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COMP3152/9152
Lecture 4
Properties of Knowledge and Time
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Reading, FHMV Ch 8

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Axioms for Linear Time: LT

T1. $\bigcirc(\varphi) \wedge \bigcirc(\varphi \Rightarrow \psi) \Rightarrow \bigcirc\psi$

T2. $\bigcirc(\neg\varphi) \Leftrightarrow \neg\bigcirc\varphi$

T3. $\varphi U \psi \Leftrightarrow \psi \vee (\varphi \wedge \bigcirc(\varphi U \psi))$

RT1. If φ then $\bigcirc\varphi$

RT2. If $\varphi' \Rightarrow \neg\psi \wedge \bigcirc\varphi'$ then $\varphi' \Rightarrow \neg(\varphi U \psi)$

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Properties of systems

sync: A system $\mathcal{R}$ is *synchronous* if for all agents i, if $(r,m) \sim_i (r',m')$ then $m = m'$.

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Concordant intervals

Two intervals (possibly infinite) of two runs are *concordant* wrt agent i if agent i goes through the same sequence of local states over those intervals, not counting consecutive repeats.

E.g. if

$$r_i[19,\infty] = aaabbaacc\ldots$$

and

$$r'_i[2,\infty] = abaaaaaaaaaaaaaacc\ldots$$

then $r[19,\infty]$ and $r'[2,\infty]$ are concordant for agent i.

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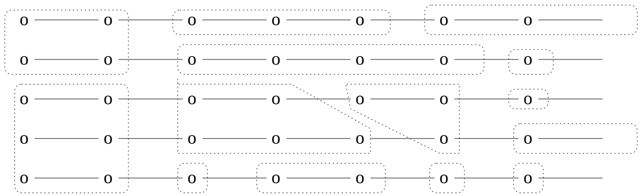
Properties of Systems (continued)

pr: A system $\mathcal{R}$ has *perfect recall* (or no forgetting) if for all points (r, m) and all agents i, if $(r, m) \sim_i (r', m')$ then the intervals $r[0, m]$ and $r'[0, m']$ are concordant wrt agent i.



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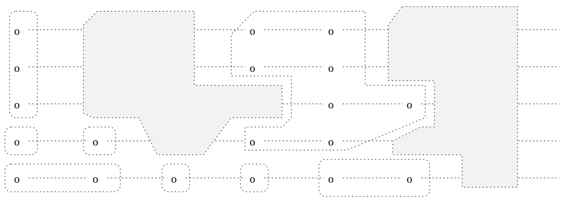
Properties of Systems (continued)

nl: A system $\mathcal{R}$ has *no learning* if for all points (r, m) and all agents i, if $(r, m) \sim_i (r', m')$ then the intervals $r[m, \infty]$ and $r'[m', \infty]$ are concordant wrt agent i.



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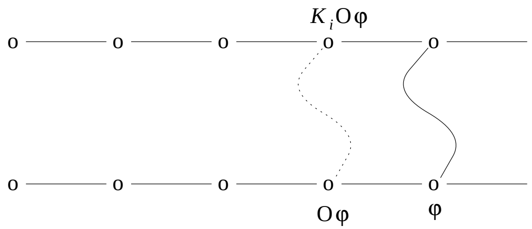
An Axiom for Synchronous Systems with Perfect Recall

$\mathrm{KT}^{pr,sync}$: $K_i \bigcirc \varphi \Rightarrow \bigcirc K_i \varphi$



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A Characterization of Perfect Recall

Let $\mathcal{I}$ be an interpreted system. Then the following are equivalent:

(a) $\mathcal{I}$ is a system with perfect recall.

(b) For all agents i, for all runs r, s and for all numbers n, m, if $(r, m+1) \sim_i (s, m)$ then either $(r, m) \sim_i (s, m)$ or there exists a number $l < m$ such that $(r, m) \sim_i (s, l)$ and for all k with $l < k \leq m$ we have $(r, m+1) \sim_i (s, k)$.

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An Axiom for Asynchronous Systems with Perfect Recall

KT^{pr}:

$$K_i\varphi_1 \wedge \bigcirc (K_i\varphi_2 \wedge \neg K_i\varphi_3) \Rightarrow \neg K_i\neg\{(K_i\varphi_1)U[(K_i\varphi_2)U\neg\varphi_3]\}$$

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An Axiom for Asynchronous Systems with No Learning

KT^{nl}: $K_i\varphi_1 U K_i\varphi_2 \Rightarrow K_i(K_i\varphi_1 U K_i\varphi_2)$

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An Axiom for Synchronous Systems with No Learning

$\mathrm{KT}^{nl,sync}$: $\bigcirc K_i\varphi \Rightarrow K_i \bigcirc \varphi$

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Class of Systems	Complete Axiomatization
$\mathcal{C}$, $\mathcal{C}^{sync}$	$\mathrm{S5(C)}_m$+LT
$\mathcal{C}^{pr}$	$\mathrm{S5}_m$+ LT +KT^{pr}
$\mathcal{C}^{pr,sync}$	$\mathrm{S5}_m$+ LT + $\mathrm{KT}^{pr,sync}$
$\mathcal{C}^{nl}$	$\mathrm{S5}_m$+ LT + KT^{nl}
$\mathcal{C}^{nl,sync}$	$\mathrm{S5}_m$+ LT + $\mathrm{KT}^{nl,sync}$
$\mathcal{C}^{pr,nl}$	$\mathrm{S5}_m$+ LT + KT^{pr}+ KT^{nl}
$\mathcal{C}^{pr,nl,sync}$	$\mathrm{S5}_m$+ LT + KT^{pr}+ $\mathrm{KT}^{nl,sync}$

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Branching Time

Extend the temporal language to a variant of CTL* (Emerson & Halpern)

if ϕ is a formula, then so is

1. $\mathtt{A}\phi$ (read "on all paths ϕ")
2. $\mathtt{E}\phi$ (read "on some path ϕ").

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Two runs r, r' are said to be equivalent to time n, if $r[0 \ldots n] = r'[0 \ldots n]$.

> $(\mathcal{I}, r, n) \models \mathtt{A}\varphi$ if for all runs r' of $\mathcal{I}$ that are equivalent to r to time n, we have $(\mathcal{I}, r', n) \models \varphi$.

(This is the *bundle* semantics (Burgess, Stirling).)

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Axioms for Branching Time: AXB

B1. $p \Rightarrow \mathtt{A}p$, where p is atomic

B2. $\exists p \Rightarrow p$, where p is atomic

B3. $\mathtt{A}\phi \Rightarrow \phi$

B4. $\mathtt{A}(\phi \Rightarrow \psi) \Rightarrow (\mathtt{A}\phi \Rightarrow \mathtt{A}\psi)$

B5. $\mathtt{A}\phi \Rightarrow \mathtt{AA}\phi$

B6. $\exists\phi \Rightarrow \mathtt{A}\exists\phi$

RB. From φ infer $\mathtt{A}\phi$.

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Interaction Axioms

FC. $\mathtt{A} \bigcirc \phi \Rightarrow \bigcirc \mathtt{A}\phi$

Theorem: AXB + LT + FC is sound and complete for $\mathcal{L}_{\{\mathtt{A}, \bigcirc, \mathcal{U}\}}$ in the class of all interpreted systems.

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An Interaction between Knowledge and Branching

KB. $K_i\phi \Rightarrow \mathtt{A}K_i\phi$

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Class of Systems	Complete Axiomatization
$\mathcal{C}$, $\mathcal{C}^{sync}$	S5(C)$_m$+ AXB + LT + FC + KB
$\mathcal{C}^{pr}$	S5$_m$+ AXB + LT + FC + KB +KT^{pr}
$\mathcal{C}^{pr,sync}$	S5$_m$+ AXB + LT + FC + KB + KT^{pr,sync}