A Performance Model of Pedestrian Dead Reckoning with Activity-based Location Updates

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***Abstract*—Advanced computing and sensing capabilities of smartphones provide new opportunities for personal indoor positioning. A particular trend is to employ human activity recognition for autonomous calibration of pedestrian dead reckoning systems thereby achieving accurate indoor positioning even in the absence of any positioning infrastructure. The basic idea is that the activity context, such as switching from a walking to a stair climbing activity gives clues about pedestrian's current position. In this paper, we have made a first attempt in developing a performance model for such systems. For an unbiased random walk, we have obtained two interesting results in closed-form expressions. First, we have demonstrated that the distance a pedestrian is expected to travel before the PDR is recalibrated is reciprocal of the density of activity switching points (ASPs) in the indoor environment. The implication of this finding is that the continuous unaided use of PDR can be curbed drastically by identifying more ASPs in a given environmental setting. Second, we have shown that false negatives of the activity detection algorithms do not have a major impact as long as they are within a reasonable range of 0-30%. The system performance however degrades rapidly if false negatives continue to grow beyond 30%.**

I. Introduction

Pedestrian dead reckoning (PDR) refers to a self-contained positioning technique that uses kinematics of human walking to estimate the current user location without any help from the infrastructure [1]. PDR uses an accelerometer to detect steps and estimate step lengths of a walking user and a compass to estimate the heading. From step length and heading information, PDR works out the current location as a displacement from a previously known position. Since modern personal mobile devices, e.g., smartphones, already provide an accelerometer and a compass, PDR is considered as a viable technique to overcome positioning difficulties in areas, e.g., indoors, dense forests, and urban canyons, where GPS signals are difficult to receive [2].

A fundamental problem with PDR is the accumulation of errors over time or distance travelled. In PDR, each new position estimate is based on the previous estimate of the last step. Therefore, if we have errors associated with step length and heading estimation of a human step, which is unavoidable with commercial grade sensors, then PDR becomes unreliable if continued to be used over a long period or distance. For this reason, PDR is often used in conjunction with another type of positioning, e.g., RFID [1], which provides frequent location updates to PDR. With such location updates, PDR has to work only for a short duration or distance before being updated with accurate positioning yielding an improved outcome.

For location updates in PDR, researchers have traditionally relied on some form of external sources. For example, Constandache et al. [2] used Assisted-GPS (AGPS) in outdoor scenarios while House et al. [3] used RFID for indoor areas as means of occasional location updates to their PDR systems. More recently, activity-based map matching (AMM) [4], [5] has been proposed as a fully self-contained solution to PDR updates that does not require input from any external sources. AMM uses the same accelerometer used by the PDR to recognize specific human activities, e.g., climbing a stair, and then matching that activity to discrete locations on a given map (which shows locations of stairs). Thus, each time the user performs certain types of activities, PDR can be autonomously updated with correct locations provided the map of the environment is available in the mobile device (note that most public complexes, e.g., shopping malls and airports, are now publishing their maps on the Internet).

Unlike some external sources, e.g., RFID, which can provide more deterministic updates by carefully designing the placement of the sensors, AMM relies on pre-existing locations in a map as a natural source of location update. Hence, AMM is inherently a stochastic system, where the actual PDR distance travelled between two updates is a random number. This is because location update in AMM is contingent on the user reaching to a specific location and performing the activity available in that location (these are spatially dependent activities). The expected distance (D) a pedestrian will travel with pure PDR before an update, therefore, becomes an important variable to study. Note that higher the D, the more error the PDR will accumulate on average before being updated, and vice versa. Hence, D would ultimately affect the 'zone of uncertainty' within which a user is localized using PDR.

Researchers have investigated other aspects of AMM, such as how to improve detection of human activity using machine

	1	2	3	4	5	6	7	8
1	0	0	0	0	0	0.25	0	0
2	0	0	0.25	0	0.25	0	0.25	0
3	0	0.25	0	0.25	0	0.25	0	0
4	0	0	0.25	0	0	0	0	0

Fig. 1. An 8×4 2D lattice with two activity switching points (shaded locations).

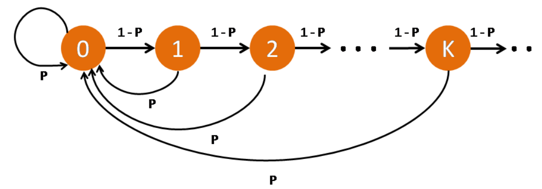

Fig. 2. The proposed Markov chain model of the PDR system with activity-based location updates.

learning [5], but to the best of our knowledge, the expected PDR distance between two updates has not been modelled before. Contributions of this paper can be summarized as:

- We model a PDR system with activity-based updates as a Markov chain and derive a closed-form expression for D as a function of the density of ASPs in a given indoor environment.
- We show that D is *reciprocal* of ASP density in a given indoor complex. The implication of this finding is that the continuous unaided use of PDR can be curbed drastically by identifying and detecting more ASPs in a given environmental setting.
- We further demonstrate that false negatives of the activity detection algorithms do not have a major impact as long as they are within a reasonable range of 0-30%. This finding is encouraging for researchers and developers considering activity recognition as a viable tool to improve PDR-based indoor positioning, especially if such systems are to be deployed over smartphones where it is more challenging to avoid false negatives.

The rest of the paper is organized as follows. We present the proposed Markov chain-based performance model and analysis in Section II. Related work is reviewed in Section III. We draw our conclusions along with discussions on future work in Section IV.

II. Performance Model and Analysis

We consider a 2D $M \times N$ square lattice, where each of the $M \times N$ squares is considered a distinct location. A pedestrian wanders in this environment following a Random Walk model as follows. At each step, the pedestrian can move only to the neigbouring locations either up, down, left or right with equal probability. A subset of total locations where activity switchings occur are called *special* locations. Figure 1 shows a 8×4 lattice with two special locations (activity switching points).

A special location is a location where a switch in user activity occurs, which creates a distinguishing change in the accelerometer pattern, which in turn can be detected using appropriate activity classification and recognition algorithms, e.g. those proposed in [5]. Hence the exact location is learned immediately when the pedestrian moves to any one of these special locations, provided the pattern recognition algorithm can detect the event. When the exact location is learned, the PDR is reset or updated with this location knowledge. Thus in this system, the PDR error grows only during the consecutive steps (PDR distance) the pedestrian travels through non-special locations. For a random walk, the PDR distance is a stochastic variable and is the focus of our study. Specifically, we want to answer the following performance questions:

1) Starting from a random location, what is the probability that the pedestrian takes exactly k consecutive steps before reaching to an ASP (probability distribution of PDR distance)?
2) What is the expected number of steps the pedestrian takes before reaching to an ASP (expected PDR distance)?

The activity detection algorithm may or may not be perfect. A perfect algorithm would detect every single arrival to special locations while an imperfect and perhaps more realistic one would miss the detection of some of these arrivals. We propose a Markov chain that can be used to study both perfect and imperfect algorithms.

A. Perfect Activity Recognition

We model the PDR system using the Markov chain shown in Figure 2. The states in this Markov chain represent the number of consecutive steps taken without being updated. For example, state (0) means the pedestrian is now standing on a special location, state (1) means the pedestrian has taken one step from a known location and is currently residing in a non-special location, state (2) means the pedestrian has taken two consecutive steps from a known location and so on.

State transitions occur at discrete time events when the pedestrian takes a step. There are only two outcomes when a step is taken, the pedestrian lands either on a special location or continues to be on a non-special location. The former leads to state (0) while the latter increments the state counter by one. Let us assume that at each step, there is a probability p that he will land on a special location and $(1-p)$ that he will land on a non-special location. Therefore, from any state in the Markov chain, there is a probability p that it will move to state (0) and $(1-p)$ to move to the next higher state.

For an unbias and uncorrelated random walk, it could be shown that

$$p = \frac{L}{M \times N} \tag{1}$$

where L is the number of special locations in the $M \times N$ lattice. Therefore, p is basically the ASP density of the indoor space. We leave the formal proof of this concept for a future communication, but provide a quick intuitive explanation using the example lattice shown in Figure 1.

The numbers inside the squares in Figure 1 represent the probabilities that the pedestrian would land on a special location in the next step if the step is taken from those squares. Since the user can be in any of the 32 locations when the next step is taken, on average the probability of landing on a special location is $\frac{8 \times 0.25}{32} = \frac{1}{16}$, which is equal to $\frac{L}{M \times N}$ (note that $L = 2$ in this example).

Probability Distributions

From the Markov chain of Figure 2, we have the following probability equations:

$$\begin{aligned} \pi(0) &= p \sum_{i=0}^{\infty} \pi(i) && (2) \\ \pi(k) &= \pi(0)(1-p)^k \text{ for } k = 1, 2, \ldots && (3) \\ \sum_{i=0}^{\infty} \pi(i) &= 1 && (4) \end{aligned}$$

By solving the above equations, we obtain the steady state probability distributions as:

$$\pi(k) = \begin{cases} p & \text{for } k = 0 \\ p(1-p)^k & \text{for } k > 0 \end{cases}$$

We observe the following properties for $\pi(k)$. First, it can be seen that $\pi(k)$ decreases exponentially with k for any values of p (see Figure 3). This is intuitive because not finding a special location after the nth step is conditional to not finding this in all of the previous $(n-1)$ steps.

Our second observation is less intuitive. Intuitively, it may seem that $\pi(k)$ would always decrease for an increasing p, because increasing p means there are more chances that the pedestrian would find a special location in the next step. However, it can be shown (proof omitted) that $\pi(k)$ is a concave function of p, which means that we have a single maximum for $\pi(k)$ in the interval $0 < p < 1$. This result is shown graphically in Figure 4. By equating the first derivative $\pi'(k)$ to zero, we obtain a unique solution of $p = \frac{1}{k+1}$. This means that the maximum is reached at $p = \frac{1}{k+1}$ and the maximum value of $\pi(k)$ is

$$\pi^{\max}(k) = \frac{k^k}{(k+1)^{k+1}}$$

.

The reason that we have a maximum is because for small values of p, i.e., $p < \frac{1}{k+1}$, the pedestrian is likely to take more steps before finding a special location, but with increasing p some of the system probabilities from 'higher order steps' (greater than k) are shifted to $\pi(k)$. For example, in Figure 4, $\pi(5)$ is decreasing while $\pi(2)$ is increasing for $\frac{1}{6} < p < \frac{1}{3}$.

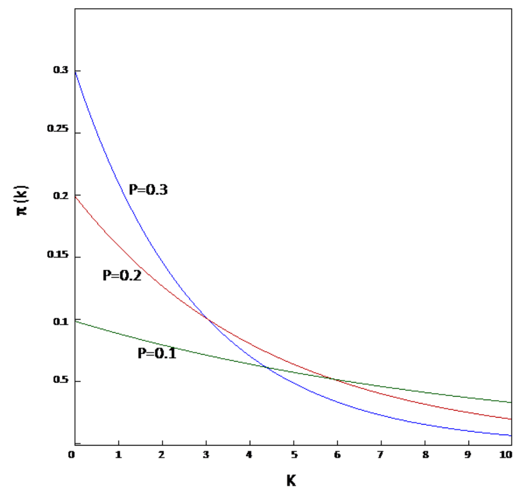

Fig. 3. Steady state probability as a function of number of consecutive steps k.

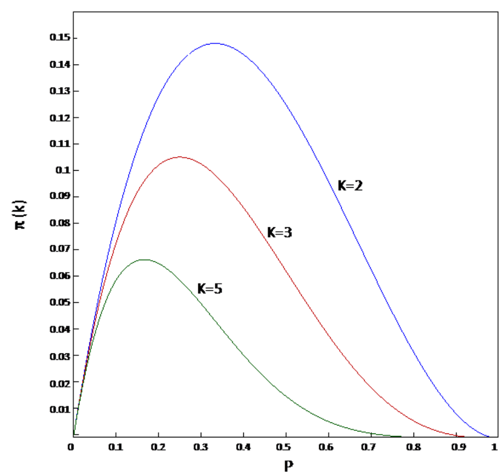

Fig. 4. Steady state probability as a function of ASP density p.

Expected PDR Distance

The expected number of PDR steps taken (D) before reaching to a special location can be obtained from the steady state probabilities as:

$$D = \sum_{i=1}^{\infty} i\pi(i) \tag{5}$$

Replacing $\pi(i)$ with $p(1-p)^i$ and expanding the summation into a geometric series, we obtain:

$$D = p[(1-p) + 2(1-p)^2 + 3(1-p)^3 + \ldots] \tag{6}$$

The geometric series inside the square bracket in the right hand side is an infinite sum of $\sum_{i=1}^{\infty} ix^i$, which adds up to $\frac{x}{(x-1)^2}$ when $|x| < 1$ (the proof is omitted). In our case, we replace x with $(1-p)$ to obtain $\frac{1-p}{p^2}$ provided the series variable

TABLE I
REDUCTION IN AVERAGE PDR DISTANCE

p	Average PDR Distance (steps)
1%	99
10%	9
20%	4

$(1-p)$ fulfils the condition $|1-p| < 1$, which is indeed the case for us as p is a probability. Therefore we finally obtain:

$$D = p\left[\frac{(1-p)}{p^2}\right] = \left(\frac{1}{p} - 1\right) \qquad 0 < p < 1 \tag{7}$$

We observe that the average number of consecutive steps taken from a known position is inversely proportional to p. Figure 6 shows this relationship. Table I shows how the average PDR distance can be reduced dramatically from 99 steps to only 4 steps by increasing the density of activity switching points from 1% to 20%.

Model Validation with Simulation

Using MATLAB, we have simulated random walk on a wrap-around 3×3 lattice, where the edges are assumed to be connected to the corresponding squares of the opposite edge. The wrap-around allows the pedestrian to move to any of the four directions with equal probability even at the edges without falling off the lattice.

We have conducted two sets of simulations, each corresponding to a different number of special locations (see Figure 5). The first set corresponds to a single special location ($p = \frac{1}{9}$), while the other corresponds to two special locations ($p = \frac{2}{9}$). In each set of simulation, we run 100 experiments for each of the 9 locations as the starting location for the pedestrian. Each experiment starts on a given location and ends when the pedestrian reaches one of the special locations, at which time the total number of consecutive steps to reach the special location is recorded. Figure 5 shows the average of 100 runs for each location, when that location was used as a starting point. The average PDR distance, i.e., the average of all 9 locations is shown at the bottom of each sub-figure. Note that our model predicts D as $\frac{1}{p} - 1$, which gives 8 for a single special location and 3.5 for the lattice with 2 special locations. The simulation outcomes are 8.09 and 3.87, respectively, when averaged over all possible starting points. Thus, our model provides a very accurate prediction for the expected PDR distance for these experiments.

B. Activity Recognition with Errors

We assume that our activity recognition module has non-negligible false negative errors (we leave the treatment of false positive error as a future work). Let us denote the false negative error rate by β. It means that when the pedestrian reaches a special location, the detection of this event, i.e., activity switch event, is missed with a probability of β and detected with probability $(1-\beta)$.

The effect of false negatives can be captured in our Markov chain by replacing p with $p(1-\beta)$. Thus the expected PDR distance with false negatives is obtained as:

Fig. 5. Simulation of random walk on a wrap-around 3×3 lattice, where special locations are shaded. The number inside each square is PDR distance D, averaged over 100 experiments, if that location was chosen as the starting point. The overall average over all 9 starting locations is shown below the lattice.

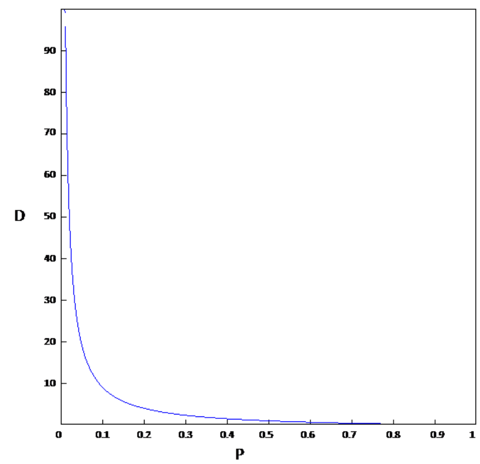

Fig. 6. Expected PDR distance D as a function of special location density p.

$$D^{\epsilon} = \frac{1 - p(1-\beta)}{p(1-\beta)} \tag{8}$$

It can be shown, as also expected intuitively, that $D^{\epsilon} > D$ for $0 < \beta < 1$. We derive the relative extension in expected PDR distance due to false negative errors as:

$$\frac{D^{\epsilon}}{D} = \frac{1 - p(1-\beta)}{(1-p)(1-\beta)} \tag{9}$$

which is strictly greater than 1 for $\beta > 0$.

Figure 7 shows that relative extension increases slowly (almost linearly) for reasonable values of β (for about $\beta < 0.3$). It grows exponentially only when β becomes too large. This is encouraging because it means that false negatives do not become a performance bottleneck as long as they remain within a reasonably large range. This is particularly important for smartphone accelerometers, which (1) contain much weaker signal than those inertial measurement units fixed to specific human body parts, and (2) yield much noisier signals due to free movement of the smartphone.

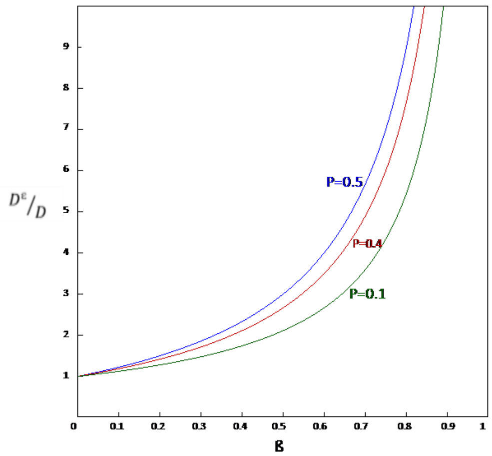

Fig. 7. PDR distance extension as a function of false negative rate β.

III. Related Work

We are witnessing a growing trend in the literature focussing on PDR-based indoor positioning, which has a potential to provide effective positioning even when no external infrastructure is available. Because PDR is plagued with accumulated error drift over time, researchers are trying to find new ways to control or reduce the PDR drift as much as possible. There is a growing consensus that it is now relatively easy to detect human steps and estimate their lengths using accelerometers, but heading estimation using consumer compass remains a primary source of drift for PDR [6], [7]. While some researchers are working on better heading estimators [7], others have proposed using external sources to update or recalibrate a drifting PDR. For example, Constandache et al. [2] used Assisted-GPS (AGPS) in outdoor scenarios while House et al. [3] used RFID for indoor areas as means of occasional location updates to their PDR systems.

In a recent work, Gusenbauer et al. [4] proposed the idea of activity-based map matching for autonomous recalibration of PDR. They built a basic prototype to demonstrate the idea in an indoor environment involving stairs and elevators. When the stair climbing or elevator riding was detected, the current position was matched to the nearest stair (elevator) to correct the drift in PDR. This work was followed up in more depth by [5], where detailed experiments were conducted involving many different types of activities including spiral stairs. Although the experiments were based on sensors fixed to human body parts, they indicate that recognition of such human activities using smartphone accelerometer may be within reach. These researchers conducted their experiments in specific environments with real subjects under prescribed mobility and focussed their performance evaluation to the comparison of PDR systems augmented with activity recognition against pure PDR that does not use activity recognition. Our focus in this paper is to develop a generic performance model assuming unprescribed random pedestrian mobility, which can be used to study fundamental properties of PDR systems that are autonomously calibrated using an environmental map and activity recognition.

Random walk models have been studied extensively in the literature in many different domains [8]. The main performance measure derived for a walker on a 1D lattice (walker is allowed to move either right or left with equal probability) is the probability that the walker will be a given distance from origin after k steps. 2D random walks are also studied, but again with the goal of determining the probability of destination locations after n steps. The problem studied in this paper is new. We studied the probability distribution $\pi(k)$, i.e., the probability that it takes exactly k steps to reach one of the multiple special locations in a 2D lattice.

IV. Conclusion and Future Work

In this paper, we have made a first attempt in developing a performance model for PDR systems with activity-based location updates. For an unbiased and uncorrelated random walk, we have obtained two interesting results in closed-form expressions. First, we have demonstrated that the distance a pedestrian is expected to travel using a pure unaided PDR is reciprocal of the density of activity switching points in an indoor space. Therefore, the continuous reliance on PDR can be curbed drastically by identifying more activity switching points in a given environmental setting. Second, we have shown that false negatives of the activity detection algorithms do not have a major impact as long as they are within a reasonable range of 0-30%. The system performance however degrades rapidly, i.e., the expected PDR distance increases exponentially, if false negatives continue to grow beyond 30%.

There is significant opportunity to extend the current work from several different angles. We have considered only the basic unbiased and uncorrelated random walks. While useful to gain significant insight to the problem, such random walks do not capture some of the dynamics exhibited by real pedestrians in practical environments, such as shopping malls and airports. While there are some randomness in the mobility of pedestrians, they also exhibit some correlation and bias. For example, while a pedestrian is heading towards a particular shop, his steps would be biased towards a certain direction until reaching that shop. However, after that shop, the new direction would be random. Therefore, one obvious extension of the current work would be to consider a biased random walk and derive the performance metrics as a function of some bias parameters. One could also consider more advanced pedestrian movement models, such as those based on the principles of gas diffusion [9], [10], to more accurately model realistic scenarios. These advanced models would be, however, more challenging to capture mathematically, but finding an approximate or non-closed-form solution may be possible. One could even consider simulation to explore the effect of these mobility models.

Another interesting avenue to explore would be to study the effect of false positives (commonly known as *false alarms*) of the activity recognition algorithms. In the current work, we considered only false negatives and assumed that false positives are negligible. However, if false positives are not negligible for a given system, then the PDR performance can be adversely affected. Modelling false positives can be more challenging, because the system would incorrectly think that it had reached an ASP and update the PDR with a wrong location. Depending on which ASP is incorrectly detected, the error in PDR can vary.

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