A Rule-based Theorem Prover: an Introduction to Proofs in Secondary Schools

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The introduction of automated deduction systems in secondary schools faces several bottlenecks. Beyond the problems related with the curricula and the teachers, the dissonance between the outcomes of the geometry automated theorem provers and the normal practice of conjecturing and proving in schools is a major barrier to a wider use of such tools in an educational environment.

Since the early implementations of geometry automated theorem provers, applications of artificial intelligence methods, synthetic provers based on inference rules and using forward chaining reasoning are considered to be best suited for education proposes.

Choosing an appropriate set of rules and an automated method that can use those rules is a major challenge. We discuss one such rule set and its implementation using the geometry deductive databases method (GDDM). The approach is tested using some chosen geometric conjectures that could be the goal of a 7th year class ($\approx$12-year-old students). A lesson plan is presented, its goal is the introduction of formal demonstration of proving geometric theorems, trying to motivate students to that goal.

1 Introduction

The introduction of automated deduction systems in secondary schools faces several bottlenecks. The absence of the subject geometry itself, of rigorous mathematical demonstrations, not to mention formal proofs, in many of the national curricula, the lack of knowledge (and/or training) by the teachers about the subject [25], the dissonance between the outcomes of the available Geometry Automated Theorem Provers (GATP) and the normal practice of conjecturing and proving in the secondary education [23], are the most important in our opinion.

According to [5], the structuring of thinking stands out as one of the major purposes for the teaching of mathematics. In order to achieve it, teaching should be based on sequential learning, to build knowledge in the classroom, a hierarchy of concepts with the systematic study of their properties, fostering clear and precise arguments. These are the basic elements of hypothetical-deductive reasoning, which is par excellence the mathematical reasoning. Inductive reasoning also plays an important role in mathematics since it allows the establishment of conjectures, which can then be proved using deduc-

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tive reasoning. All forms of reasoning have had, and continue to have, their place in the mathematics curriculum [13].

The new curriculum documents [7, 8] also reinforce the importance of promoting and mobilising computational thinking, assuming the ability to analyse and define algorithms, allowing a structuring of thinking and providing students with more tools to solve problems and prove results. The use of technological resources is unavoidable and the learning of mathematics can benefit from their use. This work unites all of those purposes.

The paper is organised as follows: first, in Sec. 2, some dynamic geometry tools and geometry automated theorem provers will be highlighted. In Sec. 3, the algorithm of a rule-based geometry automated theorem prover and the set of rules that will be used in the examples will be presented. In Sec. 4, two examples and excerpts of the formal proofs done by a rule-based geometry automated theorem prover will be analysed. In Sec. 5 an outline of a lesson plan to address one of the examples will be proposed. Finally, in Sec. 6 conclusions are drawn and future work will be discussed.

2 Geometric Reasoning

Learning geometry involves some cognitive complexity. According to [6] there are three cognitive processes involved in learning geometry: visualisation (relating to spatial representation), construction (using tools) and reasoning (in particular the discursive processes to broaden the knowledge processes, for demonstration and for interpretation). These processes can be performed separately, that is, the visualisation does not depend on the construction. Even if construction precedes visualisation, construction processes only depend on connections between mathematical properties and tool constraints. If visualisation is an intuitive aid useful to find a proof, it can, in some cases, be misleading. The validity of the proof rely solely on the corpus of propositions (definitions, axioms, theorems) that are available.

As Duval points out "these three kinds of cognitive processes are closely connected and their synergy is cognitively necessary for proficiency in geometry" [6, p. 38]. This work aims to make contributions to the complex question of how to get ≈ 12-year-old students to understand and see the connections between these processes, in a given content of their curriculum.

2.1 Dynamic Geometry Tools

Dynamic geometry systems (DGS) have stimulated investigations into students' conceptions of mathematical demonstrations. There are studies that prove that this contribution of the DGS is twofold. First, they provide environments in which students can experiment freely, easily verifying their intuitions and conjectures in the process of looking for patterns, general properties, etc. Second, they provide non-traditional ways for students to learn and understand mathematical concepts and methods [21]. One of the advantages of DGS consists of carrying out tasks, not only for exploring geometric situations, but also for investigating situations that the tool itself promotes when moving objects, thus providing valuable support, both for students and for teachers. From the several DGS available to support geometry learning, we can highlight: Cabri [20], C.a.R. [12], Cinderella [24], GeoGebra [14], GeometerSketchpad [15] and JGEx [26].

2.2 Geometry Automated Theorem Provers

Automated deduction in geometry has been, since 1960s, an important field in the area of automated reasoning. Various methods and techniques have been studied and developed for automatically proving and discovering geometric theorems [22]. Focusing in the DGS/GATP platforms, i.e., platforms that combine the DGS with one (or several) GATP(s) we can highlight (some have been mentioned already): Cinderella, with a *randomised prover*; GCLC [16], which include several provers (area method, Wu's method and Gröbner basis method) [17]; GeoGebra, which include several algebraic provers [18, 19] and JGEx which include several provers (area method, full-angle method, deductive databases method, Wu's method and Gröbner basis method). In the last of these systems, the JGEx (*Java Geometry Expert*), three components can be identified: the dynamic geometry component, the automated deduction component, and the most distinctive component, the generation of visually dynamic presentation of proofs in plane geometry.

The JGEx will be used in the following sections, specifically its GATP based in the deductive database method.

3 A Rule-based Geometry Automated Theorem Prover

Since the early attempts, linked to artificial intelligence, synthetic provers based on inference rules and using forward chaining reasoning has been seen has a more suited approach for education. Using an appropriated set of rules and using forward chaining they can more easily mimic the expected behaviour of a student when developing a proof [2, 22].

The JGEx geometry deductive database method prover is an efficient implementation of a synthetic method, based on a set of inference rules. For a given geometric configuration, the program can find its fix-point with respect to a fixed set of geometric inference rules, in other words, it can find all the properties of the geometric construction (conjecture) that can be deduced using those rules [4].

After finding the fix-point, if the conjecture is among all the deduced facts, the conjecture is proved. A synthetic proof can then be generated with a natural language and visual renderings. The method is not complete, if the conjecture is not among the deduced facts it does not mean that it is not a theorem, it means that a different method, a decision procedure (e.g. the area method), must be used.

The algorithm is simple, a data-based search strategy is used, a list of "new data" is kept and for each new data the system searches the rule set to find and apply the rules using this data (see Figure 1) [4].

In the original implementation [4] and in a new implementation being finished [1], a rule set based on the full-angle method [3] is used. Both implementation are open source software, available in GitHub servers.[1]

A different set of rules can be used, keeping the efficient database search strategy. In the following, the application of a set of rules [9, 10], more adapted to the 7th year ($\approx$ 12-year-old students) in the Portuguese curricula [5, 7, 8] is presented and its application to the demonstration of formal proof of some geometric theorems is described.

3.1 A Set of Rules for the 7th Year

The set of rules presented here is contained in the set of rules implemented in the tutorial system *QED-Tutrix* [9, 10, 11].[2] Its formalisation in the *TPTP FOF-format* is presented here[3]. This set of rules is to be added to the set of rules used to implement the deductive database prover [1, 4], notice that some of them

[1]JGEx: `https://github.com/yezheng1981/Java-Geometry-Expert`, OGP-GDDM: `https://github.com/opengeometryprover/OpenGeometryProver`

[2]Only the rules needed in the proofs were chosen.

[3]`http://tptp.cs.miami.edu/TPTP/QuickGuide/Problems.html`

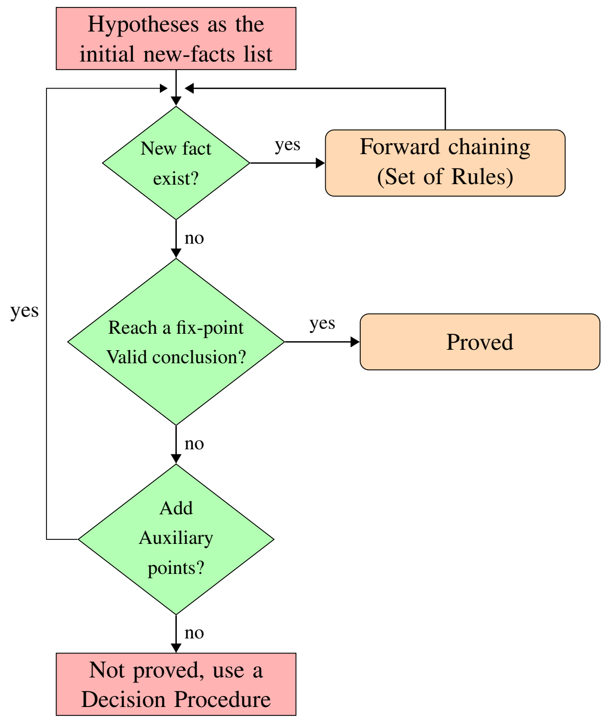

Figure 1: Rule-based Geometry Automated Theorem Prover

are already taken (coincide) with rules presented in that set of rules. The *FOF format* is chosen because it is the format used in the new implementation of the deductive database method already mentioned above [1]. The geometric predicates[4] are those described in [4], with some additions whose meaning should be clear.

R1 (definition of parallelogram) A quadrilateral $[ABCD]$ is a parallelogram iff AB is parallel to CD and BC is parallel to AD.

```
fof(ruleR1,axiom,(![A,B,C,D] :
          (para(A,B,D,C) & para(A,D,B,C) => parallelogram(A,B,C,D)) )).
fof(ruleR1a,axiom,(![A,B,C,D] :
          (parallelogram(A,B,C,D) => para(A,B,D,C)) )).
fof(ruleR1b,axiom,(![A,B,C,D] :
          (parallelogram(A,B,C,D) => para(A,D,B,C)) )).
```

R2 If two lines are parallel, the alternate interior angles determined by a transversal are equal. This is rule D40 of the deductive database method [4].

```
fof(ruleD40,axiom,(![A,B,C,D,P,Q] :
           (para(A,B,C,D) => eqangle(A,B,P,Q,C,D,P,Q)) )).
```

R3 (a.s.a. criterion of equality of triangles). This is rule D61 of the deductive database method [4].

```
fof(rulerD61,axiom,(![A,B,C,P,Q,R] :
            (simtri(A,B,C,P,Q,R) & cong(A,B,P,Q) => contri(A,B,C,P,Q,R)) )).
```

[4]para = parallel; perp = perpendicular; eqangle = equal-(full)angles; cong = congruent segments; simtri = similar triangles; contri = congruent triangles; coll = collinear

R4 Given two equal triangles $[ABC]$ and $[DEF]$, the sides and the corresponding angles are equal $\overline{AB} = \overline{DE}$, $\overline{BC} = \overline{EF}$, $\overline{CA} = \overline{FD}$, and for the angles $B\widehat{A}C = E\widehat{D}F$, $C\widehat{B}A = F\widehat{E}D$ and $A\widehat{C}B = D\widehat{F}E$.

```
fof(ruleR4,axiom,(![A,B,C,P,Q,R] :
            (cong(A,B,P,Q) & cong(A,C,P,R) & cong(B,C,Q,R)
            => contri(A,B,C,P,Q,R)) )).
fof(ruleR4a,axiom,(![A,B,C,P,Q,R] :
            (contri(A,B,C,P,Q,R) => cong(A,B,P,Q) ) )).
fof(ruleR4b,axiom,(![A,B,C,P,Q,R] :
            (contri(A,B,C,P,Q,R) => cong(A,C,P,R)) )).
fof(ruleR4c,axiom,(![A,B,C,P,Q,R] :
            (contri(A,B,C,P,Q,R) => cong(B,C,Q,R)) )).
```

R5 (definition of rectangle) A quadrilateral $[ABCD]$ is a rectangle iff the interior angles are all right angles.

```
fof(ruleR5,axiom,(![A,B,C,D] :
            (rightangle(D,A,A,B) & rightangle(A,B,B,C) &
             rightangle(B,C,C,D) & rightangle(C,D,D,A)
            => rectangle(A,B,C,D)) )).
fof(ruleR5a,axiom,(![A,B,C,D] :
            (rectangle(A,B,C,D) => rightangle(D,A,A,B)) )).
fof(ruleR5b,axiom,(![A,B,C,D] :
            (rectangle(A,B,C,D) => rightangle(A,B,B,C)) )).
fof(ruleR5c,axiom,(![A,B,C,D] :
            (rectangle(A,B,C,D) => rightangle(B,C,C,D)) )).
fof(ruleR5d,axiom,(![A,B,C,D] :
            (rectangle(A,B,C,D) => rightangle(C,D,D,A)) )).
fof(ruleR5e,axiom,(![A,B,C,D] :
            (rectangle(A,B,C,D) => para(A,B,D,C)) )).
fof(ruleR5f,axiom,(![A,B,C,D] :
            (rectangle(A,B,C,D) => para(A,D,B,C)) )).
```

R6 If the angle ABC is a right angle then the lines AB and BC are perpendicular.

```
fof(ruleR6,axiom,(![A,B,C] :
            (rightangle(A,B,B,C) => perp(A,B,B,C)) )).
```

R7 Two lines perpendicular to a third line are parallel to each other. This is rule D9 of the deductive database method [4].

```
fof(ruleD9,axiom,(![A,B,C,D,E,F] :
            (perp(A,B,E,F) & perp(C,D,E,F) => para(A,B,C,D) )).
```

R8 Two right triangles with two equal legs have equal hypotenuses.

```
fof(ruleR8,axiom,(![A,B,C,D,E,F] :
             (rightangle(A,B,B,C) & rightangle(D,E,E,F) &
              cong(A,B,D,E) & cong(B,C,E,F) => cong(A,C,D,F)) )).
```

D58 A rule, from the deductive database method [4], needed for the proof of theorem 1.

```
fof(ruleD58,axiom,(![A,B,C,P,Q,R] :
            (eqangle(A,B,B,C,P,Q,Q,R) & eqangle(A,C,B,C,P,R,Q,R) &
             ~coll(A,B,C)) => simtri(A,B,C,P,Q,R)) )).
```

4 Some 7th Year Geometry Problems

In a step-wise learning, it is intended that there will be a progressive proficiency in the use of hypothetical-deductive reasoning and mathematical argumentation in primary and secondary school. It is expected that, by the 7th year, students will be able to elaborate, with some accuracy, small demonstrations [5].

To put this into practice the most appropriate domain is "Geometry and Measurement" [5] or "Geometry" in the new curriculum document [8]. In the seventh grade, as part of the study of quadrilaterals, some properties of quadrilaterals and their diagonals are studied. Using some concepts and results already learned, equality of triangles, internal alternate angles, corresponding angles, among others, students can and should be able to demonstrate some of these properties, once they are armed with all the necessary tools. The following theorems illustrate some of these properties, and are part of a larger set of results that can be achieved.

It should also be noted that in this topic the use of technological tools can be very valuable, in the drawing, in the execution of rigorous constructions, in the visualisation of objects and their properties, both by teachers and by students, when this use is done in a thoughtful way.

The notation and terminology used in theorems and their proofs are that defined in [5] and currently used in Portuguese textbooks. The geometric constructions, conjectures and proofs were made with JGEx[5] (see Fig. 2–3), the informal demonstrations and the corresponding excerpts of formal demonstrations were written by the authors, it is expected that those demonstrations can be produced automatically by the new GATP being finished (see Sec. 6). Given that the proofs were done using the full-angle rules [3, 4], there are two different notions of angles in use, the "normal" angle, defined by two semi-lines, are used in the informal rules, the full-angles, defined by two lines, are used in the formal rules. The conversion between the two definitions are to be dealt by the process of rendering the formal proof in an informal natural language form. Nevertheless this conversion it is not without problems, for example, the known rule of triangles congruence, side-angle-side (s.a.s.) rule, is not correct if full-angles are used [4].

Theorem 1 (Opposite Sides of a Parallelogram)**.** *If* $[ABCD]$ *is a parallelogram then the opposite sides are equal, i.e.* $\overline{AB} = \overline{CD}$ *and* $\overline{AD} = \overline{BC}$ *(see Figure 2).*

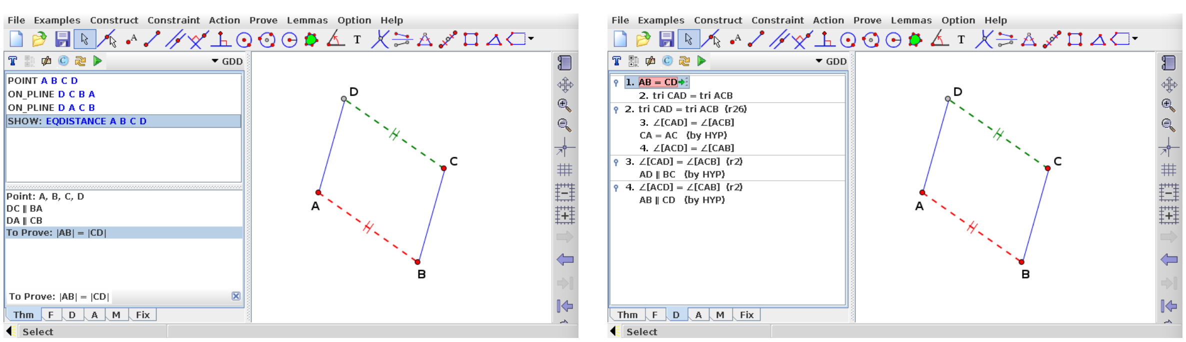

Figure 2: Opposite Sides of a Parallelogram, in JGEx

[5]The rule numbering used by the JGEx prover, e.g. "(r26)" are in no direct correspondence with the numbering of the rules described in this paper.

Proof. $[ABCD]$ is a parallelogram, by rule R1 (parallelogram definition), the lines AB and CD are parallel and the lines AD and BC are also parallel.

By rule R2, since the lines AB and CD are parallel, the angles BAC and DCA are equal.

By rule R2, since the lines AD and BC are parallel, the angles ACB and CAD are equal.

Since BAC and DCA are equal angles, ACB and CAD are equal angles and, $\overline{AC} = \overline{CA}$, by rule R3 (a.s.a. criterion of equality), triangles $[ABC]$ and $[CDA]$ are equal.

Finally, using rule R4, we have $\overline{AB} = \overline{CD}$ and $\overline{BC} = \overline{DA}$. □

Excerpt of a Formal Proof done by a Rule-based GATP. As described in section 3 a deductive database method prover works by forward chaining on the set of rules, starting on the conjecture and managing two sets of facts, "new facts" and "old (already known) facts".

The following table tries to illustrate the way the prover would work in order to reach the intended conclusion. Many other facts would be generated by the application of the deductive database method rules, in its way to reach the fix-point. The presented chain of facts and rules would be obtained working backwards, from the conclusion to the conjecture, and it would be used to produce a natural-language, informal proof description. To avoid a longer list of facts then necessary, the old facts list was written cumulatively from the start, i.e. only the new additions to the list are written.

```
include('geometryDeductiveDatabaseMethod.ax').
fof(theorem1,conjecture,(![A,B,C,D] :
                    parallelogram(A,B,C,D) => cong(A,B,C,D) & cong(A,D,B,C) )).
```

New Facts	Rules	Already Known Facts	ndg.[6]
parallelogram(A,B,C,D)	by hyp.		
para(A,B,C,D) para(A,D,B,C)	R1a,R1b	parallelogram(A,B,C,D)	
eqangle(A,B,B,C,C,D,D,A) eqangle(A,C,B,C,C,A,D,A)	D40 (×2)	para(A,B,C,D) para(A,D,B,C)	
simtri(A,B,C,C,D,A) cong(B,D,B,D)[7]	D58	eqangle(A,B,B,C,C,D,D,A) eqangle(A,C,B,C,C,A,D,A)	¬coll(A,B,C)
contri(A,B,C,C,D,A)	D61	simtri(A,B,C,C,D,A)	¬coll(A,B,C)
cong(A,B,C,D) cong(D,A,B,C)	R4a,R4b	contri(A,B,C,C,D,A)	¬coll(A,B,C)

□

The formal proof, done by an actual implementation of the deductive database method would have more steps, e.g. trivial steps, changing the order of the letters to be able to apply the rules. The non-degenerated conditions (ndgs.) would be added automatically by the prover, they must be added to the hypothesis to ensure that the conjecture is true. All this would lead to an informal (rigorous) language rendering of the formal proof, hiding the trivial steps (maybe with an explanation), explaining the need of the non-degenerate conditions, and the conversion of the formal rules in a «normal» secondary schools language, e.g. rule R3/D61 wrote as the angle-side-angle (a.s.a.) rule for the congruence of triangles.

Theorem 2 (Diagonals of a Rectangle). *If $[ABCD]$ is a rectangle then its diagonals are equal, i.e. $\overline{AC} = \overline{BD}$ (see Figure 3).*

[6] ndg. — non-degenerated conditions.

[7] Trivial fact that it is automatically added to apply the inference rule.

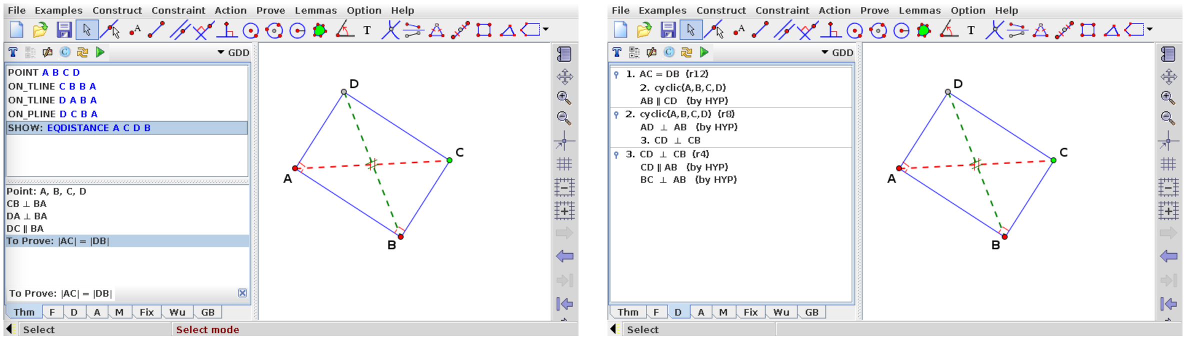

Figure 3: Diagonals of a Rectangle, in JGEx

Proof. If $[ABCD]$ is a rectangle by rule R5 the angles BAD, CBA, DCB and ADC are all right angles.

By rule R6, the lines AB and AD are perpendicular, the lines BC and AB are perpendicular, the lines BC and CD are perpendicular and also the lines AD and CD are perpendicular.

By rule R7, since AB and CD are both perpendicular to AD, AB and CD are parallel, and since AD and BC are both perpendicular to AB, AD and BC are parallel. ([ABCD] is a parallelogram, by rule R1)

Repeating the steps of the proof of theorem 1 proof, we have $\overline{AD} = \overline{BC}$.

Since $\overline{AD} = \overline{BC}$, BAD and CBA are both right angles and $\overline{AB} = \overline{BA}$, by rule R8 (s.a.s. criterion of equality of right triangles), the triangles $[ABD]$ and $[BAC]$ are equal.

Finally, using R4, we have $\overline{AC} = \overline{BD}$. □

```
include('geometryDeductiveDatabaseMethod.ax').
fof(theorem2,conjecture,(![A,B,C,D] : rectangle(A,B,C,D) => cong(A,C,B,D) )).
```

Excerpt of a Formal Proof done by a GDDM prover.

New Facts	**Rules**	**Already Known Facts**	**ndg.**
`rectangle(A,B,C,D)`	by hyp.		

`rightangle(D,A,A,B)`
`rightangle(A,B,B,C)`
`para(A,B,D,C)`
`para(A,D,B,C)`
	R5a,R5b,R5e,R5f	`rectangle(A,B,C,D)`	

`rightangle(D,A,A,B)`
`rightangle(A,B,B,C)`
`parallelogram(A,B,C,D)`
	R1	`para(A,B,D,C)`
		`para(A,D,B,C)`	

`rightangle(D,A,A,B)`
`rightangle(A,B,B,C)`
`cong(A,B,C,D)`
`cong(A,D,B,C)`
`cong(A,B,A,B)`[8]
	Th. 1	`parallelogram(A,B,C,D)`	

`cong(A,B,C,D)`
`cong(A,C,B,D)`
	R8	`rightangle(D,A,A,B)`
		`rightangle(A,B,B,C)`
		`cong(A,D,B,C)`
		`cong(A,B,A,B)`	

□

As can be seen in figure 3 the proof done by JGEx uses a different approach. It uses the fact that the points *A*, *B*, *C* and *D* are concyclic, and the diagonals of the rectangle are diameters of the circle. This is not an approach suitable for the intended school level, but, if JGEx is used that would be the proof produced by it. An implementation of the deductive database method should allow the selection of a given set of rules, such possibility will be discussed in the final section (see Section 6).

5 Lesson Plan

A lesson plan must reflect what will be proposed to students. Here an outline of a lesson plan to address theorem 1 and its formal proof by students in a classroom is presented. See appendix A for the lesson plan in full details.

The essential learning goals [8] that this plan aims at are: explore, in a dynamic geometry environments (e.g. GeoGebra), convex polygons with different numbers of sides; Formulate conjectures, generalisations and justifications, based on the identification of regularities common to the objects under study; establish conjectures in dynamic geometry environments and their exploration.

From those goals follow a set of tasks in which the student can conjecture about the various properties that are explored in the DGS.

Task 1 - Quadrilaterals, length of sides and measure of angles

- Build a quadrilateral.
- Determine the length of the sides.
- Determine the measure of angles.

Task 2 - Parallelogram

- Build three parallelograms.
- Determine lengths of sides of those parallelograms
- What can you conjecture about the sides of a parallelogram?

Task 3 - Prove the conjecture. Follow the steps below:

- Build a parallelogram [*ABCD*]
- Construct a diagonal of the parallelogram.
- Use the parallelism of the sides of the parallelogram and indicate the angles that have the same measure (mark these angles in your construction and indicate their measure)
- Remember the a.s.a. criterion of equality of triangles, in the figure which triangles can you conclude are equal?

[8]Trivial fact that it is automatically added to apply the inference rule.

- Mark in the figure with the same colour the segments that have the same length.

As already said, the goal is to explore geometric conjectures and their proofs, with the help of DGS and GATP computational tools.

In the final part of the lesson and before its conclusion, teacher will show to the students the full proof of the result using JGEx. Using the preview windows it is possible to follow the demonstration step by step, see the illustration of each step, and see how the formal rules are being used. The teacher will have to adapt the formal language and the rules themselves into informal language closer to the language used by the students.

6 Conclusions and Future Work

This work aims to contribute to the complex question of how to get a $\approx$12-year-old student to understand and see the connections between visualisation, construction and reasoning, in particular, the discursive processes to broaden the knowledge processes, for demonstration and for interpretation.

The rules presented here (see Sec. 3.1) were written in a rigorous mathematical (natural) language, appropriated to be used in a classroom setting. Alongside those rules a corresponding set of formal rules, appropriated to a rule base theorem prover were also presented. Using the set of rules found by Font [9] and the set of rules presented by Chou et al. [4] as a starting point, our intention is to build a consistent set of rules that can narrow the dissonance between the outcomes of the geometry automated theorem provers and the normal practice of conjecturing and proving in schools.

The use of a rule-based theorem prover being developed within our research group [1] will allow to implement the set of rules which deem appropriated for the automation for one side and its use with the intended target audience. There are many challenges ahead: the implementation of an efficient prover; the possibility to separate the set of rules from the inference mechanisms, i.e. the possibility to adapt the set or rules to the task at hand without the need, each time, to build a different GATP; the natural language rendering, i.e. the transformation of the formal proof into a rigorous proof, appropriated to be used in a classroom setting; the connection between GeoGebra (a well-known, by the students and teachers, DGS) and the GATP, i.e. the possibility of a visual connection between the construction, the conjecture and the proof.

The lesson plan presented (see Sec. A) could be applied at an actual classroom. It is our intention to put into practice an experience in a classroom, first through an experiment with pre-service teachers and afterwords in an actual 7th year classrooms.

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A Lesson Plan

Subject: Geometry

Subtopic: Plane Figures

Summary: Properties of quadrilaterals: relationships between the lengths of the sides of a parallelogram. Using a dynamic geometry program to explore the tasks. Problem solving. Formal proof.

Contents: Quadrilaterals and interior angles.

Background

- Parallelogram definition.
- Angles determined by two parallel lines intersected by a secant (alternate interior angles).
- Triangle equality criteria.
- Knowledge of using various tools including digital technology, namely specific computer programs (GeoGebra), in exploring properties of plane figures.

Essential Learning

Explore, in dynamic geometry environments, e.g. GeoGebra, convex polygons with different numbers of sides.
Formulate conjectures, generalisations and justifications, based on the identification of regularities common to the objects under study.
Establish conjectures in dynamic geometry environments, e.g. GeoGebra, and their exploration.

Methodology/Strategy

Dialogue exposition. Exploration of dynamic geometry environments. Generate individual and group work opportunities.

Teaching resources/materials

- Computer
- Dynamic geometry system
- Projector
- PowerPoint to support tasks to be proposed to students
- Activity sheet "Let's explore the parallelogram"

Classroom organisation
The organisation of the classroom space is in a computer room.

Class Description - Class Strategies and Development

- Presentation of the task and the way of working in class (10 minutes)

After the students enter the classroom, the lesson is opened and the summary is written on the board. Students record the summary in their daily notebook. Then the teacher will propose to them the task "Let's explore the parallelogram". The teacher will invite students to form pairs and choose a computer to work on. The teacher explains to the students that they will solve tasks, in a previously available form, in a group on the computer.

The dynamic geometry environment will be explored by performing the following tasks (total of 50 minutes):

Task 1 - Quadrilaterals, length of sides and measure of angles (10 minutes).

- Build a quadrilateral.
- Determine the length of the sides.
- Determine the measure of angles.

Task 2 - Parallelogram (10 minutes).

- Build three parallelograms.
- Determine lengths of sides of those parallelograms
- What can you conjecture about the sides of a parallelogram?

Task 3 - Prove the conjecture (30 minutes). Follow the steps below:

- Build a parallelogram $[ABCD]$.
- Construct a diagonal of the parallelogram.
- Use the parallelism of the sides of the parallelogram and indicate the angles that have the same measure (mark those angles in your construction and indicate their measure).
- Remember the a.s.a. criterion for equality of triangles, in the figure which triangles can you conclude are equal?
- Mark with the same colour, in the figure, the segments that have the same length.

Complete the following sentence:

"In a parallelogram, . . . sides are . . . "

- Task discussion (10 minutes)
- Synthesis and formal proof (20 minutes)

A synthesis will be carried out in the last part of carrying out the tasks. In fact, this will be done at the same time as the discussion.

In the final part of the lesson and before its conclusion, the teacher will show to the students the full proof of the result using JGEx. Through the preview windows it is possible to follow the demonstration step by step, see the illustration of each step, and see how the formal rules are being used. The teacher will have to adapt the formal language and the rules themselves into informal language closer to the language used by the students.

Time schedule

- Task introduction: 10 minutes
- Autonomous task work: 50 minutes
- Discussion: 10 minutes
- Synthesis and formal proof: 20 minutes