                checki(I,JNC).

checki(I,J)  :- near(I,J,f),
                succ(J,JNC),
                checki(I,JNC).

share_place(X,Y) :- iat(X,L1),
                    iat(Y,L2),
                    have_common_place(L1,L2).

have_common_place([H1|T1],L2) :-  member(H1,L2).

have_common_place([H1|T1],L2) :-  have_common_place(T1,L2).
```

C Prolog Program to verify Nearness Relation s-near1 considering determinate values only

```
%Facts of Nearness Relations between significant places
near(1,1,t).
[...]
near(22,22,t).

%Facts of Influence Areas of Significant Places only including true Relations
iat(1,[1,21]).
iat(2,[2,5,6]).
iat(3,[3]).
iat(4,[4,7,10,18,20]).
iat(5,[2,5]).
iat(6,[6,17,21]).
iat(7,[4,7,20,22]).
iat(8,[8,14,18]).
iat(9,[9,22]).
iat(10,[4,10,17,20]).
iat(11,[11,13]).
iat(12,[12,18]).
iat(13,[11,13,14,15,21]).
iat(14,[8,13,14,15]).
iat(15,[13,14,15]).
iat(16,[16]).
iat(17,[6,10,17,21]).
iat(18,[4,8,12,18]).
iat(19,[19]).
iat(20,[4,7,10,20]).
iat(21,[1,6,13,17,21]).
iat(22,[7,9,22]).

%--------------------------------------------------------
%Loop through all significant places - outer loop 1 to 22
%--------------------------------------------------------
checko(22) :- checki(22,1).

checko(I)  :- checki(I,1),
              succ(I,INC),
              checko(INC).

%--------------------------------------------------------
%Loop through all significant places - inner loop 1 to 22
%--------------------------------------------------------

checki(I,22) :- share_place(I,22),
                near(I,22,t).

checki(I,22) :- share_place(I,22),
                near(I,22,i).

checki(I,22) :- share_place(I,22),
                near(I,22,f),
                write('Contradiction for: '), write(I),write(' and '), write(22),nl.

checki(I,22).

checki(I,J)  :- share_place(I,J),
                near(I,J,t),
                succ(J,JNC),
                checki(I,JNC).
```

```
checki(I,J)  :- share_place(I,J),
                near(I,J,i),
                succ(J,JNC),
                checki(I,JNC).

checki(I,J)  :- share_place(I,J),
                near(I,J,f),
                write('Contradiction for: '),write(I),write(' and '),write(J),nl,
                succ(J,JNC),
                checki(I,JNC).

checki(I,J)  :- succ(J,JNC),
                checki(I,JNC).

share_place(X,Y) :- iat(X,L1),
                    iat(Y,L2),
                    have_common_place(L1,L2).

have_common_place([H1|T1],L2) :-  member(H1,L2).

have_common_place([H1|T1],L2) :-  have_common_place(T1,L2).
```