Manipulating Sequential Allocation: an Overview

HARIS AZIZ, SYLVAIN BOUVERET, AND JÉRÔME LANG

Sequential allocation is a simple allocation mechanism in which agents are given pre-specified turns. An agent whose turn comes can freely choose and take any item among those that remain. It is well-known that sequential allocation is not strategyproof. We present an overview of computational results on manipulation of sequential allocation.

Categories and Subject Descriptors: F.2.2 [**Analysis of Algorithms and Problem Complexity**]: Nonnumerical Algorithms and Problems; I.2.11 [**Distributed Artificial Intelligence**]: Multiagent Systems; J.4 [**Computer Applications**]: Social and Behavioral Sciences—*Economics*

General Terms: Theory, Algorithms, Economics

Additional Key Words and Phrases: Fair allocation, Solutions Concepts, Multiagent resource allocation, Computational Complexity

1. INTRODUCTION

A simple but popular mechanism to allocate indivisible items is *sequential allocation* [see e.g, Aziz et al., 2015, Bouveret and Lang, 2011, Brams and Straffin, 1979, Kalinowski et al., 2013a,b, Kohler and Chandrasekaran, 1971, Levine and Stange, 2012]. In sequential allocation, a sequence specifies the turns of the agents. For example, for sequence 1212, agents 1 and 2 alternate with agent 1 taking the first turn. In sequential allocation, the agents do not have to report their preferences explicitly. At her turn, an agent just chooses an item among those that have not been allocated yet, and takes it. In some sense, the only information that an agent has to provide about her preferences is to name her most preferred item at her turn. An alternative version of this mechanism can also be used in a centralized context: in this case, each agent reports a linear order over the items to the central authority, and, given the sequence and the orders, the central authority just allocates to each agent her most preferred item (according to the reported order) at her turn.

It is folklore that, in spite of the small amount of information needed from the agents, sequential allocation is not strategyproof (except when the agents are not interleaved in the sequence): an agent can decide not to pick her most preferred item at her turn and end up with a better allocation. Let us consider for instance the situation where the sequence is 1212 and the agents preferences are as follows:

agent 1: $a \succ b \succ c \succ d$
agent 2: $b \succ c \succ d \succ a$

If both agents act sincerely, the final allocation will be (ac, bd) since agent 1 will first pick a, then 2 b, then 1 c, and 2 d. However, if agent 1 acts strategically and chooses at her first turn to rather take item b, then the final allocation will be (ab, cd), which is strictly better to agent 1: in other word, agent 1 can successfully manipulate to get a better allocation.

Authors' addresses: `haris.aziz@data61.csiro.au`, `sylvain.bouveret@imag.fr`, `lang@lamsade.dauphine.fr`

The manipulability of sequential allocation motivates the natural problem of computing best responses (also referred to as manipulations). In other words, the problem we consider is that of an agent (the manipulator) taking the appropriate picks or submitting an ordinal ranking that gives her the best possible allocation. The problem is equivalent whether we view sequential allocation as a centralized or decentralized mechanism.

In one of the earliest papers on this topic, Kohler and Chandrasekaran [1971] focused on the the case in which there are two agents and the the sequence is *strict alternating* (121212..). They presented a polynomial-time algorithm to compute the optimal manipulation of an agent. They also considered optimal strategies when both players strategise. Budish and Cantillion [2012] study a variant of the model (course allocation to students with a mechanism which is a randomized version of a picking sequence) and show that not only it is manipulable in theory, but that it also is manipulated by students in practice. Sequential allocation was formally studied in a more general and systematic way by Bouveret and Lang [2011], and further by Kalinowski et al. [2013b] who present a game-theoretic study of picking sequences. There have been several followup works [see e.g, Aziz et al., 2017b, 2015, Bouveret and Lang, 2011, Brams and Straffin, 1979, Kalinowski et al., 2013a, Levine and Stange, 2012] on strategic aspects of sequential allocation since the work of Kohler and Chandrasekaran [1971].

In this note, we present an overview of results concerning manipulation in sequential allocation. In the setting we consider, we highlight two assumptions. Firstly, the manipulating agent has complete information about the reported preferences of the other agents. More precisely, in the centralized context, the manipulating agent knows all the linear orders submitted by the other agents. In the distributed context, we assume that the agents pick the objects as if they had a linear order in mind and always take their most preferred object, and the manipulating agent knows all these linear orders. Secondly, although agents report strict and ordinal preferences, we assume that the manipulator has additive cardinal utilities for the items. The first is a worst case assumption often made in computer science and economics. The second assumption of additive utilities is also standard in this research area.

2. COMPLEXITY OF MANIPULATION PROBLEMS

We consider natural problems concerning manipulation of sequential allocation. As mentioned earlier, for all the problems, we assume that the manipulator has complete information about the reported preferences of the other agents.

BestResponse: can the manipulator get at least a specified utility?

BetterThanTruthResponse: can the manipulator get more utility than the allocation under her truthful report?

UnambiguouslyBetterThanTruthResponse: can the manipulator get more utility than the allocation under her truthful report for all utility functions consistent with her truthful ordinal preference?

AllocationResponse: can the manipulator get a specified allocation (set of items)?

Note that BestResponse is at least as hard as BetterThanTruthResponse. For the problems in which agent wants to maximize utility, we also consider lexicographic utilities as well as binary utilities.

The results are summarized in Table I. The results are based on the papers published in the Proceedings of IJCAI'11 [Bouveret and Lang, 2011], ECAI'14 [Bouveret and Lang, 2014], and AAAI'17 [Aziz et al., 2017a].

	General problem	$n = 2$	Lex utilities	binary utilities
BestResponse	NP-complete	in P	in P	in P
BetterThanTruthResponse	NP-complete	in P	in P	in P
UnambiguouslyBetterThanTruthResponse	in P	in P	in P	in P
AllocationResponse	in P	in P	in P	in P

Table I. Computational complexity of manipulation in sequential allocation.

Bouveret and Lang [2011] first showed that when there are two agents, it can be checked via a greedy procedure whether a manipulator can get any particular set of items (AllocationResponse). The same result can be generalized for AllocationResponse for any number of agents by providing an interesting reduction to the same problem for the case of two agents [Bouveret and Lang, 2011]. As a consequence, BestResponse under lexicographic preferences can be solved in polynomial time because we can iteratively build up the allocations of increasing sizes that are lexicographically most preferred and also achievable [Bouveret and Lang, 2011].

Note that a polynomial-time algorithm for AllocationResponse does not imply a polynomial-time algorithm for BestResponse. Interestingly, when $n = 2$, it is sufficient to solve BestResponse under lexicographic preferences which gives the best possible allocation to the manipulator for any additive utilities consistent with the ordinal preferences. This observation was made by Bouveret and Lang [2014].

On the other hand, both BestResponse and BetterThanTruthResponse are NP-complete in general. The result was proved recently [Aziz et al., 2017a] and the reduction is from a restricted variant of 3SAT. Just as for lexicographic utilities, there is a polynomial-time algorithm for BestResponse when the manipulator has binary utilities [Aziz et al., 2017a].

Finally, one can consider a stronger notion of manipulation whereby the untruthful outcome yields more utility than the truthful outcome for all utilities consistent with the ordinal preferences. For this notion and its corresponding problem UnambiguouslyBetterThanTruthResponse, there is a polynomial-time algorithm [Aziz et al., 2017a].

Sequential allocation is a widely used allocation mechanism because of its simplicity. In this note we gave a high level overview of some of the algorithmic and complexity results from the literature.

REFERENCES

H. Aziz, T. Walsh, and L. Xia. Possible and necessary allocations via sequential mechanisms. In *Proceedings of the 24th International Joint Conference on Artificial Intelligence (IJCAI)*, pages 468–474, 2015.

H. Aziz, S. Bouveret, J. Lang, and S. Mackenzie. Complexity of manipulating sequential allocation. In *Proceedings of the 31st AAAI Conference on Artificial Intelligence (AAAI)*, pages 328–334, 2017a.

H. Aziz, P. Goldberg, and T. Walsh. Equilibria in sequential allocation. In *Proceedings of the 5th International Conference on Algorithmic Decision Theory (ADT)*, 2017b.

S. Bouveret and J. Lang. A general elicitation-free protocol for allocating indivisible goods. In *Proceedings of the 22nd International Joint Conference on Artificial Intelligence (IJCAI)*, pages 73–78. AAAI Press, 2011.

S. Bouveret and J. Lang. Manipulating picking sequences. In *Proceedings of the 21st European Conference on Artificial Intelligence (ECAI)*, pages 141–146, 2014.

S. J. Brams and P. D. Straffin. Prisoners' dilemma and professional sports drafts. *The American Mathematical Monthly*, 86(2):80–88, 1979.

E. Budish and E. Cantillion. The multi-unit assignment problem: Theory and evidence from course allocation at Harvard. *American Economic Review*, 102(5): 2237–2271, 2012.

T. Kalinowski, N. Narodytska, and T. Walsh. A social welfare optimal sequential allocation procedure. In *Proceedings of the 22nd International Joint Conference on Artificial Intelligence (IJCAI)*, pages 227–233. AAAI Press, 2013a.

T. Kalinowski, N. Narodytska, T. Walsh, and L. Xia. Strategic behavior when allocating indivisible goods sequentially. In *Proceedings of the 27th AAAI Conference on Artificial Intelligence (AAAI)*, pages 452–458. AAAI Press, 2013b.

D. A. Kohler and R. Chandrasekaran. A class of sequential games. *Operations Research*, 19(2):270–277, 1971.

L. Levine and K. E. Stange. How to make the most of a shared meal: Plan the last bite first. *The American Mathematical Monthly*, 119(7):550–565, 2012.