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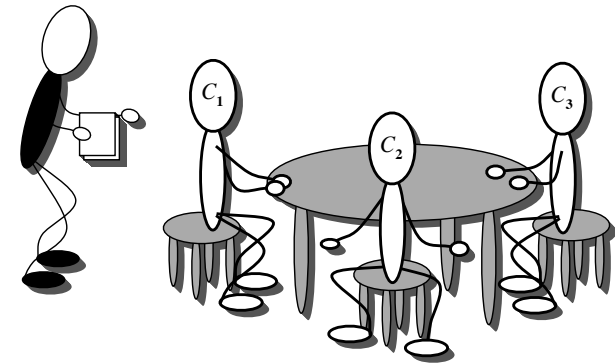
COMP3152/9152  
Lecture 10  
Applications to Security  
Ron van der Meyden

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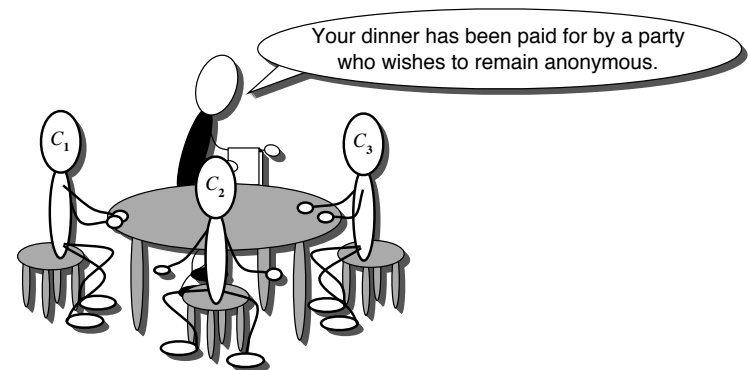
**Chaum's Dining Cryptographers protocol**

(Symbolic Model Checking the Knowledge of the Dining Cryptographers R. van der Meyden and K. Su, 17th IEEE Computer Security Foundations Workshop, Asilomar, June 2004, pp. 280-291)

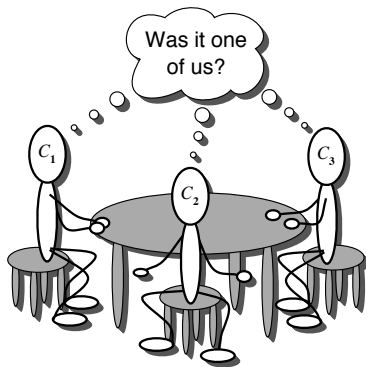
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## Knowledge Theoretic Specification

For Cryptographer C1:

$$\neg \text{paid}(C1) \Rightarrow$$

$$\text{Knows } C1 \text{ paid}(NSA)$$

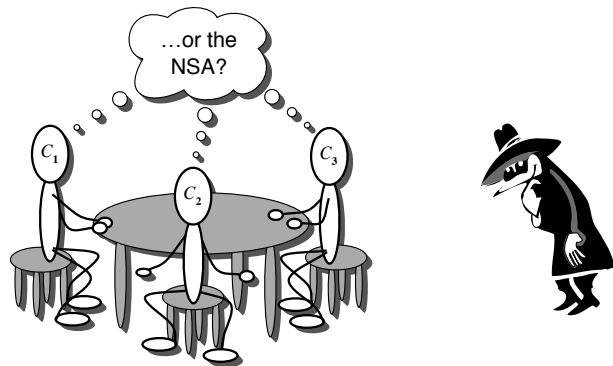
$$\vee [\text{Knows } C1 (\text{paid}(C2) \vee \text{paid}(C3)) \wedge$$

$$\neg \text{Knows } C1 \text{ paid}(C2) \wedge$$

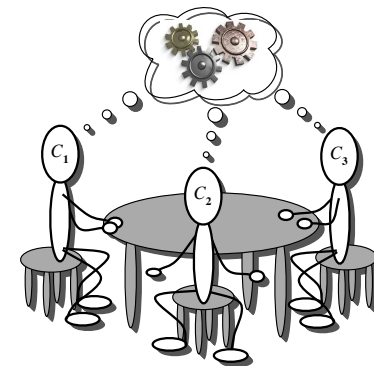
$$\neg \text{Knows } C1 \text{ paid}(C3)]$$

Similarly for the others ...

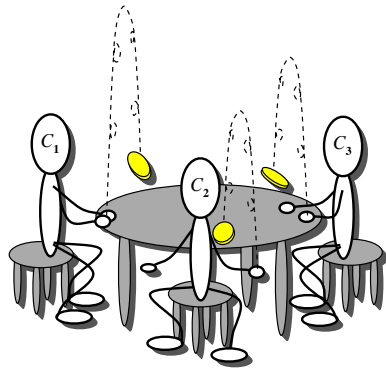
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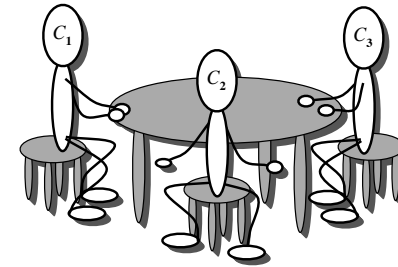


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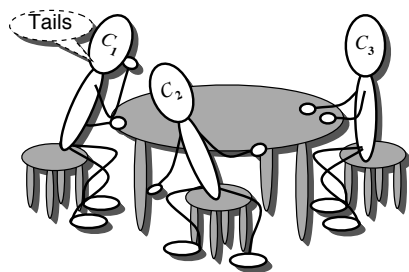
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2. Each  $C_i$  announces whether the two coin tosses are equal – *unless* he paid.



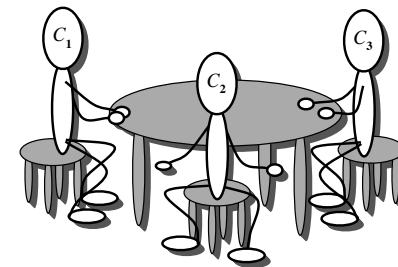
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1. Each  $C_i$  tells *only* his right neighbour what he tossed.



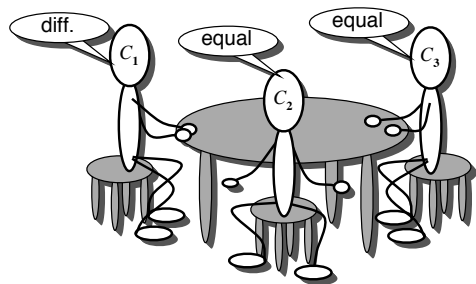
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2. Each  $C_i$  announces whether the two coin tosses are equal – *unless* he paid.



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3. An **odd** number of “diff.” indicates one of the  $C_i$  paid.



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No. of Cryptographers:	3	4	5	10	20
fixed ordering	0.3	2.3	26.8	-	-
with sifting	.7	1.8	6.9	66	519

Table 1: Runtimes of Dining Cryptographers Verification (Seconds)

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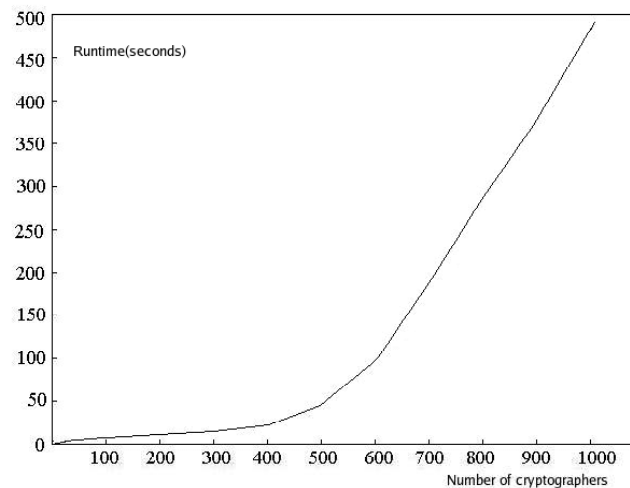
## An observation

Given the protocol, the pattern of variable values observed by cryptographer 1 over time is very predictable:

- $\text{paid}[1]$  is constant
- $\text{said}[i]$  changes only in the final step, for  $i = 1, \dots, n$
- $\text{coin}[\text{left}]$  changes in step 2, then is constant
- $\text{coin}[\text{right}]$  changes in step 3, then is constant

Upshot: we can reduce the representation of  $o_0, \dots, o_5$  from 5 copies of the above variables to 1.

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### Another Security Protocol Example: Oblivious Transfer

Specification:

Alice has two messages  $m_0, m_1 \in \{0, 1\}^k$ , unknown to Bob.

Bob selects whether he wants to receive  $m_0$  or  $m_1$ .

Bob should learn only the message he selected.

Alice should not learn which message Bob selected.

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### Rivest's solution, using an offline trusted third party

1. *Setup.* Ted chooses  $r_0, r_1 \in \{0, 1\}^k$  randomly and sends these values to Alice.  
Ted chooses  $d \in \{0, 1\}$  and sends  $d$  and  $r_d$  to Bob.
2. *Request.* Bob computes  $e = c \oplus d$ , where  $\oplus$  denotes exclusive or, and sends it to Alice.
3. *Reply.* Alice computes  $f_0 = m_0 \oplus r_e$  and  $f_1 = m_1 \oplus r_{1-e}$  and sends  $f_0$  and  $f_1$  to Bob.
4. *Result.* Bob computes  $m = f_c \oplus r_d$ .

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### Intransitive Noninterference

What, indeed, is intransitive noninterference?, R. van der Meyden,  
Proc. European Symposium on Research in Computer Security,  
Dresden, Sept 2007, LNCS Vol. 4734, pp. 235-250.

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### Noninterference

Proposed by Goguen and Meseguer 1982

Context: Multi-level secure systems

partially ordered security levels  $\Rightarrow$  transitive policies

Haigh and Young 87: extension to intransitive policies, deterministic systems

Rushby 1992: further results and corrections to Haigh and Young

van der Meyden 2007: improvement of Rushby theory

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## Noninterference policies

Let  $D$  be a set of security domains.

A noninterference policy is a reflexive relation  $\succrightarrow \subseteq D \times D$

$u \succrightarrow v$  means

“actions of domain  $u$  are permitted to interfere with domain  $v$ ”,  
or

“information is permitted to flow from domain  $u$  to domain  $v$ ”

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## Example

Public  $\succrightarrow$  Secret  $\succrightarrow$  Top-Secret

Public  $\succrightarrow$  Top-secret

but

Secret  $\not\succrightarrow$  Public, Top-Secret  $\not\succrightarrow$  Secret, Top-Secret  $\not\succrightarrow$  Public

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## Semantics for Transitive Policies

For each  $u \in D$  define the function  $\text{purge}_u : A^* \rightarrow A^*$  such that  $\text{purge}_u(\alpha)$  is the subsequence of all actions  $a$  in  $\alpha$  such that  $\text{dom}(a) \succrightarrow u$ .

The system  $M$  is said to be *secure with respect to the policy*  $\succrightarrow$  when for all  $\alpha \in A^*$  and domains  $u \in D$ , we have

$$\text{obs}_u(s_0 \cdot \alpha) = \text{obs}_u(s_0 \cdot \text{purge}_u(\alpha)).$$

An equivalent formulation:

For all sequences  $\alpha, \alpha' \in A^*$  such that  $\text{purge}_u(\alpha) = \text{purge}_u(\alpha')$ , we have  $\text{obs}_u(s_0 \cdot \alpha) = \text{obs}_u(s_0 \cdot \alpha')$ .

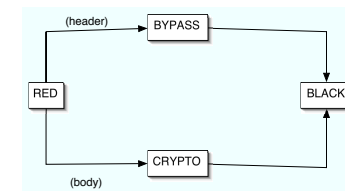
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## Motivation for Intransitive Policies

Downgrading:

$$H \succrightarrow D \succrightarrow L$$

Channel Control:



## Deterministic System Model

Machines of the form  $\langle S, s_0, A, \rightarrow, \text{obs}, \text{dom} \rangle$  where

1.  $S$  is a set of states,
2.  $s_0 \in S$  is the *initial state*,
3.  $A$  is a set of actions,
4.  $\text{dom} : A \rightarrow D$  associates each action to an element of the set of security domains  $D$ ,
5.  $\rightarrow : S \times A \rightarrow S$  is a deterministic transition function, and
6.  $\text{obs} : S \times D \rightarrow O$  associates an observation in some set  $O$  with each security domain.

Notation:  $s \cdot \alpha$  for the state reached by performing the sequence of actions  $\alpha \in Actions^*$  from state  $s$ .

## Haigh and Young's Semantics (1987)

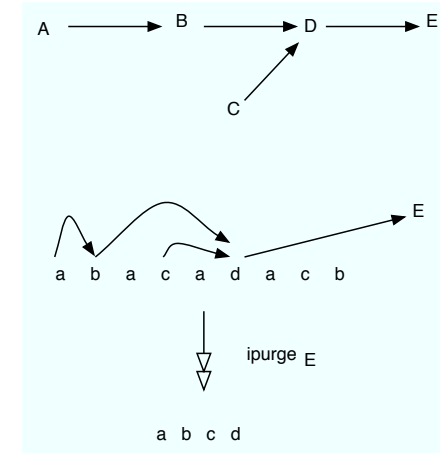
Given a sequence of actions  $a_1 \dots a_n \in Actions^*$  and domain  $u$ , the *intransitive purge*  $\text{ipurge}_u(\alpha)$  is the subsequence of all actions  $a_i$  such that there exists

$$i = i_1 < i_2 < \dots < i_k$$

with

$$\text{dom}(a_{i_1}) \twoheadrightarrow \text{dom}(a_{i_2}) \twoheadrightarrow \dots \twoheadrightarrow \text{dom}(a_{i_k}) \twoheadrightarrow u$$

**Example:**



### Haigh and Young's definition: IP-security

A system  $M$  is IP-secure with respect to a (possibly intransitive) policy  $\triangleright$  if for all  $u \in D$  and all sequences  $\alpha, \alpha' \in A^*$  with  $\text{ipurge}_u(\alpha) = \text{ipurge}_u(\alpha')$ , we have  $\text{obs}_u(s_0 \cdot \alpha) = \text{obs}_u(s_0 \cdot \alpha')$ .

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## (Perfect Recall) Knowledge in Asynchronous Systems

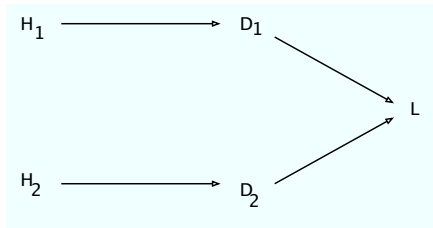
Define the view of domain  $u$  with respect to a sequence  $\alpha \in A^*$  to be the sequence of all observations of  $u$  and actions of  $u$  while running  $\alpha$ , with stuttering observations reduced to a single occurrence:

E.g., if running  $\alpha = hhlh$  produces  $o_1o_1o_1lo_2o_2$  at  $L$  then then  $\text{view}_L(\alpha) = o_1lo_2$

define  $\sim_u$  on sequences of actions by  $\alpha \sim_u \alpha'$  if  $\text{view}_u(\alpha) = \text{view}_u(\alpha')$ .  
 $\alpha \models K_u \phi$  if  $\alpha' \models \phi$  for all  $\alpha' \sim_u \alpha$

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## Example



Define the system  $M$  with

1. actions:  $h_1, h_2, d_1, d_2, l$  of domains  $H_1, H_2, D_1, D_2, L$  respectively.
2. states: the set of all strings in  $A^*$ .
3. transitions:  $\rightarrow (\alpha, a) = \alpha a$  for  $\alpha \in A^*$  and  $a \in A$ ,
4.  $\text{obs}_u(\alpha) = [\text{ipurge}_u(\alpha)]$ .

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Let  $\alpha_1 = h_1h_2d_1d_2$

Then  $\text{obs}_L(\alpha_1) = [\text{ipurge}_L(\alpha_1)] = [\alpha_1]$

Let  $p = \text{"there was an } h_1 \text{ before an } h_2\text{"}$

$p$  is a fact about  $H_1, H_2$ .

$\alpha_1 \models K_L p$

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But

$$\begin{aligned}
 \text{view}_{D_1}(\alpha_1) &= \text{view}_{D_1}(h_1h_2d_1d_2) \\
 &= [\epsilon] \circ [h_1] \circ [h_1] \circ d_1 \circ [h_1d_1] \circ [h_1d_1] \\
 &= [\epsilon] \circ [\epsilon] \circ [h_1] \circ d_1 \circ [h_1d_1] \circ [h_1d_1] \\
 &= \text{view}_{D_1}(h_2h_1d_1d_2)
 \end{aligned}$$

Similarly,  $\text{view}_{D_2}(\alpha_1) = \text{view}_{D_2}(h_2h_1d_1d_2)$

So

$$\alpha_1 \models K_L p \wedge \neg D_{\{D_1, D_2\}} p$$



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### An alternative definition - TA security

Given a policy  $\rightarrow$ , define, for each agent  $u \in D$ , the function  $\mathbf{ta}_u$ , with domain  $A^*$ , inductively by  $\mathbf{ta}_u(\epsilon) = \epsilon$ , and, for  $\alpha \in A^*$  and  $a \in A$ ,

$$\mathbf{ta}_u(\alpha a) = \begin{cases} \mathbf{ta}_u(\alpha) & \text{if } \text{dom}(a) \not\rightarrow u \\ (\mathbf{ta}_u(\alpha), \mathbf{ta}_{\text{dom}(a)}(\alpha), a) & \text{if } \text{dom}(a) \rightarrow u \end{cases}$$

Define a system  $M$  to be TA-secure with respect to a policy  $\rightarrow$  if for all agents  $u$  and all  $\alpha, \alpha' \in A^*$  such that  $\mathbf{ta}_u(\alpha) = \mathbf{ta}_u(\alpha')$ , we have  $\text{obs}_u(s_0 \cdot \alpha) = \text{obs}_u(s_0 \cdot \alpha')$ .

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### How these definitions are related

#### Theorem 1

1.  $P\text{-secure} \Rightarrow TA\text{-secure} \Rightarrow IP\text{-secure}$ .
2. If  $\rightarrow$  is transitive then  $P\text{-secure} = TA\text{-secure} = IP\text{-secure}$ .

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### Unwinding and Access Control Models

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### Access Control

A *system with structured state* is a machine  $\langle S, s_0, A, \rightarrow, \text{obs}, \text{dom} \rangle$  together with

1. a set  $N$  of *names*,
2. a set  $V$  of *values*, and functions
3.  $\text{contents} : S \times N \rightarrow V$ , with  $\text{contents}(s, n)$  interpreted as the value of object  $n$  in state  $s$ ,
4.  $\text{observe} : D \rightarrow \mathcal{P}(N)$ , with  $\text{observe}(u)$  interpreted as the set of objects that domain  $u$  can observe, and
5.  $\text{alter} : D \rightarrow \mathcal{P}(N)$ , with  $\text{alter}(u)$  interpreted as the set of objects whose values domain  $u$  is permitted to alter.

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For a system with structured state, when  $u \in D$  and  $s$  is a state, define  $\mathbf{state}_u(s) : \mathbf{observe}(u) \rightarrow V$  by  $\mathbf{state}_u(s)(n) = \mathbf{contents}(s, n)$  for  $n \in \mathbf{observe}(u)$ .

Define a binary relation  $\sim_u^{oc}$  of *observable content equivalence* on  $S$  for each domain  $u \in D$ , by  $s \sim_u^{oc} t$  if  $\mathbf{state}_u(s) = \mathbf{state}_u(t)$ .

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### Rushby's Reference Monitor Assumptions

- RM1. If  $s \sim_u^{oc} t$  then  $\mathbf{obs}_u(s) = \mathbf{obs}_u(t)$ .
- RM2. If  $s \sim_{\mathbf{dom}(a)}^{oc} t$  and either  $\mathbf{contents}(s \cdot a, n) \neq \mathbf{contents}(s, n)$  or  $\mathbf{contents}(t \cdot a, n) \neq \mathbf{contents}(t, n)$  then  $\mathbf{contents}(s \cdot a, n) = \mathbf{contents}(t \cdot a, n)$
- RM3. If  $\mathbf{contents}(s \cdot a, n) \neq \mathbf{contents}(s, n)$  then  $n \in \mathbf{alter}(\mathbf{dom}(a))$ .

RM2 is equivalent to the following: For all states  $s$ , either

1. for all  $t \sim_{\mathbf{dom}(a)}^{oc} s$ , we have  $\mathbf{contents}(t \cdot a, n) = \mathbf{contents}(t, n)$ , or
2. for all  $t \sim_{\mathbf{dom}(a)}^{oc} s$ , we have  $\mathbf{contents}(s \cdot a, n) = \mathbf{contents}(t \cdot a, n)$

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Consistency of an access control system with a policy:

AOI. If  $\mathbf{alter}(u) \cap \mathbf{observe}(v) \neq \emptyset$  then  $u \mapsto v$ .

**Proposition 1** (*Rushby 92*) Suppose  $M$  is a system with structured state that satisfies RM1-RM3 and AOI. Then  $M$  is IP-secure for  $\mapsto$ .

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### A weaker notion of Access Control

[RM2'] For all actions  $a$  states  $s, t$  and names  $n \in \mathbf{alter}(\mathbf{dom}(a))$ , if  $s \sim_{\mathbf{dom}(a)}^{oc} t$  and  $\mathbf{contents}(s, n) = \mathbf{contents}(t, n)$  we have  $\mathbf{contents}(s \cdot a, n) = \mathbf{contents}(t \cdot a, n)$ .

Example:  $n$  is a block of memory,  $a$  writes to a single location

Say  $M$  a system with structured states is a *weak access control system* compatible with  $\mapsto$  if it satisfies RM1, RM2', RM3 and AOI.

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**Proposition 2** *If  $M$  is a weak access control system compatible with  $\rightarrow$  then  $M$  is TA-secure (hence IP-secure) for  $\rightarrow$ .*

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### Unwinding Conditions

Suppose we have for each domain  $u$  an equivalence relation  $\sim_u$  on the states of  $M$ .

OC: If  $s \sim_u t$  then  $O_u(s) = O_u(t)$ . (Output Consistency)

SC: If  $s \sim_u t$  then  $s \cdot a \sim_u t \cdot a$ . (Step Consistency)

LR: If not  $\text{dom}(a) \rightarrow u$  then  $s \sim_u s \cdot a$ . (Left Respect)

If these conditions are satisfied then  $M$  is secure with respect to a transitive policy (Goguen & Meseguer 84).

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### Completeness of Unwinding (Transitive Policies)

**Proposition 3** *(Rushby 92) Suppose  $M$  is P-secure with respect to the transitive policy  $\rightarrow$ . Then there exist equivalence relations  $\sim_u$  on the states of  $M$  with respect to which  $M$  satisfies OC, SC and LR.*

(Specifically,  $s \approx_u t$  if for all strings  $\alpha$  in  $A^*$  we have  $O_u(s \cdot \alpha) = O_u(t \cdot \alpha)$ .)

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### Unwinding Intransitive Noninterference

WSC: If  $s \sim_u t$  and  $s \sim_{\text{dom}(a)} t$  then  $s \cdot a \sim_u t \cdot a$ .  
(Weak Step Consistency)

**Proposition 4** *(Rushby 92) Suppose that  $\sim_u$  are equivalence relations on the states of a system  $M$  that satisfy OC, WSC and LR. Then  $M$  is IP-secure for  $\rightarrow$ .*

(But no completeness result.)

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## Unfolding a system

Given a system  $M = \langle S, s_0, \rightarrow, \text{obs}, \text{dom} \rangle$  with actions  $A$ , define the system  $\text{uf}(M) = \langle S', s'_0, \rightarrow', \text{obs}', \text{dom} \rangle$  with actions  $A$  by

1.  $S' = A^*$
2.  $s'_0 = \epsilon$
3.  $\rightarrow'(\alpha, a) = \alpha a$ , for  $\alpha \in S'$  and  $a \in A$
4.  $\text{obs}'_u(\alpha) = \text{obs}_u(s_0 \cdot \alpha)$  (RHS in  $M$ )

$\text{uf}(M)$  is *bisimilar* to  $M$  (in the expected sense)

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Say that a system  $M$  with states  $S$  admits a *weak access control interpretation compatible with  $\succrightarrow$*  if there exists

1. a set of names  $N$
2. a set of values  $V$  and functions
3.  $\text{observe} : D \times S \rightarrow \mathcal{P}(N)$ ,
4.  $\text{alter} : D \times S \rightarrow \mathcal{P}(N)$  and
5.  $\text{contents} : N \times S \rightarrow V$

with respect to which  $M$  is a weak access control system compatible with  $\succrightarrow$ .

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**Theorem 2** *The following are equivalent*

1.  $M$  is TA-secure with respect to  $\succrightarrow$
2.  $\text{uf}(M)$  admits a weak access control interpretation compatible with  $\succrightarrow$ ;
3. there exist equivalence relations  $\sim_u$  on the states of  $\text{uf}(M)$  satisfying OC, WSC and LR;

(So, weak unwinding incomplete for IP-security on two counts: unwinding is complete for the stronger TA-security, wrt  $\text{uf}(M)$  rather than  $M$ ).