Ring Withdrawals

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Outline


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Varities of Conservatism

- Metaphysics: *Ex Nihilo Nihil Fit*
  - Nothing Comes from Nothing
  - Dual: Nothing Vanishes to Nothing
- Physics: *Conservation of (Mass-)Energy*
  - Energy can neither be created nor be destroyed
  - It can only be transformed
  - $\Rightarrow$ (Quantum-)Information cannot be created or . . .
- Epistemology: *Principle of Minimality (Sufficient Reason)*
  - Beliefs cannot be acquired from Nothing
  - Beliefs cannot be lost *irrevocably*

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Epistemic Conservatism

- Beliefs cannot be acquired from Nothing
  - *A piece of evidence can only lead to beliefs whose veracity it can guarantee jointly with the old beliefs*
  - *Inclusion*: $\mathbf{K} * \alpha \subseteq \mathbf{K} + \alpha$
- Beliefs cannot be lost *irrevocably*
  - All information lost through removal of a belief α can be regained through reinstating α
  - *Recovery:* $\mathbf{K} \subseteq (\mathbf{K} - \alpha) + \alpha$
- Principle of Minimal Change
  - If coherence demands change in beliefs, that change must be as little as one can get away with

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Contraction Postulates

$K - \alpha = Cn(K - \alpha)$.	**(Closure)**
$K - \alpha \subseteq K$.	**(Inclusion)**
If $\alpha \notin K$, then $K - \alpha = K$.	**(Vacuity)**
If $\nvdash \alpha$, then $\alpha \notin K - \alpha$.	**(Success)**
If $\vdash \alpha \leftrightarrow \beta$, then $K - \alpha = K - \beta$.	**(Extensionality)**
$K \subseteq (K - \alpha) + \alpha$.	**(Recovery)**
$(K - \alpha) \cap (K - \beta) \subseteq K - (\alpha \wedge \beta)$.	**(Conjunctive overlap)**
$K - (\alpha \wedge \beta) \subseteq K - \alpha$ whenever $\alpha \notin K - (\alpha \wedge \beta)$.	**(Conjunctive inclusion)**

Contraction Postulates – Rationale

$(K - \alpha) \cap (K - \beta) \subseteq K - (\alpha \wedge \beta)$. **(Conjunctive overlap)**

- To remove $\alpha \wedge \beta$, remove at least one of α, β
- Suppose x gets discarded through removing $\alpha \wedge \beta$
- $\Rightarrow$ x will be lost via removal of α, or via removal of β
- So if $x \notin K - (\alpha \wedge \beta)$, then $x \notin (K - \alpha) \cap (K - \beta)$

$K - (\alpha \wedge \beta) \subseteq K - \alpha$ whenever $\alpha \notin K - (\alpha \wedge \beta)$. **(Conj. Incl.)**

- Suppose $\alpha \notin K - (\alpha \wedge \beta)$
- removal of $\alpha \wedge \beta$ is sufficient to remove α . . .

  no more information loss is mandated
- So $K - (\alpha \wedge \beta) \subseteq K - \alpha$

Contraction Postulates

$K - \alpha = Cn(K - \alpha)$. **(Closure)**
$K - \alpha \subseteq K$. **(Inclusion)**
If $\alpha \notin K$, then $K - \alpha = K$. **(Vacuity)**
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Against Recovery [Hansson, 1999]

I entertained two beliefs

- α: George is a criminal
- β: George is a mass murderer

New information led me to suspend belief in α...

- $\beta \notin (\mathbf{K} - \alpha)$ since $\beta \models \alpha$

Then I learned δ: George is a shoplifter...

$\Rightarrow$ new belief set: $(\mathbf{K} - \alpha) + \delta$

- $(\mathbf{K} - \alpha) + \alpha \subseteq (\mathbf{K} - \alpha) + \delta$, since $\delta \models \alpha$
- By recovery, $\beta \in \mathbf{K} \subseteq (\mathbf{K} - \alpha) + \alpha \subseteq (\mathbf{K} - \alpha) + \delta$
- *Shop_lifter(george)* $\mid\!\not\sim$ *Mass_murderer(george)*

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Enter Withdrawals

Definition (Makinson, 1987)

Let **K** be a belief set. An operation $\div$ for **K** is a *withdrawal* operation if and only if it satisfies closure, inclusion, vacuity, success and extensionality.

- Recovery is no longer mandated.
- Contraction operation is a withdrawal operation that also satisfies Recovery

Possible Worlds

- language $\mathcal{L}$ (a set of sentences)
- a world ω is a maximal consistent subset of $\mathcal{L}$
- $\mathcal{M}_{\mathcal{L}}$: set of all such possible worlds ω
- $\|R\| = \{\omega \in \mathcal{M}_{\mathcal{L}} : R \subseteq \omega\}$, for all $R \subseteq \mathcal{L}$
  - $\|R\|$: set of worlds ω that satisfy **every sentence** in R
  - If R is inconsistent, $\|R\| = \emptyset$
  - R-world: a world ω in $\|R\|$
  - $\|\alpha\|$ abbreviates $\|\{\alpha\}\|$, for any sentence $\alpha \in \mathcal{L}$
  - α-world abbreviates $\{\alpha\}$-world

Possible Worlds

Observation (Grove,1988)

Assume:

- *belief sets* **K** *and* **H**
- *sentences* α *and* β
- *sets of possible worlds U and V*

(a) *If* $\mathbf{K} \subseteq \mathbf{H}$, *then* $\|\mathbf{H}\| \subseteq \|\mathbf{K}\|$.

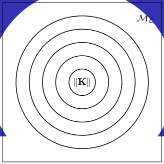
(b) $Th(V) = \bigcap V$ *is a belief set.*

(c) $Th(\|\mathbf{K}\|) = \mathbf{K}$ *(if the underlying logic is compact).*

(d) *If* $U \subseteq V$, *then* $Th(V) \subseteq Th(U)$.

(e) *For any* $\alpha \in \mathcal{L}$, $Th(V \cap \|\alpha\|) = Cn(Th(V) \cup \{\alpha\})$.

System of Spheres



Definition (Grove,1988)

- belief set **K**
- system of spheres $\mathbb{S}$ centred on $\|\mathbf{K}\|$
  - a collection of sets of worlds $\omega \in \mathcal{M}_{\mathcal{L}}$

($\mathbb{S}$1) $\mathbb{S}$ is totally ordered (with respect to $\subseteq$)

($\mathbb{S}$2) $\|\mathbf{K}\| \in \mathbb{S}$, and $\|\mathbf{K}\|$ is the $\subseteq$-minimum of $\mathbb{S}$

($\mathbb{S}$3) $\mathcal{M}_{\mathcal{L}}$(set of all worlds) is the largest element of $\mathbb{S}$

($\mathbb{S}$4) If an element (sphere) in $\mathbb{S}$ intersects $\|\alpha\|$ for $\alpha \in \mathcal{L}$, then:

- there is a smallest sphere in $\mathbb{S}$ that intersects $\|\alpha\|$
- $\mathbb{S}_\alpha$ denotes that smallest sphere

Contraction based on System of Spheres

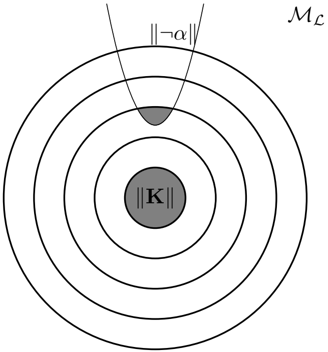


Figure: Representation of $\|\mathbf{K}\| \div_{\mathbb{S}} \|\alpha\|$ (shaded)

System of Spheres-based Contraction

Definition (Grove,1988)

- Let $\mathbb{S}$ be an SOS centred on $\|\mathbf{K}\|$
- $\div_{\mathbb{S}}$ is an $\mathbb{S}$-based contraction on $\|\mathbf{K}\|$:
  - for all $\alpha \in \mathcal{L}$

$$\|\mathbf{K}\| \div_{\mathbb{S}} \|\alpha\| = \begin{cases} \|\mathbf{K}\| \cup (\mathbb{S}_{\neg\alpha} \cap \|\neg\alpha\|) & \text{if } \|\neg\alpha\| \neq \emptyset \\ \|\mathbf{K}\| & \text{otherwise} \end{cases}$$

  - To remove α, incorporate closest $\neg\alpha$-worlds to $\|\mathbf{K}\|$
- operator $\div$ on $\mathbf{K}$ is an SOS-based contraction iff it is generated from some $\mathbb{S}$-based contraction $\div_{\mathbb{S}}$ on $\|\mathbf{K}\|$
  - $\mathbf{K} \div \alpha = Th(\|\mathbf{K}\| \div_{\mathbb{S}} \|\alpha\|)$, for all sentences $\alpha \in \mathcal{L}$
- AGM Contraction is SOS-based

Severe Withdrawals

Definition (Rott, Pagnucco, 1999)

- let SOS $\mathbb{S}$ be centered on $\|\mathbf{K}\|$
- severe withdrawal operator $\sim_{\mathbb{S}}$ on **K**:

$$\|\mathbf{K}\| \sim_{\mathbb{S}} \|\alpha\| = \begin{cases} \mathbb{S}_{\neg\alpha} & \text{if } \nvdash \alpha \\ \|\mathbf{K}\| & \text{otherwise} \end{cases}$$

- operator $\sim$ on **K** is analogously defined:
  - $\mathbf{K} \sim \alpha = \mathit{Th}(\|\mathbf{K}\| \sim_{\mathbb{S}} \|\alpha\|)$, for all sentences $\alpha \in \mathcal{L}$

Story line: to remove α incorporate some $\neg\alpha$-world to $\|\mathbf{K}\|$

- closest $\neg\alpha$-world is in $\mathbb{S}_{\neg\alpha}$
- no world in $\mathbb{S}_{\neg\alpha}$ is worse (farther) than any $\neg\alpha$-world
- incorporate into $\|\mathbf{K}\|$ all worlds in $\mathbb{S}_{\neg\alpha}$

Severe Withdrawals

Observation (Rott, Pagnucco, 1999)

An operator $\sim$ *for* **K** *is a severe withdrawal if and only if it satisfies closure, inclusion, vacuity, success, and*

If $\vdash \alpha$, then $\mathbf{K} = \mathbf{K} \sim \alpha$ **(Failure)**
If $\alpha \notin \mathbf{K} \sim \beta$, then $\mathbf{K} \sim \beta \subseteq \mathbf{K} \sim \alpha$ **(Strong inclusion)**

Also satisfies **Expulsiveness:**
If $\nvdash \alpha$ and $\nvdash \beta$, then either $\alpha \notin \mathbf{K} \sim \beta$ or $\beta \notin \mathbf{K} \sim \alpha$

Rather presumptive:

- Strong justification structure among non-trivial beliefs
- no $\alpha, \beta \in \mathbf{K}$ are epistemically independent of each other

  $\Rightarrow$ Excessive loss of information

Ring Withdrawals

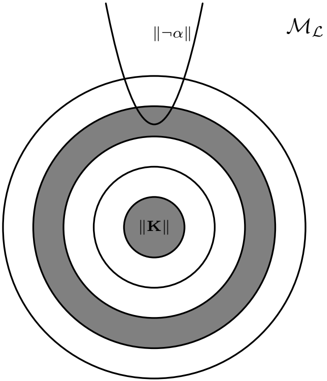


Figure: $\|\mathbf{K} \div \alpha\|$, ring withdrawal of **K** by α (shaded)

Ring Withdrawals

Definition (Fermé, Garapa, Nayak, Reis, 2024)

- let SOS $\mathbb{S}$ be centered on $\|\mathbf{K}\|$
- ring withdrawal operator $\div_{\mathbb{S}}$ on **K**:

$$\|\mathbf{K}\| \div_{\mathbb{S}} \|\alpha\| = \begin{cases} \|\mathbf{K}\| \cup (\mathbb{S}_{\neg\alpha} \setminus \bigcup\{S : S \subsetneq \mathbb{S}_{\neg\alpha}\}) & \text{if } \|\neg\alpha\| \neq \emptyset \\ \|\mathbf{K}\| & \text{otherwise} \end{cases}$$

- operator $\div$ on **K** is analogously defined:
  - $\mathbf{K} \div \alpha = Th(\|\mathbf{K}\| \div_{\mathbb{S}} \|\alpha\|)$, for all sentences $\alpha \in \mathcal{L}$

Story line: to remove α incorporate some $\neg\alpha$-world to $\|\mathbf{K}\|$

- closest $\neg\alpha$-worlds are in $\mathbb{S}_{\neg\alpha}$
- incorporate into $\|\mathbf{K}\|$ **all worlds** that are equally close

Ring Withdrawals

Theorem (Fermé, Garapa, Nayak, Reis, 2024)

An operator $\div$ for **K** *is a ring withdrawal iff it satisfies:*

- *closure, inclusion, vacuity, success and extensionality,* **and**
- **Recuperation:**

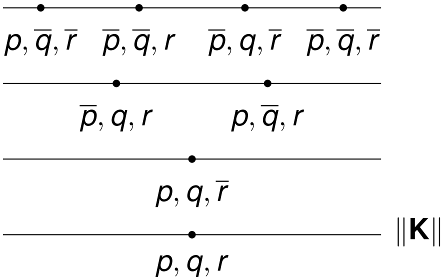
  If $\alpha \in (\mathbf{K} - \beta)$, *then* $\mathbf{K} \subseteq Cn(\mathbf{K} - \alpha \cup \mathbf{K} - \beta)$

  if α didn't (epistemically) depend on β, then nothing that depends on β depends on α
- **Strong Conjunctive Inclusion:**

  If $\alpha \notin \mathbf{K} - (\alpha \wedge \beta)$, *then* $\mathbf{K} - (\alpha \wedge \beta) = \mathbf{K} - \alpha$

  if α is the weaker of the two, then α gets to be jettisoned

Example



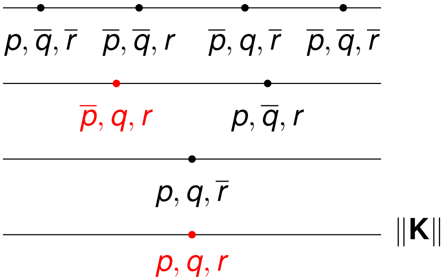
AGM Contraction

$$\mathbf{K} - p = Cn(\{q, r\})$$

Ring Withdrawal

$$\mathbf{K} \div p = Cn(\{r, p \vee q\})$$

Example



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Severe Withdrawal

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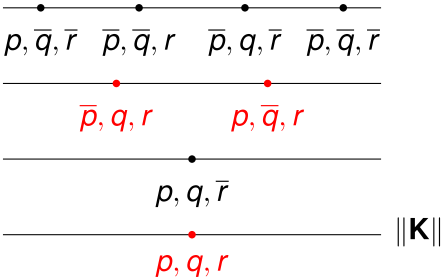

AGM Contraction

$$\mathbf{K} - p = Cn(\{q, r\})$$

Ring Withdrawal

$$\mathbf{K} \div p = Cn(\{r, p \vee q\})$$

Severe Withdrawal

$$\mathbf{K} \sim p = Cn(\{(p \wedge q) \vee (p \wedge r) \vee (q \wedge r)\})$$

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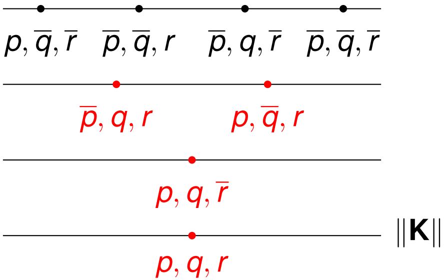

AGM Contraction

$$\mathbf{K} - p = Cn(\{q, r\})$$

Severe Withdrawal

$$\mathbf{K} \sim p = Cn(\{(p \wedge q) \vee (p \wedge r) \vee (q \wedge r)\})$$

Ring Withdrawal

$$\mathbf{K} \dot{-} p = Cn(\{r, p \vee q\})$$

Example

AGM Contraction

$$\mathbf{K} - p = Cn(\{q, r\})$$

Ring Withdrawal

$$\mathbf{K} \div p = Cn(\{r, p \vee q\})$$

Severe Withdrawal

$$\mathbf{K} \sim p = Cn(\{(p \wedge q) \vee (p \wedge r) \vee (q \wedge r)\})$$

$$\mathbf{K} \sim p \subseteq \mathbf{K} \div p \subseteq \mathbf{K} - p$$

Three ways for removing beliefs

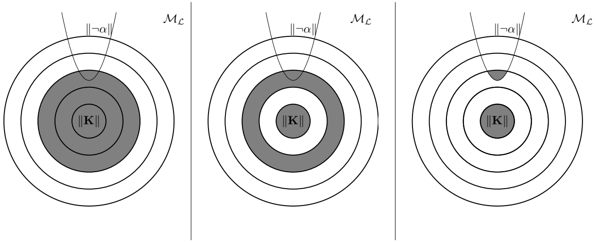

Figure: Severe withdrl (L),Ring withdrl (C) and AGM con (R)

(Lindström and Rabinowicz, 1991) interpolation thesis:

- Any reasonable belief removal operation should fall between severe withdrawals and AGM contractions.

Three ways for removing beliefs

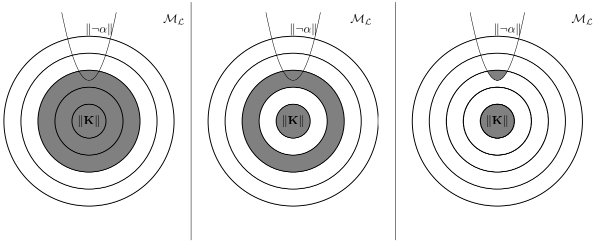

Figure: Severe withdrl (L),Ring withdrl (C) and AGM con (R)

(Lindström and Rabinowicz, 1991) interpolation thesis:

- Any reasonable belief removal operation should fall between severe withdrawals and AGM contractions.

Revision Equivalent Withdrawals

Definition (Makinson, 1987))

Let $\div$ and $\div'$ be two withdrawal operations on **K**

They are revision equivalent iff

- $(\mathbf{K} \div \neg\alpha) + \alpha = (\mathbf{K} \div' \neg\alpha) + \alpha$

Note the Levi Identity: $(\mathbf{K} * \alpha) = (\mathbf{K} - \neg\alpha) + \alpha$

Severe Withdrawal to AGM Contraction, and Back

(Rott, Pagnucco, 1999)

Defining AGM Contractions from Severe Withdrawals

$$\mathbf{K} - \alpha = ((\mathbf{K} \sim \alpha) + \neg\alpha) \cap \mathbf{K} \qquad (\text{Def} - \text{from} \sim)$$

Observation

Let operation $-$ be obtained from a severe withdrawal operator $\sim$ as shown, via (Def $-$ from $\sim$). Then,

- *$-$ is an AGM contraction operator*
- *$-$ is revision equivalent to $\sim$*
- $\mathbf{K} \sim \alpha \subseteq \mathbf{K} - \alpha$*, for all* $\alpha \in \mathcal{L}$

Severe Withdrawal to AGM Contraction, and Back

(Rott, Pagnucco, 1999)

Defining Severe Withdrawals from AGM Contractions

$$\mathbf{K} \sim \alpha = \begin{cases} \{\beta : \beta \in \mathbf{K} - (\alpha \wedge \beta)\} & \text{if } \nvdash \alpha \\ \mathbf{K} & \text{otherwise} \end{cases} \quad \text{(Def } \sim \text{ from } -\text{)}$$

Observation

Obtain $\sim$ from an AGM contraction operation $-$ as shown

- *$\sim$ is severe withdrawal operator*
- *$\sim$ is revision equivalent to $-$*
- $\mathbf{K} \sim \alpha \subseteq \mathbf{K} - \alpha$*, for all* $\alpha \in \mathcal{L}$

Ring Withdrawal to Severe Withdrawal, and Back

(Fermé, Garapa, Nayak, Reis, 2024)

Definining Ring Withdrawals from Severe Withdrawals

$$\mathbf{K} \div \alpha = \bigcap\{(\mathbf{K} \sim \alpha + \neg\beta) \cap \mathbf{K} : \mathbf{K} \sim \alpha = \mathbf{K} \sim \beta\}$$

(Def $\div$ from $\sim$)

Observation

Obtain $\div$ from severe withdrawal operation $\sim$ as shown

- *$\div$ is a ring withdrawal operation*
- *$\div$ is revision equivalent to $\sim$*
- $\mathbf{K} \sim \alpha \subseteq \mathbf{K} \div \alpha$, *for all $\alpha \in \mathcal{L}$*

Ring Withdrawal to Severe Withdrawal, and Back

(Fermé, Garapa, Nayak, Reis, 2024)

Defining Severe Withdrawals from Ring Withdrawals

$$\mathbf{K} \sim \alpha = \begin{cases} \{\beta : \beta \in \mathbf{K} \div (\alpha \wedge \beta)\} & \text{if } \nvdash \alpha \\ \mathbf{K} & \text{otherwise} \end{cases} \qquad \text{(Def } \sim \text{ from } \div\text{)}$$

Observation (Feré, Garapa, Nayak, Reis, 2024)

Obtain $\sim$ from ring withdrawal operation $\div$ as shown

- *$\sim$ is a severe withdrawal operation*
- *$\sim$ is revision equivalent to $\div$*
- *$\mathbf{K} \sim \alpha \subseteq \mathbf{K} \div \alpha$, for all $\alpha \in \mathcal{L}$*

Ring Withdrawal to AGM Contraction, and Back

(Fermé, Garapa, Nayak, Reis, 2024)

Definining Ring Withdrawals from AGM Contractions

$$\mathbf{K} \div \alpha = \begin{cases} \bigcap\{\mathbf{K} - (\alpha \wedge \beta) : \mathbf{K} - (\alpha \wedge \beta) \cap \{\alpha, \beta\} = \emptyset\} & \text{if } \alpha \in \mathbf{K}, \not\vdash \alpha \\ \mathbf{K} & \text{otherwise} \end{cases}$$

(Def $\div$ from $-$)

Observation

Obtain $\div$ from an AGM contraction operation $-$ as shown

- *$\div$ is a ring withdrawal operation*
- *$\div$ is revision equivalent to $-$*
- *$\mathbf{K} \div \alpha \subseteq \mathbf{K} - \alpha$, for all $\alpha \in \mathcal{L}$*

Ring Withdrawal to AGM Contraction, and Back

(Fermé, Garapa, Nayak, Reis, 2024)

Definining AGM Contractions from Ring Withdrawals

$$\mathbf{K} - \alpha = ((\mathbf{K} \sim \alpha) + \neg\alpha) \cap \mathbf{K} \qquad (\text{Def} - \text{from} \div)$$

Observation

Obtain $-$ from a ring withdrawal operation $\div$ as shown

- $-$ *is an AGM contraction operation*
- $-$ *is revision equivalent to* $\div$
- $\mathbf{K} \div \alpha \subseteq \mathbf{K} - \alpha$, *for all* $\alpha \in \mathcal{L}$

Thanks!

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