The Team Order Problem: Maximizing the Probability of Matching Being Large Enough

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Abstract. We consider a matching problem, which is meaningful in team competitions, as well as in information theory, recommender systems, and assignment problems. In the competitions which we study, each competitor in a team order plays a match with the corresponding opposing player. The team that wins more matches wins. We consider a problem where the input is the graph of probabilities that a team 1 player can win against the team 2 player, and the output is the optimal ordering of team 1 players given the fixed ordering of team 2. Our central result is a polynomial-time approximation scheme (PTAS) to compute a matching whose winning probability is at most ε less than the winning probability of the optimal matching. We also provide tractability results for several special cases of the problem, as well as an analytical bound on how far the winning probability of a maximum weight matching of the underlying graph is from the best achievable winning probability.

1 Introduction

Bipartite matching underpins several impactful problems in allocation and market design problems including kidney allocation, adword auctions, on demand taxi allocation, refugee assignment, or school choice (see, e.g., [11]). We consider a fundamental matching problem with an underlying weighted bipartite graph where each edge weight has weight between 0 and 1. Instead of focusing on the classical objective of maximizing the total weight of the matching, we focus on a different objective with a probabilistic interpretation: We want to compute a matching that maximizes the probability of reaching a target size. This problem can model several scenarios, including that of the so called *team order problem*.

One of the most relevant applications of our setting is the rivalry between teams of contestants. Consider team competitions in which both teams put forward an ordering of their players. The contestants then play matches against the corresponding contestants from the opposing team. The team that wins more matches wins the overall competition. Such competitions are not only held in various inter-club tennis competitions, the same format is also used in international table tennis and badminton competitions, such as the Corbillon Cup, Swaythling Cup, Thomas Cup, and the Olympic Games. We focus on the problem in which one team's order is fixed (as is the case in many situations where the home team commits to an ordering) and the other team wants to compute the optimal ordering. As the ordering of one team is fixed, the problem of computing the other team's ordering is essentially a competitor matching problem.

The problem of finding a way to maximize the number of achieved goals by setting an appropriate line-up is not limited to sport competitions. Indeed, it admits several other motivations in competitive contexts such as politics (fielding political candidates in different constituencies against candidates of a rival party). Our problem also provides a perspective into finding *durable matchings*. Suppose that we are given the probability of success of various partnerships. For example a partnership could represent a job placement or allocation of refugee family to a council (see, e.g., [7, 2]). A typical objective could be maximizing the expected number of partnerships. However, another meaningful objective that is centred around a particular target could be to maximize the probability of having a target number of successful partnerships, which maps to the objective that we study. Another potential application of our research relates to *information networks* (see, e.g., [21]). Suppose that we are given such a network, represented by a flow network. There, each edge has a reliability probability of a message reaching the other side, and we want to find a flow maximizing probability of delivering a target number of messages. Finally, our research is motivated by its applications in *recommendation systems* (see, e.g., [25]). Suppose that a ranked list of recommendations needs to be displayed with each item having a probability of being clicked depending on its position in the ranking list. One may want to maximize the probability of having a target number of items being clicked, which can be captured by our problem. We explore the following questions.

> *How hard is the team order problem? Under what conditions is it easy to solve? What are reasonable approximation approaches for the problem?*

We note that the problem that we study in this paper is closely related to the maximum-weight matching problem. There, we are given a bipartite graph, where each edge is assigned a weight, and the objective is to find a matching with the maximum sum of weights. In fact, our results reflect that finding the solution to that problem provides a good approximation of the optimal solution. However, the problem we study is substantially more complex. Indeed, for an instance of the team order problem to be positive, we require that the weights in a selected matching are large enough for some subset of edges, instead of maximizing their global sum. Furthermore, given the strategic games interpretation of our setting, our results concern the computation of the optimal response to the opponent choice, which is an important step towards the study of equilibria in this setting.

Contributions. We first show that the winning probability of a given matching (line-up) can be computed in polynomial time (Proposition 1). Subsequently, we show that in certain settings computing an optimal line-up is tractable. In particular, when the input winning probability of each partnership takes its value from a size-three set $\{\alpha, \beta, 0\}$ we show that the optimal matching can be computed in polynomial time (Theorem 1). While we conjecture that the team order problem is hard in the general case, we show that it is tractable for practical purposes. Our central result is a *polynomial-time approximation scheme (PTAS)*[4] to compute a matching whose winning probability is at most ε less than the winning probability of the optimal matching (Theorem 3). Although the winning probability is not a linear objective, we show that the general problem of computing an optimal matching can be solved via integer linear programming. Also, we provide an analytical bound on how far the winning probability of a maximum weight matching is from the best achievable winning probability.

2 Related Work

Our results are relevant to a number of research direction in multi-agent systems.

Matching Theory. Matching problems have been widely studied in combinatorial optimization. The standard objectives typically focus on maximizing the weight of the matching (see, e.g., [9, 24]). In our context, maximizing the weight of the underlying weighted bipartite graph gives us a matching maximizing the expected number of matches won. Our objective is different as we want to maximize the probability of winning a target number of matches. The paper most relevant to our work is by Tang et al. [28], which concerns the same setting but considered different problems. It takes an economic design approach and presents necessary and sufficient conditions, ensuring that truthful reporting and maximal effort in matches are equilibrium strategies. We note that the probabilistic approach in matching has been previously studied. E.g., Aziz et al. [6] studied the stable matching problem with uncertain preferences.

Manipulation of Competitions. Within the wider topic of manipulations in competitions, there have been several papers on identifying conditions or manipulations under which a certain team or player can win. A notable example is manipulating the draw of a balanced knockout tournament to maximize the probability of a certain player winning, i.e., the *tournament fixing problem* [30, 5, 31]. Similarly, there has also been algorithmic research on round-robin formats to understand which teams have a chance to win the overall tournament [17, 4].

Colonel Blotto Game. Furthermore, the team line-up setting bears resemblance to Colonel Blotto Games which are two-player zero-sum games in which two armies fight in n battle fields with each battle being won by the army that had more troops in the battle (see, e.g., [26, 27]). The armies are interested in maximizing a weighted sum of utilities from the battlefields where they gain victories. Although the team-line-up setting is similar in that each battle corresponds to

[4] A PTAS is a scheme which, for every instance of a problem and $\varepsilon > 0$, provides an approximate solution based on ε.

a match, in Colonel Blotto games, the armies have more flexibility in shuffling their troops around. Secondly, in Colonel Blotto games the outcome of a battle depends on the *number* of troops of each army whereas in the team line up setting, the outcome of a match depends on the identities of the respective players. Independent of our work, Gaonkar et al. [15] considered a version of Blotto games in which every resource is unique and non-interchangeable which makes it close to our setting. They motivate the problem as *derby games* in which teams assign each resource to a particular round and wins a payoff corresponding to that round if they win the round. We note, however, that our work differs significantly from their results. In particular, they examine Nash equilibria, which are not the focus of our study. Furthermore, they do not take the information on winning probabilities into account and do not focus on algorithmic issues.

Sequential Games. Games between teams of players in which the ordering of contestants matters gained a substantial interest in recent literature. Fu et al. [14] studied the scenario in which teams compete in a number of games between pairs of players. Within this setting they investigated how the sequencing of those matches impacts the result. We note that, in contrast to our study, the games they considered are also based on private rewards for the individual players. Furthermore, Konishi et al. [18] studied the problem of whether the equilibrium winning probability in such games depends on whether matches are held simultaneously, or sequentially. Also, Fu and Lu [13] explored the topic of how teams can strategically assign contestants to time-slots of a sequential competition. Let us further note that in contrast to our work the discussed papers on sequential games do not focus on computational complexity.

Nominee Selection. Our setting is also related to the literature on strategic selection of group members participating in a competition. In social choice theory, this problem relates to the process of selecting representative for the elections (see, e.g., [12, 3]). Regarding sport competitions, our problem relates to choosing a coalition member to participate in a tournament (see, e.g., [23, 22]).

3 The Team Order Problem

We consider the following problem setting.

- Two teams T_1 and T_2 are to play a team competition.
- Each team T_i has n contestants $t_i^1, \dots, t_i^n$.
- We have information about the winning probability $p(t_i^a, t_j^b)$ of any contestant t_i^a against any other contestant t_j^b. The instance is said to be *degenerate* if all the winning probabilities are 0 or 1.

In the competition each team is required to report a line-up, i.e., an ordering $i_1, \dots, i_n$ of its contestants, which is a permutation of $1, \dots, n$. Then each contestant $t_i^{i_k}$ plays a match with the corresponding contestant $t_j^{j_k}$. The team that wins at least $\lfloor \frac{n}{2} \rfloor + 1$ matches wins the competition. All of our results hold equally well if the target $\lfloor \frac{n}{2} \rfloor + 1$ is replaced by some generic target L that is higher or lower than $\lfloor \frac{n}{2} \rfloor + 1$.

We will consider computational problems related to strategic aspects of deciding on a line-up of players of a team. Our primary consideration is the following problem of computing the best response to a given line-up of the opposing team.

Team Order	
Input:	A target probability $q \in [0,1]$ and a finite set Team Order instance, and a (deterministic) line-up of team T_2.
Question:	Does there exists a line-up for team T_1 under which the probability of T_1 winning against T_2 is at least q?

Without loss of generality, we can assume that the line-up of T_2 is fixed to $t_2^1, \ldots, t_2^n$ when dealing with the Team Order problem. From a graph theoretic perspective, it can be captured by a weighted and complete bipartite graph $G = (T_1 \cup T_2, E, p)$. The weight of an edge (t_i^a, t_j^b) is winning probability $p(t_i^a, t_j^b)$ of any contestant t_i^a against any other contestant t_j^b. We will call G the *corresponding graph*. The line-ups of the two teams correspond to a perfect matching in G, which pairs up every player in T_1 with a unique player in T_2. Assuming that matches are independent, we are interested in computing a perfect matching M whose edge weights maximize the winning probability:

$$\sum_{\substack{S \subseteq \{1,\ldots,n\} \\ |S| \geq \lfloor \frac{n}{2} \rfloor + 1}} \prod_{i \in S}^{n} p(t_1^i, t_2^{M(i)}) \prod_{i \notin S}^{n} \left(1 - p(t_1^i, t_2^{M(i)})\right),$$

where $M(i)$ denotes the index of the player in T_2 who is matched with t_1^i, and each S is an outcome of the competition represented as the set of players in T_1 who win against their opponents. For simplicity, we will also write the probabilities as $p_{i,j} = p(t_1^i, t_2^j)$.

In fact, even when the line-ups of both teams are given, it is not immediately clear that the winning probability of M can be computed efficiently, since there are exponentially (in n) many possible outcomes of the competition. One way that leads to a polynomial-time algorithm to compute this probability is via dynamic programming, which results in the proposition below.

Proposition 1. *Given the line-ups of* T_1 *and* T_2*, the winning probability of each team can be computed in time* $O(n^2)$*.*

We present an example below to illustrate the problem.

Example 1. Take an instance with the input winning probabilities as in Table 1. Also, Team T_1 has 3! different line-ups $O_1, \ldots, O_6$ as illustrated in Figure 1.

Suppose that T_2 uses the line-up (t_2^1, t_2^2, t_2^3). If T_1 responds with (t_1^3, t_1^1, t_1^2) (underlined entries), the probability that they beat T_2 is 1, as they will win

	t_2^1	t_2^2	t_2^3
t_1^1	0.9^*	$\underline{1}$	1
t_1^2	0.5	0.9^*	$\underline{1}$
t_1^3	$\underline{0}$	0.5	0.9^*

Table 1: Each entry (i, j) is the probability $p(t_1^i, t_2^j)$.

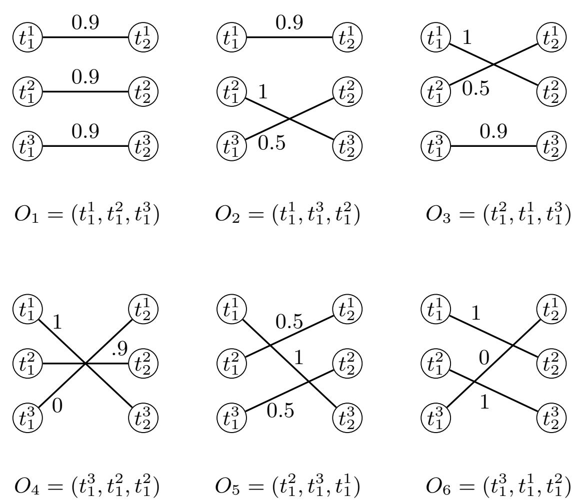

Fig. 1: Graph theoretic view of Example 1. There are 3! different line-ups for T_1 and each line-up is a perfect matching and has its own winning probabilities illustrated on the edges.

two matches with certainty. On the other hand, if T_1 responds with (t_1^1, t_1^2, t_1^3) (starred entries), their winning probability becomes

$$\underbrace{0.9 \times 0.9 \times 0.9}_{\text{prob. of winning all the matches}} + \underbrace{0.9 \times 0.9 \times (1 - 0.9) \times 3}_{\text{prob. of winning exactly two matches}} = 0.972.$$

Indeed, in the above example, the line-up (t_1^1, t_1^2, t_1^3) corresponds to the perfect matching with the maximum total weight in this instance. This demonstrates that weight maximizing matchings may not be optimal solutions to Team Order. The next example shows that such matchings fail to even provide any approximation guarantee to Team Order.

Example 2. Suppose that $n = 7$ and the input winning probabilities are given in Table 2. The maximum weight matching gives the guarantee of winning three matching with certainty but losing all the others, and hence probability 0 of winning the competition. On the other hand, the matching that gives probability 0.5 of winning four matches wins the competition with a non-zero probability.

	t_2^1	t_2^2	t_2^3	t_2^4	t_2^5	t_2^6	t_2^7
t_1^1	0	0	0	0.5	1	1	1
t_1^2	0	0	0	0	0.5	1	1
t_1^3	0	0	0	0	0	0.5	1
t_1^4	0	0	0	0	0	0	0.5
t_1^5	0	0	0	0	0	0	0
t_1^6	0	0	0	0	0	0	0
t_1^7	0	0	0	0	0	0	0

Table 2: Each entry (i, j) is the probability $p(t_1^i, t_2^j)$.

The example also shows that the maximum weight matching cannot approximate the highest winning probability within any multiplicative factor.

In the above example, the better solution has more balanced winning probabilities over the matches. In view of this, one may conjecture that a *leximin-maximizing* matching is optimal for the TEAM ORDER problem.[5] However, the next example disproves this conjecture: a leximin-maximizing matching may not be optimal, even when it is also maximum weight matchings.

	t_2^1	t_2^2	t_2^3
t_1^1	0.9	0.5	1
t_1^2	0.5	0.1	1
t_1^3	0	0	1

Table 3: Each entry (i, j) is the probability $p(t_1^i, t_2^j)$.

Example 3. Suppose that $n = 3$ and one match is guaranteed to be won as shown in Table 3. The edge weights of the maximum weight matchings are (1) $0.5, 0.5, 1$, or (2) $0.1, 0.9, 1$, and the first one is a leximin-maximizing matching. However, the winning probabilities of these two matchings are $1 - 0.25 = 0.75$ and $1 - 0.09 = 0.89$, respectively.

4 Tractable Variants

In this section we show that TEAM ORDER is tractable if there are only two values of probabilities which are greater than 0 in an instance. Moreover, we

[5] A vector x is leximin-greater than a vector y if x and y are in non-decreasing order and x is lexicographically greater than y.

ALGORITHM 1: ITERATIVE ALGORITHM

Input: a TEAM ORDER instance $G = (T_1 \cup T_2, E, p)$ where $p_{i,j} \in \{\alpha, \beta, 0\}$, $\alpha > \beta > 0$.
Output: an optimal solution to TEAM ORDER.

```
Remove all zero-weight edges of G;
opt ← 0;
for s = ⌊n/2⌋ + 1, ... n do
    M_s ← maximum weight matching of size s; // polynomial-time solvable
    if M_s ≠ ∅ then
        p_s ← winning probability of line-up M_s; // see Proposition 1
        if p_s > opt then
            opt ← p_s;
            M* ← M_s;
        end
    end
end
return M*.
```

demonstrate that checking if a team can win with a non-zero probability can be done in polynomial time. Finally, we show that finding the line-up maximizing winning all the matches is tractable. Our reasoning in this section is closely related to the MAXIMUM WEIGHT MATCHING problem. We note that it can be solved in $O(n^3)$ time via the Hungarian algorithm [19].

MAXIMUM WEIGHT MATCHING	
Input:	A bipartite graph G, weight $w(e) \in \mathbb{R}_+$ for each edge e on G.
Question:	Compute a perfect matching M of G that maximizes $w(M) := \sum_{e \in M} w(e)$.

4.1 When Input Probabilities Have Three Values (Including 0)

Let us consider the case in which the input probabilities are from a set $\{\alpha, \beta, 0\}$ and, without loss of generality, assume that $\alpha > \beta > 0$. We note that the problem appears closely connected to a COLORED BIPARTITE MATCHING problem with two types of colors: given a bipartite graph with red and blue edges, does there exists a matching with (exactly) a certain number of red edges? Although the complexity of this red-blue matching problem is open [32], we show that the optimal line-up problem can be solved in polynomial-time via Algorithm 1. We also remark that with this probability set $\{\alpha, \beta, 0\}$ the problem still remains different from MAXIMUM WEIGHT MATCHING, as we demonstrated via Example 1.

Theorem 1. *Suppose that $G = (T_1 \cup T_2, E, p)$ is a* TEAM ORDER *instance with $p_{i,j} \in \{\alpha, \beta, 0\}$ for all $i, j \in \{1, \ldots, n\}$. Then an optimal line-up can be computed in polynomial time.*

Proof. Suppose that M^* denotes an optimal line-up. Let X denote a random variable counting the number of games won by T_1 corresponding to M^*. Then X follows a Poisson Binomial (PB) distribution:

$$X \sim PB(\underbrace{\alpha, \dots, \alpha}_{x}, \underbrace{\beta, \dots, \beta}_{y}, \underbrace{0, \dots, 0}_{z}) = PB(\underbrace{\alpha, \dots, \alpha}_{x}, \underbrace{\beta, \dots, \beta}_{y}),$$

where x, y and z are non-negative integers. Let us remove all 0-weight edges from G and call the resulting graph G'. Then, M^* is a matching of size $x + y$ in G'. Also, any maximum weight matching of size $x + y$, say M, has at least x α-weight edges. Notice that if M has at least $x + 1$ α-weight edges, then Poisson binomial random variable Y corresponding to M stochastically dominates X contradicting the fact that M^* is an optimal line-up. The argument also suggests that searching through all matchings of various sizes will hit the optimal line-up. Note that finding a maximum weight matching of a given size is polynomially solvable. For example, the Hungarian algorithm computes a maximum weight matching of a bipartite graph for each target size [20].

Similar approaches based on Maximum Weight Matching also lead to efficient algorithms for two variants of Team Order. First, if the goal is to decide whether T_1 can beat T_2 with non-zero probability, the problem can be solved in polynomial time. Specifically, for an instance represented as a graph G, we can consider the corresponding graph G' in which edges with weight 0 are removed. Then, T_1 can beat T_2 with non-zero probability if and only if G' has a matching of size $\lfloor \frac{n}{2} \rfloor + 1$. We state this result below.

Corollary 1. *Given the line-up of T_2, it can be decided in polynomial time whether there exists a line-up of T_1 that beats T_2 with a non-zero probability.*

Second, if the goal is to maximize the probability of winning *all* the matches, the problem reduces to computing a weight maximizing matching, where the weights are the logarithm of the non-zero winning probabilities.

Proposition 2. *Given the line-up of T_2, the line-up of T_1 that maximizes the probability of winning* all *the matches can be computed in polynomial time.*

5 Approximation Algorithm for Team Order

As we have seen, in several cases finding a solution to Team Order is tractable. However, even though it resembles Maximum Weight Matching, its exact solutions are far more nuanced, which suggests its hardness. In this section, we address the practical solvability of our problem by providing a PTAS for Team Order. Assuming the input probabilities are bounded away from 0 and 1 by any arbitrary constant $\varepsilon > 0$, the PTAS computes a solution to Team Order whose winning probability is at most ε less than that of the optimal solution.

5.1 High-level Ideas

For any perfect matching $M = \{e_1, \ldots, e_n\}$ of $G = (T_1 \cup T_2, E, p)$, let X_M be a random variable counting the number of matches won by T_1. One may observe that X_M follows a Poisson binomial distribution $PB(p_{e_1}, \ldots, p_{e_n})$. Furthermore, Team Order can be written as the following optimization problem.

$$\min_{M} \quad \mathbf{Pr}\left[X_M \leq \lfloor \frac{n}{2} \rfloor\right]$$
$$\text{subject to:} \quad M \text{ is a perfect matching of } G = (T_1 \cup T_2, E, p)$$

The main idea of our algorithm is as follows. First, we note that the number of matchings M with $\mathbf{Var}[X_M] < \varepsilon^{-2}$ is bounded from above by a polynomial in n, when ε is a constant. Hence, we can search over all such matchings to find out the optimal one among them. For the other matchings M with a high variance $\mathbf{Var}[X_M] \geq \varepsilon^{-2}$, we use $\Phi\left(\frac{\lfloor \frac{n}{2} \rfloor - \mathbf{E}[X_M]}{\sqrt{\mathbf{Var}[X_M]}}\right)$ to approximate the objective function, where $\Phi(x) = (\frac{1}{\sqrt{2\pi}}) \int_{-\infty}^{x} \mathrm{e}^{\frac{-y^2}{2}} dy$. Since X_M is a Poisson binomial random variable, it holds that if $\mathbf{Var}[X_M] \geq \varepsilon^{-2}$, then

$$\left| \mathbf{Pr}\left[X_M \leq \lfloor \frac{n}{2} \rfloor\right] - \Phi\left(\frac{\lfloor \frac{n}{2} \rfloor - \mathbf{E}[X_M]}{\sqrt{\mathbf{Var}[X_M]}}\right) \right| \leq \varepsilon.$$

Using the fact that $\Phi(x)$ is an increasing and continuous function in x, we get the following optimization problem as an approximation to the original one.

$$\min_{M} \quad \frac{\lfloor \frac{n}{2} \rfloor - \mathbf{E}[X_M]}{\sqrt{\mathbf{Var}[X_M]}}$$
$$\text{subject to:} \quad M \text{ is a perfect matching of } G = (T_1 \cup T_2, E, p)$$

The objective function is still non-linear though, but it can be characterized by the mean and variance of X_M. Using the fact that for every matching M we have $0 \leq \mathbf{Var}[X_M] \leq \frac{n}{4}$ and $\mathbf{E}[X_M] \leq n$, we can discretize the two dimensional space $\{(x, y) : 0 \leq x \leq \frac{n}{4} \text{ and } 0 \leq y \leq n\}$ and design a search mechanism to eventually hit a matching that is close enough to the optimal matching. The search mechanism is based on an approximation algorithm solving a matching problem that involves both budget and rewards, which we will discuss next.

5.2 Preliminary Results

We introduce necessary preliminary results for designing the PTAS. It has two main ingredients. We apply a normal distribution estimation for a Poisson binomial distribution, and an approximation algorithm for the following Budgeted/Reward Matching problem. We assume that every $p_e \notin \{0, 1\}$ is bounded away from 0 and 1. Define $\delta = \min_{e \in E, p_e \notin \{0,1\}} \min\{p_e, 1 - p_e\}$. Then, we get that $\frac{1}{\delta} = \Theta(1)$.

Approximation of Poisson Binomial Distribution. We use a normal distribution estimation for a Poisson binomial distribution to approximate $\mathbf{Pr}\left[X_M \le \lfloor \frac{n}{2} \rfloor\right]$, which is based on the following result.

Theorem 2 ([29, Theorem 3.5])**.** *Suppose that $X \sim PB(p_1, \ldots, p_n)$ is a Poisson binomial random variable. Then, for every $1 \le k \le n$,*

$$\left| \mathbf{Pr}\left[X \le k\right] - \Phi\left(\frac{k - \mathbf{E}\left[X\right]}{\sqrt{\mathbf{Var}\left[X\right]}}\right) \right| \le \frac{1}{\sqrt{\mathbf{Var}\left[X\right]}},$$

where $\Phi(x) = (\frac{1}{\sqrt{2\pi}}) \int_{-\infty}^{x} \mathrm{e}^{\frac{-y^2}{2}} dy$.

An immediate application of Theorem 2 results in to the following corollary.

Corollary 2. *Suppose that $X_M \sim PB(p_{e_1}, \ldots, p_{e_n})$ is a Poisson binomial random variable corresponding to a matching $M = \{e_1, \ldots, e_n\}$ with $\mathbf{Var}\left[X_M\right] \ge \varepsilon^{-2}$, for some $\varepsilon > 0$. Then,*

$$\left| \mathbf{Pr}\left[X_M \le \lfloor \frac{n}{2} \rfloor\right] - \Phi\left(\frac{\lfloor \frac{n}{2} \rfloor - \mathbf{E}\left[X_M\right]}{\sqrt{\mathbf{Var}\left[X_M\right]}}\right) \right| \le \varepsilon,$$

where $\Phi(x) = (\frac{1}{\sqrt{2\pi}}) \int_{-\infty}^{x} \mathrm{e}^{\frac{-y^2}{2}} dy$.

Budgeted Matching. We will use approximation algorithms for the following Budgeted Matching problem as subroutines in our algorithm.

Budgeted Matching	
Input:	A bipartite graph G, weight $w(e)$ and cost $c(e)$ for each edge e, and a budget B.
Question:	Compute a perfect matching M of G that maximizes $w(M) := \sum_{e \in M} w(e)$, subject to $c(M) := \sum_{e \in M} c(e) \le B$.

Specifically, we are interested in the following weight and cost functions. For every $e \in E$, we let $w(e) = p_e$ and $c(e) = p_e \cdot (1 - p_e)$. Hence, for every matching M, we have

$$w(M) = \mathbf{E}\left[X_M\right], \quad \text{and} \quad c(M) = \mathbf{Var}\left[X_M\right].$$

We will henceforth stick to the above weight and cost functions, unless otherwise clarified. We use $I_b(G, w, c, B)$ to denote an instance of Budgeted Matching. For convenience, we can also define a "rewarded" variant of Budgeted Matching, where we want the total cost to pass a threshold R, i.e., $c(M) = \sum_{e \in M} c(e) \ge R$, and we denote it by $I_r(G, w, c, B)$. Since $0 \le c(e) < 1$, we observe that $I_r(G, w, c, B)$ is equivalent to $I_b(G, w, c', n - R)$, where $c'(e) = 1 - c(e)$ for every $e \in E$. Berger et al. [8] designed a PTAS for the Budgeted Matching problem. Using the same idea this PTAS is based on, we get the following.

Lemma 1. *Suppose that $G = (T_1 \cup T_2, E, p)$ is a* Team Order *instance, $w(e) = p_e$ and $c(e) = p_e \cdot (1 - p_e)$ for each $e \in E$. Then, there is a polynomial-time algorithm to compute a feasible solution M to $I_b(G, w, c, B)$ (respectively, $I_r(G, w, c, R)$) such that $w(M) \geq opt - 2$, where opt is the weight of optimal solution of $I_b(G, w, c, B)$ (respectively, $I_r(G, w, c, R)$).*

Small / Large Variance Matchings. We partition the set of edges into edges with fractional and binary weights; let $F = \{e \in E : p_e \notin \{0, 1\}\}$ and $\overline{F} = \{e \in E : p_e \in \{0, 1\}\}$. Fix an arbitrary constant $\varepsilon \in (0, 1]$ and define $\mathcal{M}^+(\varepsilon) = \left\{N \subset F : \ N \text{ is a minimum size matching with } c(N) > \varepsilon^{-2}\right\}$, and

$$\mathcal{M}^-(\varepsilon) = \left\{N \subset F : \ N \text{ is a matching with } c(N) \leq \varepsilon^{-2}\right\}.$$

Clearly, for every perfect matching M on G, if $c(M) > \varepsilon^{-2}$, then there exists $N \in \mathcal{M}^+(\varepsilon)$ such that $M \cap N = N$. Similarly, if $c(M) \leq \varepsilon^{-2}$, there exists $N \in \mathcal{M}^-(\varepsilon)$ such that $M \cap N = N$.

For every matching $N \subset E$ and every subset of edges $E' \subseteq E$, let $E'_N = \{e \in E' : e \cap N = \emptyset\}$, i.e., E'_N is the set of all edges in E' that do not share all endpoints with N. We now define two families of bipartite graphs as follows. First, $\mathcal{G}^+(\varepsilon) = \{H = (T_1 \cup T_2, N \cup E_N, p) : N \in \mathcal{M}^+(\varepsilon)\}$. Intuitively, we fix the matching N and leave the unmatched part of the graph G free. Then, we define $\mathcal{G}^-(\varepsilon) = \{H = (T_1 \cup T_2, N \cup \overline{F}_N, p) : N \in \mathcal{M}^-_\varepsilon\}$. This differs from $\mathcal{G}^+(\varepsilon)$, as we only consider 0/1-edges in the unmatched part of G. Note that for every perfect matching M of G, if $c(M) > \varepsilon^{-2}$, there is $H \in \mathcal{G}^+(\varepsilon)$ such that $M \subset H$. Similarly, if $c(M) \leq \varepsilon^{-2}$, then there is $H \in \mathcal{G}^-(\varepsilon)$ such that $M \subset H$. Next, we show that the size of these families of graphs is polynomially bounded.

Lemma 2. *It holds that $|\mathcal{G}^+(\varepsilon)| \leq n^{4\delta^{-1}\varepsilon^{-2}}$ and $|\mathcal{G}^-(\varepsilon)| \leq n^{4\delta^{-1}\varepsilon^{-2}}$.*

5.3 The Algorithm

Now we discuss our approximation algorithm, Algorithm 2. Theorem 3 shows that the algorithm produces an ε-approximate solution to Team Order in polynomial time. The proof relies on Lemma 1 and Lemma 2.

Theorem 3. *Algorithm 2 computes an ε-approximate solution to* Team Order *and runs in time $n^{O(\delta^{-1}\varepsilon^{-2})}$, where $\delta = \min_{p_e \notin \{0,1\}} \min_{e \in E}\{p_e, 1 - p_e\}$.*

6 Winning Probability of a Maximum Weight Matching

In this section we investigate the winning probability of a maximum weight matching. Our result provides a lower bound for the winning probability of any maximum weight matching compared with that of the optimal line-up. In particular, the result shows that a sufficiently large/small maximum weight matching performs almost as well as the optimal line-up. In what follows, we view $G = (T_1 \cup T_2, E, p)$ as a weighted bipartite graph where for every $e \in E$, p_e is

ALGORITHM 2: ε-Approximation Algorithm

Input: a Team Order instance $G = (T_1 \cup T_2, E, p)$.
Output: an ε-approximate solution to Team Order.

```
ε ← ε/4;
for H ∈ 𝒢⁻(ε) do
    M* ← a maximum weight perfect matching of H;
    if M* exists and has a higher winning probability than M_ε^(1) (or M_ε^(1) = null)
     then
        M_ε^(1) ← M*;
    end
end
x_i ← ε^(-2) + i/n for each i = 0, ..., n²/4;
for H ∈ 𝒢⁺(ε) do
    for i = 1, ..., n²/4 do
        M_i* ← a solution to I_b(H, w, c, x_i) such that w(M_i*) > opt − 2; // Lemma 1
        if M_i* exists and w(M_i) < ⌊n/2⌋ then
            M_i* ← a solution to I_r(H, w, c, x_{i−1}) such that w(M_i*) > opt − 2;
                // Lemma 1
        end
        if M_i* exists and (⌊n/2⌋ − w(M_ε^(2)))/√c(M_ε^(2)) > (⌊n/2⌋ − w(M_i*))/√c(M_i*) (or M_ε^(2) = null) then
            M_ε^(2) ← M_i*;
        end
    end
end
return M_ε^(1) or M_ε^(2) whichever has the higher winning probability.
```

the weight of e. For every matching M, we use $w(M)$ to denote its weight. In addition, we assume that the size of G is sufficiently large.

Theorem 4. *Let M^* be a maximum weight matching and let O be an optimal line-up. Then,*

(1) If $w(M^*) = \frac{n}{2} \pm f(n)\sqrt{n}$, where $f(n) \in [1, \frac{\sqrt{n}}{2}]$ is any non-decreasing function in n, then $\mathbf{Pr}[T_1 \, wins \; under \; O] \leq \mathbf{Pr}[T_1 \, wins \; under \; M^*] + \mathrm{e}^{-2f^2(n)}$.
(2) If $w(M^*) \in [\frac{n}{2} - \sqrt{n \log n}, \; \frac{n}{2} + \sqrt{n \log n}]$, then

$$\begin{aligned} &\mathbf{Pr}[T_1 \, wins \; under \; O] \\ \leq &\mathbf{Pr}[T_1 \, wins \; under \; M^*] + \frac{(4 + o(1))}{n+1} \sum_{e \in M^*} (p_e - \frac{1}{2})^2. \end{aligned}$$

In the proof of Theorem 4 we rely on the following results.

Theorem 5 ([10]). *Suppose that n is a given positive integer and let $X \sim PB(p_1, \ldots, p_n)$ be a Poisson binomial random variable. Then, we have that $\mathbf{Pr}[X \geq \mathbf{E}[X] + \delta] \leq e^{\frac{-2\delta^2}{n}}$, and $\mathbf{Pr}[X \leq \mathbf{E}[X] - \delta] \leq e^{\frac{-2\delta^2}{n}}$.*

Theorem 6 ([29, Theorem 2.1]). *Let $X \sim PB(p_1, \ldots, p_n)$ and let $\bar{p} = \sum_{i=1}^{n} \frac{p_i}{n}$. Define $Y \sim Bin(n, \bar{p})$. Then, (1) for every $0 \leq k \leq n\bar{p} - 1$, $\mathbf{Pr}[X \leq k] \leq \mathbf{Pr}[Y \leq k]$, and (2) for every $n\bar{p} \leq k \leq n$, $\mathbf{Pr}[X \leq k] \geq \mathbf{Pr}[Y \leq k]$.*

Theorem 7 ([1, Theorem 1]). *Suppose that $X \sim PB(p_1, \ldots, p_n)$, and $\bar{p} = \sum_{i=1}^{n} p_i/n$. Also, let $Y \sim Bin(n, \bar{p})$ is a binomial probability distribution. Then,*

$$\max_{A \subseteq \{0,\ldots,n\}} |\mathbf{Pr}[X \in A] - \mathbf{Pr}[Y \in A]| \leq \frac{1 - \bar{p}^n - (1-\bar{p})^n}{(n+1)\bar{p}(1-\bar{p})} \sum_{i=1}^{n} (p_i - \bar{p})^2.$$

We prove the first and the second parts of the theorem separately next.

Part 1. When $w(M^*) = \frac{n}{2} \pm f(n)\sqrt{n}$

Proof. Let $M = \{e_1, \ldots, e_n\}$ be an arbitrary matching and X_M be a random variable that counts the number of games won by T_1 under line-up M. Then X_M follows Poisson binomial distribution $PB(p_{e_1}, \ldots, p_{e_n})$. , where $M = \{e_1, \ldots, e_n\}$. Thus, $\mathbf{E}[X_M] = w(M)$. Let us first assume that $w(M^*) = \frac{n}{2} - f(n)\sqrt{n}$. Then, for every matching M, including the optimal line-up O, we have $w(M) \leq w(M^*)$. Moreover, we have $f(n)\sqrt{n} = \frac{n}{2} - w(M^*) \leq \frac{n}{2} - w(M)$, and

$$\begin{aligned}\mathbf{Pr}[T_1 \text{ wins under } M] &= \mathbf{Pr}\left[X_M \geq \lfloor \frac{n}{2} \rfloor + 1\right] \\ &\leq \mathbf{Pr}\left[X_M \geq w(M) + (\frac{n}{2} - w(M))\right] \\ &= \mathbf{Pr}\left[X_M \geq \mathbf{E}[X_M] + f(n)\sqrt{n}\right] \leq \mathrm{e}^{-2(f(n)\sqrt{n})^2 \frac{2}{n}} = \mathrm{e}^{-2f(n)^2},\end{aligned}$$

using a concentration bound for Poisson binomial random variables (e.g., see Theorem 5). Following that upper bound, if $w(M^*) = \frac{n}{2} - f(n)\sqrt{n}$, then

$$\mathbf{Pr}[T_1 \text{ wins under } O\ ] \leq \mathrm{e}^{-2f(n)^2} \leq \mathbf{Pr}[T_1 \text{ wins under } M^*\ ] + \mathrm{e}^{-2f(n)^2}. \quad (1)$$

Next, we consider the case where $w(M^*) = \frac{n}{2} + f(n)\sqrt{n}$. Define random variable Y_{M^*} that counts the number of games lost under M^*. Then Y_{M^*} follows Poisson binomial distribution $PB(1 - p_{e_1}, \ldots, 1 - p_{e_n})$, where we let $M^* = \{e_1, \ldots, e_n\}$. One can check that $\mathbf{E}[Y_{M^*}] = n - w(M^*) = \frac{n}{2} - f(n)\sqrt{n}$.

$$\begin{aligned}\mathbf{Pr}[T_1 \text{ loses under } M^*\ ] &= \mathbf{Pr}\left[Y_{M^*} \geq \lfloor \frac{n}{2} \rfloor + 1\right] \\ &\leq \mathbf{Pr}\left[Y_{M^*} \geq \mathbf{E}[Y_{M^*}] + (\frac{n}{2} - \mathbf{E}[Y_{M^*}])\right] \leq \mathrm{e}^{-2f(n)^2},\end{aligned}$$

where we have applied the same concentration bound as the previous case. Hence,

$$\mathbf{Pr}[T_1 \text{ wins under } M^*\ ] = 1 - \mathbf{Pr}[T_1 \text{ loses under } M^*\ ] \geq 1 - \mathrm{e}^{-2f(n)^2}.$$

Thus,

$$\mathbf{Pr}[T_1 \text{ wins under } O\ ] \leq 1 \leq \mathbf{Pr}[T_1 \text{ wins under } M^*\ ] + \mathrm{e}^{-2f(n)^2} \quad (2)$$

Hence, combining (1) and (2) gives the first part of Theorem 4.

Part 2. When $w(M^*) \in [\frac{n}{2} - \sqrt{n \log n},\ \frac{n}{2} + \sqrt{n \log n}]$

Proof. Let us first consider the case where $w(M^*) \in [\frac{n}{2} - \sqrt{n \log n}, \frac{n}{2} - 1)$. Define random variables X_O and X_{M^*} that count the number games won by T_1 under O and M^*, respectively. Moreover, define binomial random variables $Z_O \sim Bin(n, \frac{w(O)}{n})$ and $Z_{M^*} \sim Bin(n, \frac{w(M^*)}{n})$. Notice that $w(O) \leq w(M^*)$ and hence Z_{M^*} stochastically dominates Z_O (i.e, $\mathbf{Pr}\left[Z_O \leq \frac{n}{2}\right] \geq \mathbf{Pr}\left[Z_{M^*} \leq \frac{n}{2}\right]$). Since $w(M^*) < \frac{n}{2}$, we apply the stochastic dominance between the Poisson and binomial random variables (e.g., see Theorem 6 (2)) and we have that

$$\mathbf{Pr}\left[T_1 \text{ loses under } O\right] = \mathbf{Pr}\left[X_O \leq \frac{n}{2}\right] \geq \mathbf{Pr}\left[Z_O \leq \frac{n}{2}\right] \geq \mathbf{Pr}\left[Z_{M^*} \leq \frac{n}{2}\right],$$

On the other hand, the optimal line-up O minimizes the losing probability of T_1 and hence, by above inequality we have that

$$\begin{aligned}&\mathbf{Pr}\left[T_1 \text{ loses under } M^*\right]\\ &\qquad = \mathbf{Pr}\left[X_{M^*} \leq \frac{n}{2}\right] \geq \mathbf{Pr}\left[T_1 \text{ loses under } O\right] \geq \mathbf{Pr}\left[Z_{M^*} \leq \frac{n}{2}\right].\end{aligned}$$

Applying the above inequality and Theorem 7 results in

$$\begin{aligned}&\mathbf{Pr}\left[T_1 \text{ wins under } O\right] - \mathbf{Pr}\left[T_1 \text{ wins under } M^*\right]\\ &\qquad = (1 - \mathbf{Pr}\left[T_1 \text{ wins under } M^*\right]) - (1 - \mathbf{Pr}\left[T_1 \text{ wins under } O\right])\\ &\qquad = \mathbf{Pr}\left[T_1 \text{ loses under } M^*\right] - \mathbf{Pr}\left[T_1 \text{ loses under } O\right]\\ &\qquad \leq \mathbf{Pr}\left[X_{M^*} \leq \frac{n}{2}\right] - \mathbf{Pr}\left[Z_{M^*} \leq \frac{n}{2}\right]\\ &\qquad \leq \frac{1 - (\bar{p})^n - (1-\bar{p})^n}{(n+1)(1-\bar{p})\bar{p}} \sum_{e \in M^*} (p_e - \bar{p})^2,\end{aligned}$$

where $\bar{p} = \frac{w(M^*)}{n}$. Since we have $n\bar{p} \in (\frac{n}{2} - \sqrt{n \log n}, \frac{\mathbf{n}}{\mathbf{2}})$, and n is an asymptotically large, we have $\bar{p} \approx \frac{1}{2}$ and thus

$$\frac{1 - (\bar{p})^n - (1-\bar{p})^n}{(n+1)(1-\bar{p})\bar{p}} \sum_{e \in M^*} (p_e - \bar{p})^2 \leq \frac{(4 + o(1))}{n+1} \sum_{e \in M^*} (p_e - \frac{1}{2})^2.$$

Therefore, if $w(M^*) \in [\frac{n}{2} - \sqrt{n \log n}, \frac{n}{2} - 1)$, then

$$\mathbf{Pr}\left[T_1 \text{ wins under } O\right] \leq \mathbf{Pr}\left[T_1 \text{ wins under } M^*\right] + \frac{(4 + o(1))}{n+1} \sum_{e \in M^*} (p_e - \frac{1}{2})^2.$$

To derive the same upper bound for the case where $w(M^*) \in [\frac{n}{2}, \frac{n}{2} + \sqrt{n \log n}]$, we define random variables that count the number of games lost by T_1 and the same technique for the above case follows.

7 Conclusion

We proposed the Team Order problem, which naturally captures several strategic scenarios in information systems and team competitions. We have shown that in the case in which the input probabilities are limited to three values (including 0) it is tractable and have shown that it is possible to efficiently compute a line-up which is close to the optimal in terms of the probability of winning, which is useful when the information about the players' relative strength is limited (e.g., if it is only known when a player is "strong" or "weak" against an opponent). One of our central results is a PTAS for the Team Order problem. We note that while we focused on the probability of winning against more than a half of opposing players, our results hold for any such threshold.

We conclude by highlighting some important directions for future work. First, the complexity of solving Team Order exactly is open. We believe that this is a challenging question that also has implications on the related problem of Colored Bipartite Matching. It is known that it is NP-complete when d is a variable [16]. However, the complexity of this problem is open if d is a constant larger than 2, or if $d = 2$ but the graph is incomplete (which corresponds to $\{\alpha, \beta, 0\}$) [32]. This motivates further study between the connections of the two discussed problems. It is also not known whether Team Order admits a fully polynomial-time approximation scheme (FPTAS). Resolving this question would be a strong improvement over our results.

While our result show the complexity of computing a best response to the opponents line-up, it is natural to study the extension in which multiple teams strategize. Regarding sport events, it would also be interesting to see if the results change under other natural assumptions, such as all of the players having an objective level of skill. For example, if a player i has better skill than a player j, then i might always have a better probability of winning against any player k than j's probability of beating k.

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