Pretty Good Strategies To Cast Your Vote Securely

Wojtek Jamroga

University of Luxembourg & Polish Academy of Sciences

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Outline

Secure Voting

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- In particular, the voter should get no receipt to show the coercer how she voted (receipt-freeness)

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- How can the voter make sure that the encryption is correct?
  $\rightsquigarrow$ **Benaloh challenge**

Benaloh Challenge

Procedure:

1. The voter chooses a value of the vote (e.g., a candidate)
2. The device commits to an encryption
3. The voter decides whether to cast the vote (without opening) or audit the encryption and spoil the vote
4. Repeat any number of times until the vote is cast

Benaloh Challenge

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This is a **game-theoretic argument**!

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- **[Culnane & Teague 2016]**: very nice analysis, BC as an inspection game
- Two players: the voter vs. the corrupt device serving the interests of the perpetrator
- The voter chooses $n_{cast} \in \{1, \ldots, n_{max}\}$
- The device selects $n_{cheat} \in \{1, \ldots, n_{max} + 1\}$

Game-Theoretic Analysis of Benaloh Challenge

Payoff table:

	Voter payoff $u_V(n_{cast}, n_{cheat})$	Device payoff $u_D(n_{cast}, n_{cheat})$	Comment
$n_{cast} < n_{cheat}$	$-(n_{cast} - 1)c_{audit} + Succ_V$	0	Voter votes as intended
$n_{cast} = n_{cheat}$	$-(n_{cast} - 1)c_{audit} - Fail_V$	$Succ_D$	Device successfully cheats
$n_{cast} > n_{cheat}$	$-n_{cheat} \cdot c_{audit}$	$-Fail_D$	Voter catches cheating device

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Claims (CC & VT 2016):

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Game-Theoretic Analysis of Benaloh Challenge

Claims (CC & VT 2016):

1. In deterministic **Nash equilibrium** strategies, the voter casts right away, and the device immediately cheats. Thus, only randomized strategies of the voter make sense (correct)
2. The same happens for randomized strategies with a bounded number of steps. Thus, only infinite audit strategies make sense (......?)
3. In an infinite Benaloh game, there is no natural Nash equilibrium strategy for the voter (correct)

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Backward Induction

Claim (CC & VT 2016):

In a finite Benaloh game, backward induction produces the following strategy profile:

1. The voter casts her vote immediately
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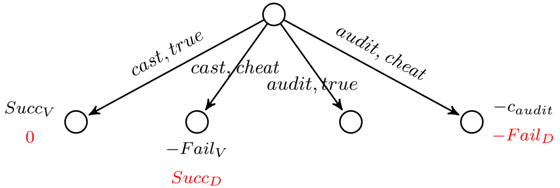
However, to see how backward induction works, we first need to switch to an extensive form game.

Game Tree for Benaloh Challenge



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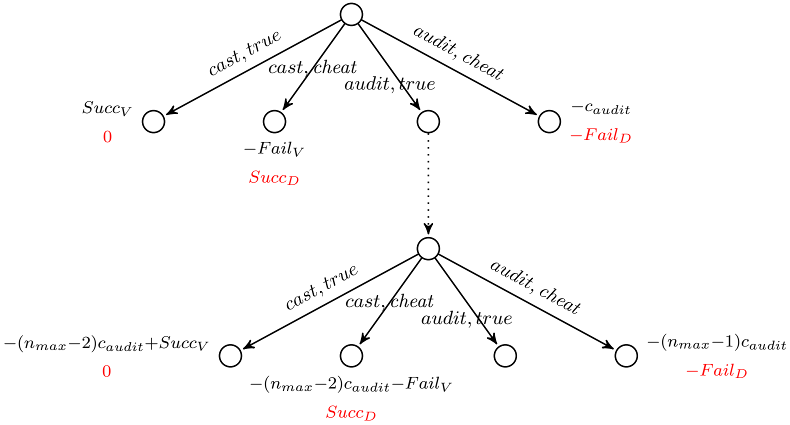

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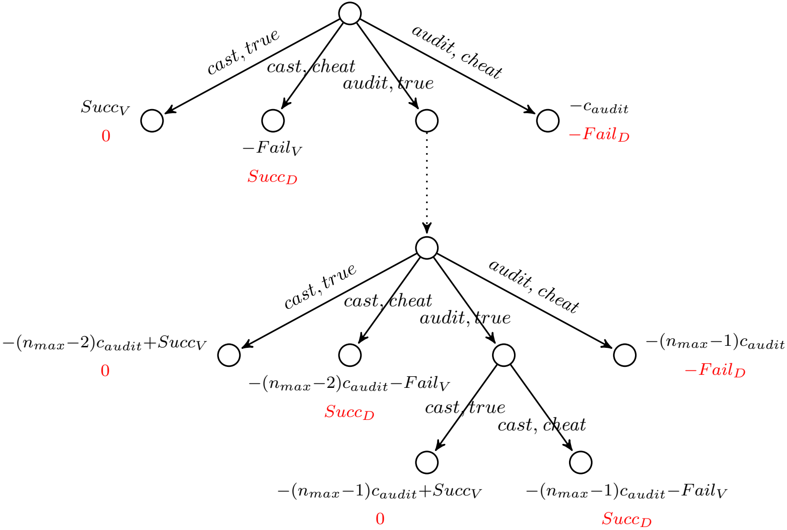
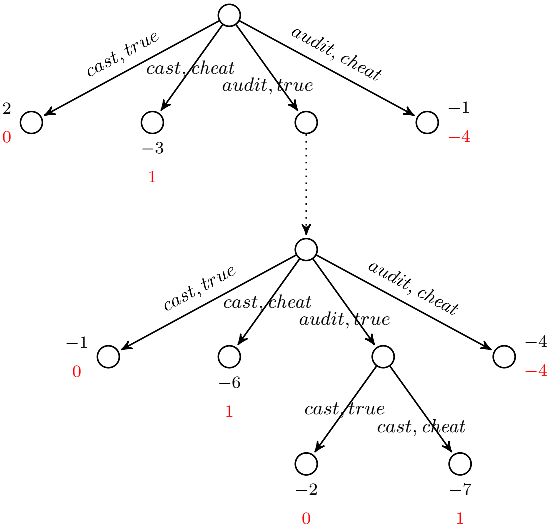


For the Sake of Presentation...

Let's fix example values of the parameters:

- $n_{max} = 5$ (at most 4 audits before casting)
- $Succ_D = 1$ and $Fail_D = 4$ (the perpetrator fears failure 4 times more than he values success)
- $Succ_V = 2$ and $Fail_V = 3$ (the voter loses slightly more by getting cheated than she gains by casting successfully)
- $c_{audit} = 1$ (the cost of audit is half of the gain from a successful vote)

Game Tree for Benaloh Challenge (Example)



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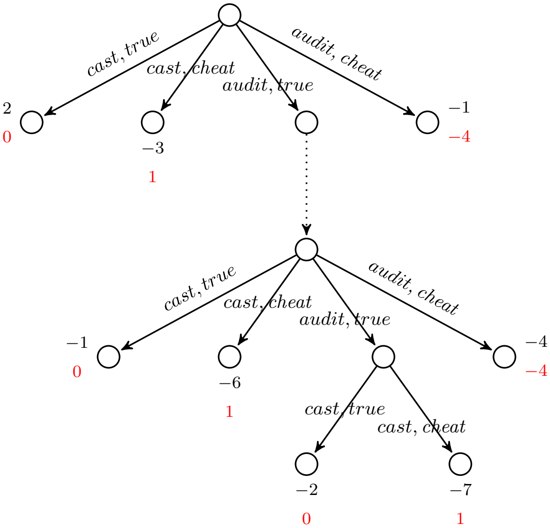

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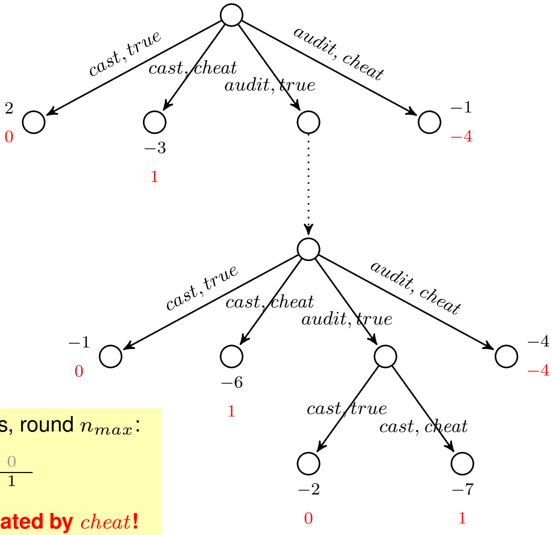

Device choices, round n_{max}:

$true$	0
$cheat$	1

$true$ **is dominated by** $cheat$**!**

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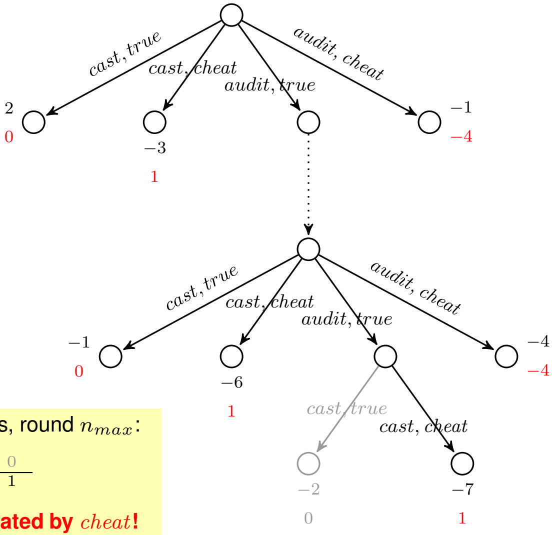

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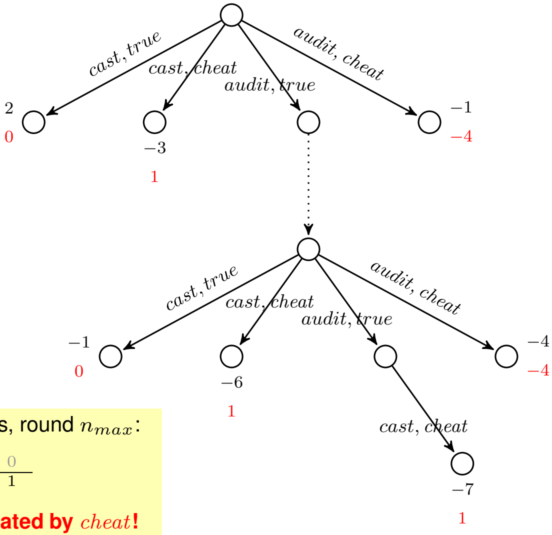

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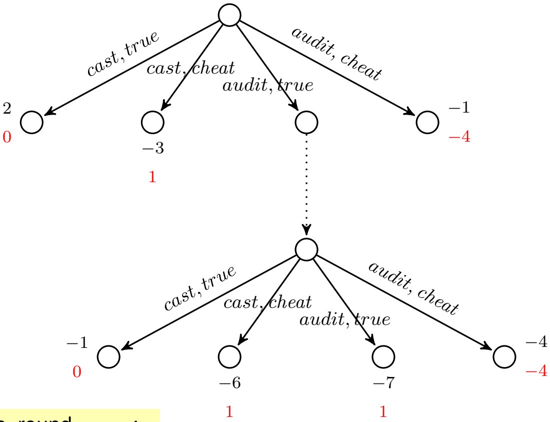

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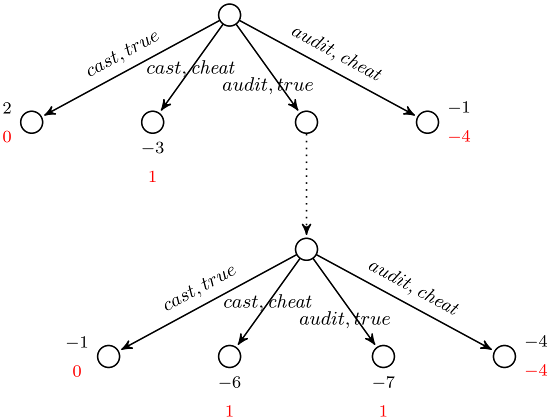

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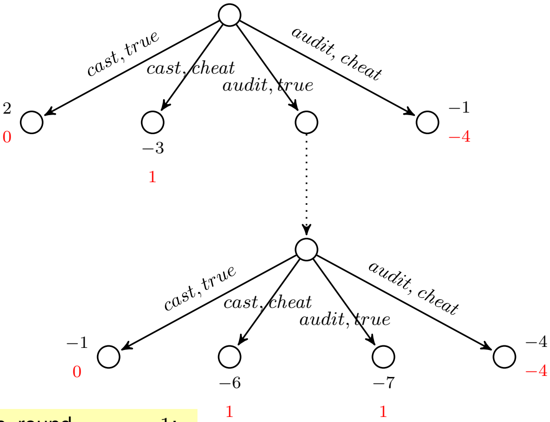

Device choices, round $n_{max} - 1$:

$true$	0	1
$cheat$	1	-4

None dominates the other one!

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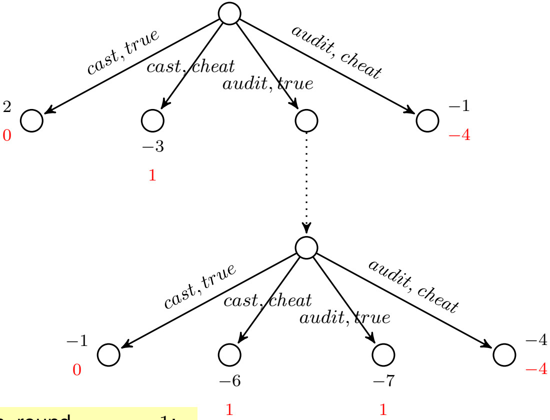

Voter's choices, round $n_{max} - 1$:

cast	-1	-6
audit	-7	-4

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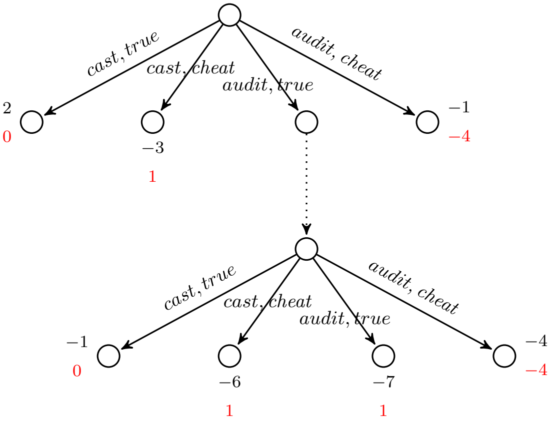

Backward induction stops here!

Thus, looking at finite audit strategies might make sense!

Nash Equilibria in Finite Randomized Strategies

Theorem (Nash equilibrium in finite Benaloh games)

The mixed voting strategy $s_V = [p_1^V, \cdots, p_{n_{max}}^V]$ is a part of Nash equilibrium iff, for every $n \in \{1, \dots, n_{max}\}$:

$$\mathbf{p_n^V} = \frac{\mathbf{(1-R)R^{n-1}}}{\mathbf{1-R^{n_{max}}}}, \qquad \text{where } R = \frac{Succ_D}{Succ_D + Fail_D}.$$

Alternatively: the behavioral voting strategy $b_V = [b_1^V, \cdots, b_{n_{max}}^V]$ is a part of Nash equilibrium iff, for every $n \in \{1, \dots, n_{max}\}$:

$$\mathbf{b_n^V} = \frac{\mathbf{1-R}}{\mathbf{1-R^{n_{max}-n+1}}}, \qquad \text{where } R = \frac{Succ_D}{Succ_D + Fail_D}.$$

Nash Equilibria in Finite Randomized Strategies

In our example, the behavioral NE strategy is

$$b_V = [0.8, 0.801, 0.81, 0.83, 1].$$

That is, the voter:

1. casts immediately with probability 0.8,
2. else audits, again, and casts with probability 0.801,
3. else audits, randomizes again, and casts with probability 0.81,
4. else audits, randomizes again, and casts with probability 0.83,
5. else audits, and finally casts with probability 1.

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That doesn't seem easy to execute...
Can we make the voter's life easier?

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Then, the unique NE strategy for the voter is:

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In our example: cast immediately with probability $\frac{5}{6}$, otherwise audit first and cast in the second round.

Towards Simple and Natural Audit Strategies

Moreover, we can often assume that $Fail_D \gg Succ_D$.
Then, p is close 1.

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Looks like a simple strategy

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Well, should it really...?

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Big question:

Is Nash equilibrium the **right solution concept** to capture the players' deliberation in Benaloh games?

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  $\rightsquigarrow$ information asymmetry in favor of the device
- GT fact: this kind of asymmetry can be exploited by the voter
  $\rightsquigarrow$ **Stackelberg equilibrium**

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- Stackelberg equilibrium captures the outcome in games where one player (the *leader*) exposes her strategy first, and the other players play their best response

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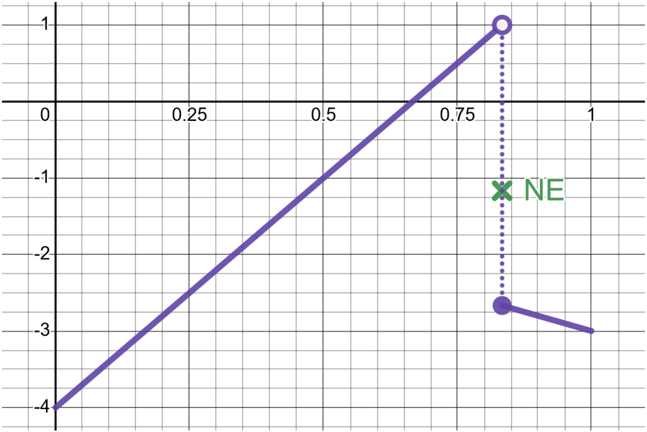
- GT fact: Stackelberg equilibrium often obtains a better payoff for the leader than Nash equilibrium

Pretty Good Strategies against Best Response



Pretty Good Strategies against Best Response

Theorem (Stackelberg equilibrium in simple Benaloh games)

The following properties hold for the Benaloh game with $n_{max} = 2$:

1. There is no Stackelberg equilibrium for V in randomized strategies.
2. The Stackelberg value of the game is $SVal_V = \frac{Succ_D(Succ_V - Fail_V - c_{audit}) + Fail_D Succ_V}{2Succ_D + Fail_D}$.
3. $SVal_V > Eu_V(p^V_{NE}, p^D_{NE})$, where (p^V_{NE}, p^D_{NE}) is the Nash equilibrium.
4. If $Fail_D \gg Succ_D$ and $Succ_V \geq aFail_V$ for a fixed $a > 0$, then $SVal_V > 0$.

Pretty Good Strategies against Best Response

In plain words...

In the simple Benaloh game:

1. Stackelberg equilibrium in randomized strategies does not exist.
2. However, the Stackelberg optimum can be approximated by the voter arbitrarily close, promising **payoff** that is **positive** and **strictly higher than NE**.

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Takeaway Advice

1. Using Benaloh challenge can be practical and beneficial to the rational voter
2. Putting a limit on the number of allowed audits makes things easier for the voter.

   Considerate election authority might design the voting system so that each voter can audit the vote encryption at most once.
3. The voters should not try to adapt to the strategy of the attacker, the way Nash equilibrium prescribes.

   Instead, they should stick to **auditing the votes with a fixed low frequency**, thus approximating the Stackelberg optimum and putting the attacker on the defensive

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Questions?