Computer-Assisted Program Reasoning Based on a Relational Semantics of Programs*

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We present an approach to program reasoning which inserts between a program and its verification conditions an additional layer, the denotation of the program expressed in a declarative form. The program is first translated into its denotation from which subsequently the verification conditions are generated. However, even before (and independently of) any verification attempt, one may investigate the denotation itself to get insight into the "semantic essence" of the program, in particular to see whether the denotation indeed gives reason to believe that the program has the expected behavior. Errors in the program and in the meta-information may thus be detected and fixed prior to actually performing the formal verification. More concretely, following the relational approach to program semantics, we model the effect of a program as a binary relation on program states. A formal calculus is devised to derive from a program a logic formula that describes this relation and is subject for inspection and manipulation. We have implemented this idea in a comprehensive form in the RISC ProgramExplorer, a new program reasoning environment for educational purposes which encompasses the previously developed RISC ProofNavigator as an interactive proving assistant.

1 Introduction

Most systems for program reasoning are based on calculi such as the Hoare Calculus or Dynamic Logic [4] where we generate from a program specification and a program implementation (which is annotated with additional meta-information such as loop invariants and termination terms) those conditions whose verification implies that the implementation indeed meets the specification. The problem is that by such an approach we gain little insight into the program before respectively independently of the verification process. In particular, if the verification attempt is a priori doomed to fail because of errors, inconsistencies, or weaknesses in the program's specification, implementation, or meta-information (which is initially the case in virtually all verification attempts), we will learn so only by unsuccessfully struggling with the verification until some mental "click" occurs. This click occurs frequently very late, because, in the heat of the struggle, it is usually hard to see whether the inability to perform a correctness proof is due to an inadequate proving strategy or due to errors or inconsistencies in the program. Actually, it is usually the second factor that contributes most to the time spent and frustration experienced; once we get the specification/implementation/meta-information correct, the verification is a comparatively small problem. We have frequently observed this fact in our own verification attempts as well as in those performed by students of computer science and mathematics in courses on formal methods.

We therefore advocate an alternative approach where we insert between a program and its verification conditions an additional layer, the denotation of the program [21] expressed in a declarative form. The program (annotated with its meta-information) is translated into its denotation from which subsequently

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the verification conditions are generated. However, even before (and independently of) any verification attempt, one may investigate the denotation itself to get insight into the "semantic essence" of the program (independently of its "syntactic surface"), in particular to see whether the denotation indeed gives reason to believe that the program has the expected behavior. Errors in the program and in the meta-information may thus be detected and fixed prior to actually performing the formal verification.

More concretely, following the relational approach to program semantics [16], we model the effect of a program (command) c as a binary relation $[\![c]\!]$ on program states which describes the possible pairs of pre- and post-states of c. Such a relation can be also described in a declarative form by a logic formula f_r with denotation $[\![f_r]\!]$. Thus a formal calculus is devised to derive from a program c a judgment $c : f_r$ such that $[\![c]\!] \subseteq [\![f_r]\!]$. For instance, we can derive `x=x+1` $: \mathsf{var}\ x = \mathsf{old}\ x + 1$ where the logic variable $\mathsf{old}\ x$ refers to the value of the program variable `x` in the prestate of the command and the logic variable $\mathsf{var}\ x$ refers to its value in the poststate. In this way, we can constrain the allowed state transitions, i.e. handle the partial correctness of programs. To capture also total correctness, we introduce the set of states $\langle\!\langle c \rangle\!\rangle$ on which the execution of c must terminate ($\langle\!\langle c \rangle\!\rangle$ is a subset of the domain of $[\![c]\!]$). Such a set can be also described in a declarative form by a logic formula (a state condition) f_c. Thus we derive a judgment $c \downarrow f_c$ such that $[\![f_c]\!] \subseteq \langle\!\langle c \rangle\!\rangle$. In this fashion, the pair of formulas f_r and f_c captures the semantic essence of c in a declarative form that is open for inspection and manipulation.

We have implemented this idea in a comprehensive form in the *RISC ProgramExplorer*[1], a new program reasoning environment for educational purposes which encompasses the previously developed *RISC ProofNavigator* as an interactive proving assistant [24]. The RISC ProgramExplorer supports reasoning about programs written in a restricted form of Java (including support for control flow interruptions such as `continue`, `break`, `return`, and `throw`, static and dynamic methods, classes and a restricted form of objects) and specified in the formula language of the RISC ProofNavigator (whose syntax is derived from PVS [20]). A first version of the system has been released under the GNU Public License in October 2011; it will be subsequently used in a regular course on formal methods.

The remainder of this paper is structured as follows: Section 2 discusses related work. Section 3 sketches the theoretical foundations of our approach, i.e. how programs are translated into their semantic essence. Section 4 presents the implementation of this approach in the RISC ProgramExplorer. Section 5 illustrates the use of the software by a detailed example. Section 6 describes how from the semantic essence the verification conditions are derived that show the correctness of a program with respect to its specification. In Section 7, we describe the typical workflow supported by the system; in Section 8, we conclude and discuss further work.

2 Related Work

There exist numerous frameworks and tools for reasoning about programs written in various languages. In the context of the programming language Java, the Java Modeling Language (JML) has become the de facto standard specification language [7]; it extends the syntax and semantics of the Java expression language to a mathematical formula language that is rich enough to formulate method contracts. Various tools aim at reasoning about JML-annotated programs in a fully automatic way, mainly in order to falsify programs (find runtime errors or violations of method preconditions) rather than to verify them. An environment for true verification is the system Why/Krakatoa [11] which translates JML-annotated programs into the input language of the verification condition generator Why that produces verification conditions for a variety of external provers such as the interactive proving assistant Coq. Likewise, the

[1] `http://www.risc.jku.at/research/formal/software/ProgramExplorer`

formal software development tool KeY [4] allows to verify JML-specified programs written in JavaCard (a subset of Java) based on the framework of dynamic logic (a modal logic whose modalities integrate program statements) using a built-in interactive prover. In contrast to these approaches, the approach presented in this paper is based on a semantic model that is visible for human inspection, reasoning is essentially based on classical predicate logic and on a formula/specification language that is independent of any programming language; the supported programming language has also Java-like syntax but is not fully object-oriented (currently no inheritance is supported).

The idea to translate computer programs to mathematical objects that describe the meaning of the programs originates from the work of Scott and Strachey in the 1960s and is now known under the name "denotational semantics". In [21], this approach is presented as a methodology for language design which also helps to understand concrete programs by investigating the mathematical objects to which they are translated. In the classical Scott/Strachey approach, programs are translated to functions; loops are translated to recursive function equations over semantic domains with special properties (pointed complete partial orders) that ensure the existence and uniqueness of the function (as the least fixed point of the defining equation). In contrast to the classical approach, in our model programs are translated to relations described by formulas that do not necessarily imply existence and uniqueness of the result (as the formula is in general derived from user-provided method contracts and loop invariants).

The idea of modeling programs as state relations is not new; it has emerged in various approaches to the formalization of programs: it is for example the core idea of the Lamport's "Temporal Logical of Actions" (TLA) [16] where the individual actions of a process are described by formulas relating pre- to post-states; the language PlusCal for the formulation of algorithms is translated to TLA specifications. Boute's "Calculational Semantics" [6] defines the behavior of a program by equations relating the pre-state to the post-state. Hoare and Jifeng's "Unifying Theories of Programming" [15] provides an integrated theory of programming based on a view of programs as relations. Apart from TLA (which provides a model checker for a final state subset of the specification language), we are not aware of any software tools or verification environments that are directly based on relational frameworks.

Also the principle of "correct by construction", which emphasizes the gradual refinement of a specification into an executable program such that at any stage of the refinement the program is provably correct, is usually based on a relational view of programs. This approach was originally pioneered by Dijkstra [9] and further elaborated by Back's "refinement calculus" [1] and other formalisms of a similar flavor [1, 18, 17, 14, 13]. In some way or another, these calculi allow declarative specifications to reside on the same language level as operational commands; the endorsed approach to program development is to gradually transform specifications to commands.

A recent variant of this principle is Back's "invariant-based programming" [2] where program invariants are constructed before the actual code is considered. This approach is implemented in the Socos environment which provides a graphical editor for the construction of invariant diagrams from which verification conditions are generated for the PVS prover. Although not directly based on these principles, also the approach presented in this paper can be applied in a top-down refinement fashion: since the transition relation of a program that executes a loop or calls a method is only constructed from the invariant of the loop respectively specification of the method, we may verify the correctness of the program before the body of the loop respectively method is implemented.

The appropriate way (didactic approach, formal calculus, tool support) of teaching formal methods is an ongoing point of debate, see e.g. the TFM conference series [8, 5, 12]. A comparative survey of formal methods courses in Europe is given in [19]; a comparison of tools for teaching program verification is presented in [10]. In general, most knowledge on formal methods education is based on personal experience reports; there is hardly any scientific evidence for the superiority of any particular approach.

3 Commands as State Relations

We base the presentation of our formalism on a simple command language without control flow interruptions and method calls. In this language, a command c can be formed according to the grammar

$$c ::= x\,\texttt{=}\,e \mid \texttt{\{var}\ x\texttt{;}\ c\texttt{\}} \mid \texttt{\{}c_1\,\texttt{;}c_2\texttt{\}} \mid \texttt{if (}e\texttt{) then}\ c \mid \texttt{if (}e\texttt{) then}\ c_1\ \texttt{else}\ c_2 \mid \texttt{while (}e\texttt{)}^{f,t}\ c$$

where x denotes a program variable, e denotes a program expression, and a while loop is annotated by an invariant formula f and termination term t. The semantics of a command c is defined, for a given set *Store* of possible states (store contents), by a binary relation $[\![c]\!] \subseteq \mathit{Store} \times \mathit{Store}$ that defines the possible state transitions of the command and by a set $\langle\!\langle c \rangle\!\rangle \subseteq \mathit{Store}$ that defines those pre-states where the command must perform a transition to some post-state; for a definition of the semantics, see [22].

In Figures 1, 2, and 3, we give rules (where the terms $\mathsf{old}\ xs$ and $\mathsf{var}\ xs$ refer to the sets of values of the program variables xs in the pre-/post-state) to derive the following three kinds of judgments:

- $c : [f_r]^{xs}_{g,h}$ denotes the derivation of a state relation f_r from command c together with the set of program variables xs that may be modified by c. The remaining arguments g and h express additional side-conditions (which may be ignored on first reading): the derived relation is correct if the derived state-independent condition g holds, and if the derived state condition h holds on the pre-state of c. The rationale for g is to capture state-independent conditions such as the correctness of loop invariants; the purpose of h is to capture statement preconditions that prevent e.g. arithmetic overflows. These side conditions have to be proved; they are separated from the transition relation f_r to make the core of the relation better understandable.
- $c \downarrow_{g_c} f_c$ denotes the derivation of a state condition (termination condition) f_c from c; the derived condition is correct, if the state-independent condition g_c holds. The purpose of this side condition is to capture that the execution of every loop body terminates and decreases the value of the termination term but does not make the value negative.
- $\mathrm{PRE}(c, f_q) = f_p$ and $\mathrm{POST}(c, f_p) = f_q$ denote derivations that compute from a command c and a condition f_q on the post-state of c a corresponding condition f_p on the pre-state, respectively from c and pre-condition f_p the post-condition f_q. The corresponding rules in Figure 3 show that these conditions can be computed directly from the transition relation of c.

The derivations use additional judgments $e \simeq_{f_e} f$ and $e \simeq_{f_e} t$ which translate a Boolean-valued program expression e into a logic formula f and an expression e of any other type into a term t, provided that the state in which e is evaluated satisfies the condition f_e (the rules for these judgments are omitted).

One should note that the rules presented in Figures 1 and 2 can be applied recursively over the structure of a command; first we determine the transition/termination formula of the subcommands, then we combine the formulas to the transition/termination formula of the whole command. Along this process, the side condition h is constructed which has to be shown separately to hold in the pre-state of the command in order to verify the correctness of the translation.

A special case is the rule for `while` loops. Here the result is only determined from the invariant formula respectively termination term by which the loop is annotated; additionally, a proof obligation g is generated to verify the correctness of the loop body with respect to invariant and termination term. In a similar way, in the full programming language calls of program methods are handled: the transition relation of the method call is derived from the specification of the method; the correctness of the implementation of the method is to be established separately. We thus yield a modular approach to the derivation of transition relations and termination conditions; the size of the derived formula is independent of the sizes of the bodies of the loops executed respectively of the methods called. Furthermore, as

$$\frac{c : [f]^{xs}_{g,h} \qquad x \notin xs}{c : [f \wedge \mathsf{var}\ x = \mathsf{old}\ x]^{xs\,\cup\,\{x\}}_{g,h}} \qquad\qquad \frac{e \simeq_h t}{x \texttt{ = } e : [\mathsf{var}\ x = t]^{\{x\}}_{\mathsf{true},h}}$$

$$\frac{c : [f]^{xs}_{g,h}}{\texttt{\{var}\ x\texttt{;}\ c\texttt{\}} : [\exists x_0, x_1 : f[x_0/\mathsf{old}\ x, x_1/\mathsf{var}\ x]]^{xs\setminus x}_{g,\forall x : h[x/\mathsf{old}\ x]}}$$

$$\frac{c_1 : [f_1]^{xs}_{g_1,h_1} \qquad c_2 : [f_2]^{xs}_{g_2,h_2} \qquad \mathrm{PRE}(c_1, h_2) = h_3}{\texttt{\{}c_1\texttt{;}c_2\texttt{\}} : [\exists ys : f_1[ys/\mathsf{var}\ xs] \wedge f_2[ys/\mathsf{old}\ xs]]^{xs}_{g_1 \wedge g_2, h_1 \wedge h_3}}$$

$$\frac{e \simeq_h f_e \qquad\qquad c_1 : [f_1]^{xs}_{g_1,h_1}}{\texttt{if (}e\texttt{) then}\ c : [\mathsf{if}\ f_e\ \mathsf{then}\ f_1\ \mathsf{else}\ \mathsf{var}\ xs = \mathsf{old}\ xs]^{xs}_{g_1, h \wedge (f_e \Rightarrow h_1)}}$$

$$\frac{e \simeq_h f_e \qquad c_1 : [f_1]^{xs}_{g_1,h_1} \qquad c_2 : [f_2]^{xs}_{g_2,h_2}}{\texttt{if (}e\texttt{) then}\ c_1\ \texttt{else}\ c_2 : [\mathsf{if}\ f_e\ \mathsf{then}\ f_1\ \mathsf{else}\ f_2]^{xs}_{g_1 \wedge g_2, h \wedge \mathsf{if}\ f_e\ \mathsf{then}\ h_1\ \mathsf{else}\ h_2}}$$

$$\frac{\begin{array}{c} e \simeq_h f_e \qquad\qquad c : [f_c]^{xs}_{g_c,h_c} \\ g \equiv \forall xs, ys, zs : f[xs/\mathsf{old}\ xs, ys/\mathsf{var}\ xs] \wedge f_e[ys/\mathsf{old}\ xs] \wedge f_c[ys/\mathsf{old}\ xs, zs/\mathsf{var}\ xs] \Rightarrow \\ h[ys/\mathsf{old}\ xs] \wedge f[xs/\mathsf{old}\ xs, zs/\mathsf{var}\ xs] \end{array}}{\texttt{while (}e\texttt{)}^{f,t}\ c : [f \wedge \neg f_e[\mathsf{var}\ xs/\mathsf{old}\ xs]]^{xs}_{g_c \wedge g, h \wedge f[\mathsf{old}\ xs/\mathsf{var}\ xs]}}$$

Figure 1: The Transition Rules

$$x \texttt{ = } e \downarrow_{\mathsf{true}} \mathsf{true} \qquad \frac{c \downarrow_g f}{\texttt{\{var}\ x\texttt{;}\ c\texttt{\}} \downarrow_g \forall x : f[x/\mathsf{old}\ x]} \qquad \frac{c_1 \downarrow_{g_1} f_1 \quad c_2 \downarrow_{g_2} f_2 \quad \mathrm{PRE}(c_1, f_2) = f_3}{\texttt{\{}c_1\texttt{;}c_2\texttt{\}} \downarrow_{g_1 \wedge g_2} f_1 \wedge f_3}$$

$$\frac{e \simeq_h f_e \qquad c \downarrow_g f}{\texttt{if (}e\texttt{) then}\ c \downarrow_g f_e \Rightarrow f} \qquad \frac{e \simeq_h f_e \qquad c_1 \downarrow_{g_1} f_1 \qquad c_2 \downarrow_{g_2} f_2}{\texttt{if (}e\texttt{) then}\ c_1\ \texttt{else}\ c_2 \downarrow_{g_1 \wedge g_2} \mathsf{if}\ f_e\ \mathsf{then}\ f_1\ \mathsf{else}\ f_2}$$

$$\frac{\begin{array}{c} e \simeq_h f_e \qquad c : [f_c]^{xs}_{g_c,h_c} \qquad c \downarrow_{g_t} f_t \\ g \equiv \forall xs, ys, zs : f[xs/\mathsf{old}\ xs, ys/\mathsf{var}\ xs] \wedge f_e[ys/\mathsf{old}\ xs] \wedge f_c[ys/\mathsf{old}\ xs, zs/\mathsf{var}\ xs] \Rightarrow \\ f_t[ys/\mathsf{old}\ xs] \wedge 0 \leq t[zs/\mathsf{old}\ xs] < t[ys/\mathsf{old}\ xs] \end{array}}{\texttt{while (}e\texttt{)}^{f,t}\ c \downarrow_{g \wedge g_t} t >= 0}$$

Figure 2: The Termination Rules

$$\frac{c : [f]^{xs}_{g,h}}{\mathrm{PRE}(c, f_q) = \forall xs : f[xs/\mathsf{var}\ xs] \Rightarrow f_q[xs/\mathsf{old}\ xs]}$$

$$\frac{c : [f]^{xs}_{g,h}}{\mathrm{POST}(c, f_p) = \exists xs : f_p[xs/\mathsf{old}\ xs] \wedge f[xs/\mathsf{old}\ xs, \mathsf{old}\ xs/\mathsf{var}\ xs]}$$

Figure 3: The Pre-/Postcondition Rules

already stated in Section 2, the approach gives rise to some sort of "correct by construction" approach: we may first develop loop invariants and method preconditions (and verify the correctness of programs executing the loops and calling the methods) before we implement the bodies of the loops and methods (and consequently verify the correctness of implementations).

Formally, the derivations satisfy the following soundness constraints.

Theorem 1 (Soundness) *For all $c \in Command, f_r, f_c, f_p, f_q, g, h \in Formula, xs \in \mathbb{P}(Variable)$, the following statements hold:*

1. *If we can derive the judgment $c : [f_r]^{xs}_{g,h}$, then we have for all $s, s' \in Store$*

$$[\![g]\!] \wedge [\![h]\!](s) \Rightarrow ([\![c]\!](s,s') \Rightarrow [\![f_r]\!](s,s') \wedge \forall x \in Variable \backslash xs : [\![x]\!](s) = [\![x]\!](s')).$$

2. *If we can (in addition to $c : [f_r]^{xs}_{g,h}$) derive the judgment $c \downarrow_{g_c} f_c$, then we have for all $s \in Store$*

$$[\![g]\!] \wedge [\![g_c]\!] \wedge [\![h]\!](s) \Rightarrow ([\![f_c]\!](s) \Rightarrow \langle\!\langle c \rangle\!\rangle(s)).$$

3. *If we can (in addition to $c : [f_r]^{xs}_{g,h}$) also derive the judgment $\mathrm{PRE}(c, f_q) = f_p$ or the judgment $\mathrm{POST}(c, f_p) = f_q$, then we have for all $s, s' \in Store$*

$$[\![g]\!] \wedge [\![h]\!](s) \Rightarrow ([\![f_p]\!](s) \wedge [\![f_r]\!](s,s') \Rightarrow [\![f_q]\!](s')).$$

The semantics $[\![f]\!](s,s')$ of a transition relation f is determined over a pair of states s, s' (and a logic environment, which is omitted for clarity); the semantics of state condition g is defined as $[\![g]\!](s) \Leftrightarrow \forall s' : [\![g]\!](s,s')$ and the semantics of a state independent-condition h is defined as $[\![h]\!] \Leftrightarrow \forall s, s' : [\![h]\!](s,s')$.

In [23], the formal semantics of commands and formulas has been defined and the soundness of (a preliminary form of) the calculus has been proved. In [22], a concise definition of the semantics, judgments, and rules of (a preliminary form of) the calculus is given.

4 The RISC ProgramExplorer

We have implemented the calculus presented in the previous section in the RISC ProgramExplorer; a screenshot of the software is given in Figure 4.

The RISC ProgramExplorer supports reasoning about programs written in a subset of Java which we call "MiniJava". This subset includes classes with class and object variables as well as class and object methods and constructors. Method bodies may execute most kinds of Java commands including those that cause interruptions of the control flow (`continue`, `break`, `return`, `throw`). The major restriction compared with full Java is that the type checker prevents sharing of objects/arrays by different variables (such that data structures can be modeled as plain values rather than as pointer structures) and that inheritance is not supported; furthermore expressions are not allowed to cause side effects (thus method calls with result values have to be written as separate commands).

The software supports a theory definition language that is derived from the language of the previously developed RISC ProofNavigator [24]; the syntax of the language is inherited from PVS [20] respectively CVC Lite [3]. The language allows to introduce new theories consisting of types, objects, functions, and predicates as well as axioms and theorems (which can be proved with the RISC ProofNavigator). Theories may be specified in separate files for reuse in different programs or may be attached to the program classes where they are used.

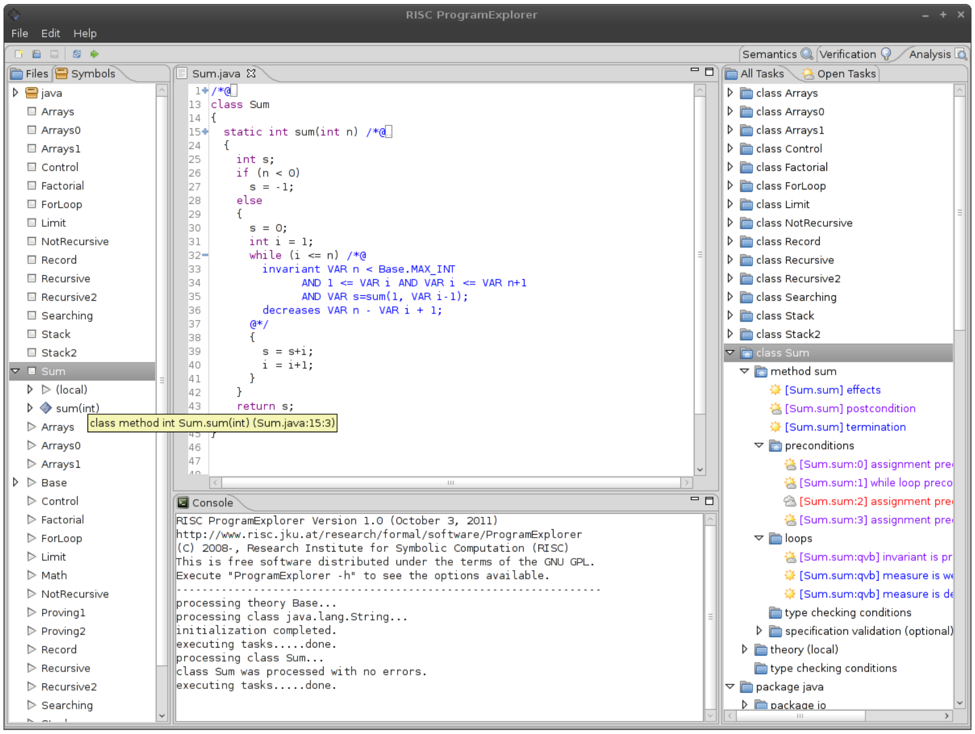


Figure 4: The RISC ProgramExplorer (Analysis View)

Based on these theories, programs may be formally specified by class invariants, method contracts, loop invariants, termination terms, and assertions in the style of the Java Modeling Language [7]. The syntax and semantics of the formula language is, however, deliberately neutral of the programming language (because we believe that mathematics precedes programming and the same mathematical definitions and specifications should be reusable for different programming languages). The specifications operate therefore directly on the semantic model of a program (e.g. every program class C is translated to a theory C that contains among other definitions a record type C; if x is a program variable that denotes an object of class C, then in a specification var x denotes a record of type C).

The RISC ProgramExplorer provides an elaborated graphical user interface that links theories, programs, semantic models, and verification tasks by three main views:

Analysis This is the central view in which mathematical theories and program classes may be developed (see Figure 4); upon saving the corresponding file, the theory respectively program is type-checked, the semantic model is constructed, and verification tasks are generated. If an error occurs during this semantic processing, the error is linked to the corresponding location in the source code; likewise the generated tasks are linked to the corresponding source locations. In detail, the following verification tasks are generated:

Effects The proof that the method does not modify any variable outside the specified set of variables and does not throw any unspecified exception.

Postcondition The proof that the transition relation derived from the body of a method implies the method's postcondition.

Termination The proof that a method's termination condition implies the termination condition

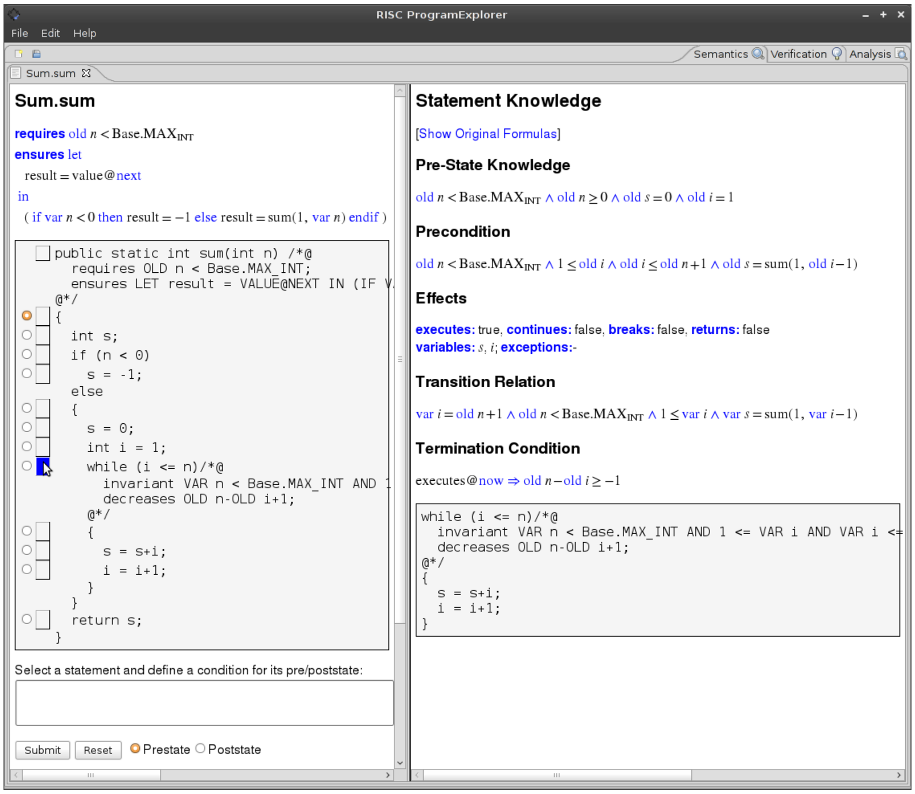


Figure 5: Semantics View

derived from the body of the method.

Preconditions The proofs that every statement is only executed in a state that satisfies the precondition of the statement.

Loops The proofs that a loop body preserves the invariant, that the execution of the body terminates and decreases the value of the specified termination term, and that the decreased value does not become negative.

Type checking conditions The proofs that all formulas are well-typed (not all type checking questions of the formula language can be statically answered).

Specification validation The (optional) proofs that a specification is satisfiable (for every argument that satisfies the precondition there is a result that satisfies the postcondition) but not trivially satisfiable (there is also a result that does not satisfy the postcondition).

Splitting the overall task of proving the correctness of a method into individual subtasks supports the gradual verification of different aspects of correctness and gives more concrete hints in the case of failed proof attempts. Some of the tasks can be fully automatically solved by the validity checker; if the checker fails, the user can start a semi-automatic interactive proof.

Semantics In this view displayed in Figure 5, the semantic model of a selected program method may be investigated. By moving the mouse pointer, the user may display for each command of the method body (respectively for the whole body)

- the transition relation of the command,

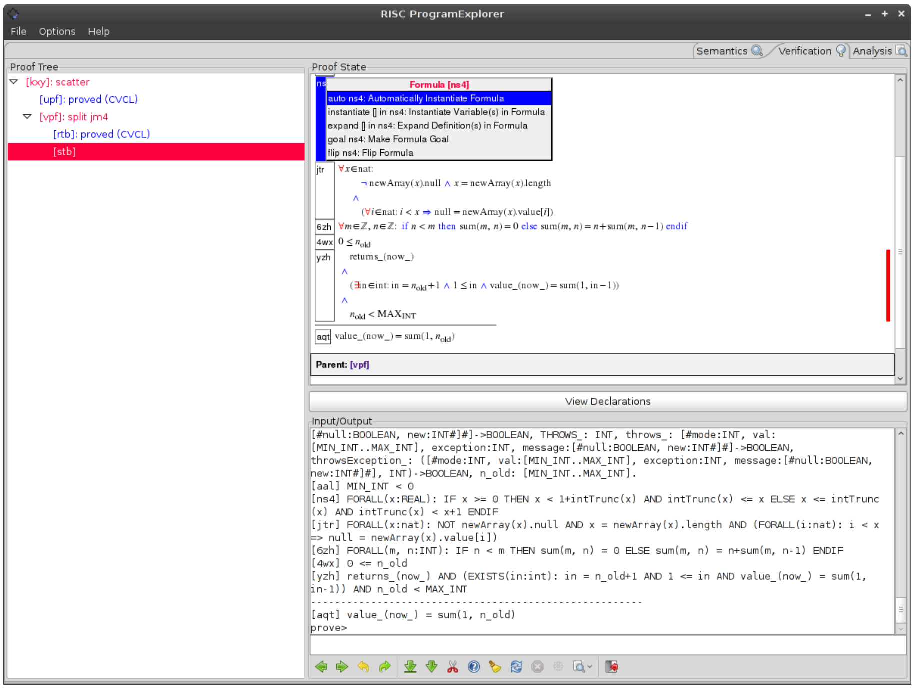


Figure 6: Verification View

- the termination condition of the command,
- the effect of the command (i.e. the set of variables potentially changed, the set of exceptions potentially thrown, the information whether the command may interrupt the control flow by executing a `continue`, `break`, `return` statement),
- the precondition of the command,
- the condition which is known to hold on the pre-state of the command.

Furthermore, the user may enter a desired condition on the pre-/post-state of a selected command; the system then determines the consequences for the pre-states of all other commands.

Verification In this view, the user may perform a semi-interactive proof of a selected verification task with the help of the RISC ProofNavigator which is embedded in the RISC ProgramExplorer. This proving assistant provides a small set of commands that implement typical proving strategies on the level of formulas (term reasoning is delegated to the validity checker CVCL [3]); the most frequently used commands are bound to buttons respectively menus attached to corresponding formulas. By the use of these commands, proof situations are gradually reduced to sub-situations with all proof situations displayed in a tree structure; the goal is to achieve such situations that can be automatically determined as valid by the validity checker. The interface has been carefully designed to make the handling of proofs as convenient as possible.

Proofs respectively proof attempts are persistently stored on disk and can be later replayed; the status of a proof and the dependencies of proofs to theory definitions or separately proved formulas are automatically managed (it is e.g. indicated whether a previously performed proof is still valid, i.e. whether no prerequisite of the proof has changed).

Figure 6 displays the verification view; for details of the RISC ProofNavigator, see [24].

5 An Example

We illustrate the use of the RISC ProgramExplorer by the following method sum which returns, for non-negative argument n, the sum of all integers from 1 to n, and for negative n, the value -1:

```
static int sum(int n) /*@
  requires VAR n < Base.MAX_INT;
  ensures
    LET result=VALUE@NEXT IN
    IF VAR n < 0
      THEN result = -1
      ELSE result = sum(1, VAR n)
    ENDIF;
@*/
{
  int s;
  if (n < 0)
    s = -1;
  else {
    s = 0;
    int i = 1;
    while (i <= n) /*@
      invariant VAR n < Base.MAX_INT
            AND 1 <= VAR i AND VAR i <= VAR n+1
            AND VAR s=sum(1, VAR i-1);
      decreases VAR n - VAR i + 1;
    @*/
    {
      s = s+i;
      i = i+1;
    }
  }
  return s;
}
```

The method is specified by a pair of pre- and postcondition; the term value@next in the postcondition refers to the return value of the function; the program type int is mapped to the specification type Base.int which denotes the set of all integers from Base.MIN_INT to Base.MAX_INT. One should note that the method is actually not completely correct with respect to the specification, since the computation of result s might yield an overflow (see the end of this section for more information on this aspect).

The specification uses a binary function $\mathsf{sum} : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ such that $\mathsf{sum}(m,n)$ denotes the sum of all integers from m to n; this function is specified in a theory which can be attached to the class in which the method is located:

```
theory {
  sum: (INT, INT) -> INT;
  sumaxiom: AXIOM
    FORALL(m: INT, n: INT):
      IF n<m
        THEN sum(m, n) = 0
        ELSE sum(m, n) = n+sum(m, n-1)
      ENDIF;
}
```

Here INT denotes the set of all integers; sum is axiomatized rather than defined, because the theory language does currently not support recursive definitions.

Based on the calculus presented in Section 3, the RISC Program Explorer translates the while loop to the following semantic model (the loop's precondition and pre-state knowledge are not shown):

Effects

executes: true, **continues:** false, **breaks:** false, **returns:** false
variables: s, i; **exceptions:**-

Transition Relation

$\text{var } i = \text{old } n+1 \wedge \text{old } n < \text{Base.MAX}_{\text{INT}} \wedge 1 \leq \text{var } i \wedge \text{var } s = \text{sum}(1, \text{ var } i-1)$

Termination Condition

$\text{executes@now} \Rightarrow \text{old } n - \text{old } i \geq -1$

Here the core of the transition relation is the formula $\mathsf{var}\ i = \mathsf{old}\ n+1 \wedge \mathsf{var}\ s = \mathsf{sum}(1, \mathsf{var}\ i-1)$ (which implies $\mathsf{var}\ s = \mathsf{sum}(1, \mathsf{old}\ n)$) while the core of termination condition is $\mathsf{old}\ n - \mathsf{old}\ i \geq -1$ (the initial value of the termination term must not be negative). The formulas derived by the plain calculus are actually (in general) much more complex; the human-friendly form shown above is derived only after performing extensive processing by a built-in simplifier (the user may freely switch between the display of the simplified formula and its original version).

From this translation, the conditional statement is translated as follows (the termination condition is automatically simplified to true and therefore not displayed any more):

Effects

executes: true, **continues:** false, **breaks:** false, **returns:** false
variables: s; **exceptions:**-

Transition Relation

```
if old n < 0 then
  var s = -1
else
    (∃in∈Base.int: in = old n+1 ∧ 1 ≤ in ∧ var s = sum(1, in-1))
  ∧
    old n < Base.MAX_INT
endif
```

The whole body of the method is translated to

Effects

executes: false, **continues:** false, **breaks:** false, **returns:** true
variables: -; **exceptions:**-

Transition Relation

```
if old n < 0 then
  returns@next ∧ value@next = -1
else
    returns@next
  ∧
    (∃in∈Base.int: in = old n+1 ∧ 1 ≤ in ∧ value@next = sum(1, in-1))
  ∧
    old n < Base.MAX_INT
endif
```

Here the "effects" clause indicates that the body of the method results in the execution of a `return` statement, which is also indicated by the formula returns@next in the transition relation. The core of the transition relation in the second branch is $\exists in : in = \text{old } n + 1 \wedge value\texttt{@next} = \text{sum}(1, in - 1)$ which implies $value\texttt{@next} = \text{sum}(1, \text{old } n)$. The transition relation denotes the semantic essence of the method which concisely describes the behavior of the method in a declarative form; from this, the correctness of the method according to its specification is quite self-evident even before the formal proof is started.

As a side effect of the translation, the RISC ProgramExplorer generates a couple of verification tasks, which will be explained in more detail in Section 6. One of them is the obligation to prove that the postcondition q is implied by the precondition p and the derived transition relation r (i.e. to prove $p \wedge r \Rightarrow q$ as described in Section 6):

$$
\begin{array}{l}
\quad n_{\text{old}} < \text{MAX}_{\text{INT}} \\
\quad \wedge \\
\quad\quad \text{if } n_{\text{old}} < 0 \text{ then} \\
\quad\quad\quad \text{returns\_(now\_)} \wedge \text{value\_(now\_)}+1 = 0 \\
\quad\quad \text{else} \\
\quad\quad\quad\quad \text{returns\_(now\_)} \\
\quad\quad\quad \wedge \\
\quad\quad\quad\quad (\exists \text{in} \in \text{int}: \text{in} = n_{\text{old}}+1 \wedge 1 \leq \text{in} \wedge \text{value\_(now\_)} = \text{sum}(1, \text{in}-1)) \\
\quad\quad\quad \wedge \\
\quad\quad\quad\quad n_{\text{old}} < \text{MAX}_{\text{INT}} \\
\quad\quad \text{endif} \\
\Rightarrow \\
\quad \text{if } n_{\text{old}} < 0 \text{ then value\_(now\_)}+1 = 0 \text{ else value\_(now\_)} = \text{sum}(1, n_{\text{old}}) \text{ endif}
\end{array}
$$

The proof of this formula proceeds by executing three commands, two triggered by pressing a button, one by selecting a command from a formula menu (see [24] for a more detailed description of the interaction with the prover). The correspondingly generated proof tree (whose nodes are labeled by the respective proof commands) is

```
▽ [kxy]: scatter
    [upf]: proved (CVCL)
  ▽ [vpf]: split jm4
      [rtb]: proved (CVCL)
    ▽ [stb]: scatter
        [mlv]: proved (CVCL)
```

This proof would be irrelevant, if the post-condition q specified by the user were "trivial", i.e., satisfied by every output value (generally indicating an error in the specification). To rule this out, an (optional) verification condition is generated that validates the specification by showing that, for every prestate that satisfies precondition p, there exists a poststate that does *not* satisfy q (i.e., $\forall x : p(x) \Rightarrow \exists y : \neg q(x,y)$ as described in Section 6):

$$
\begin{array}{l}
\forall n \in \text{int}: \\
\quad\quad n < \text{MAX}_{\text{INT}} \\
\quad \Rightarrow \\
\quad\quad (\exists \text{ns} \in \text{STATE}: \\
\quad\quad\quad \text{if } n < 0 \text{ then value\_(ns)}+1 \neq 0 \text{ else value\_(ns)} \neq \text{sum}(1, n) \text{ endif})
\end{array}
$$

The corresponding proof proceeds (as determined by the branch condition `n<0` in the program) by manual case distinction and, in each case, by an instantiation with a state that holds an incorrect return value (there also arises a third case, because we have declared the theory function sum over the integers, such that in principle a negative result might arise):

```
▽ [kxy]: scatter
  ▽ [upf]: expand value_
    ▽ [1jz]: case n_0 < 0
      ▽ [elz]: instantiate now_ WITH .val:=0 in dba
          [kaw]: proved (CVCL)
          [law]: proved (CVCL)
      ▽ [flz]: case 0 <= sum(1, n_0)
        ▽ [bq2]: instantiate now_ WITH .val:=(-1) in dba
            [fkh]: proved (CVCL)
            [gkh]: proved (CVCL)
        ▽ [cq2]: instantiate now_ WITH .val:=0 in dba
            [33j]: proved (CVCL)
            [43j]: proved (CVCL)
```

Another core verification task is the proof that the loop body with transition relation r preserves the invariant i (i.e. the proof of $i' \wedge e \wedge r \Rightarrow i''$ as described in Section 6). After the built-in logical simplification, the proof goal becomes

$$
\begin{array}{l}
\quad n_{\mathrm{old}} < \mathrm{MAX}_{\mathrm{INT}} \\
\;\wedge \\
\quad (\exists i \in \mathrm{int}: \\
\qquad\quad 1 \leq i \wedge i \leq n_{\mathrm{old}}+1 \wedge i \leq n_{\mathrm{old}} \wedge i_{\mathrm{new}} = i+1 \\
\qquad \wedge \\
\qquad\quad s_{\mathrm{new}} = \mathrm{sum}(1,\ i-1)+i) \\
\Rightarrow \\
\quad 1 \leq i_{\mathrm{new}} \wedge i_{\mathrm{new}} \leq n_{\mathrm{old}}+1 \wedge s_{\mathrm{new}} = \mathrm{sum}(1,\ i_{\mathrm{new}}-1)
\end{array}
$$

The corresponding proof is performed by two commands, one of which is the manual instantiation of the axiom defining the function sum; the corresponding proof tree is

```
▽ [kxy]: scatter
    [upf]: proved (CVCL)
  ▽ [vpf]: instantiate 1, i_new-1 in 6zh
      [rtb]: proved (CVCL)
```

The proofs of the other verification conditions proceed mostly automatically; the only exception is the proof that the increment `s = s+i` does not yield an overflow. As already stated in the beginning of this section, this is actually not true for our method. To make it true, the specification has to be extended by an additional precondition that puts an upper limit on the value of the sum (`sum(1,VAR n) <= Base.MAX_INT`); to perform the corresponding verification, then some additional lemmas about the monotonicity of sum have to be introduced and proved.

The example presented in this section only involves simple control structures and uses integer numbers as the only datatype. However, the full calculus also supports programs with commands interrupting the control flow; these are translated to formulas that involve special atomic predicates to express the behavior of a program with respect to control flow (e.g. the predicate `returns@state` shown above indicates that *state* results from the execution of a `return` statement). Furthermore, the implementation supports the usual datatypes like arrays and objects. The transition relations derived from programs involving these features are apparently more complex than the examples shown above; nevertheless they still become manageable after appropriate simplification. For this purpose, it is recommended to express method specifications and loop invariants with the help of high-level functions and predicates introduced in theory declarations. The derived transition relations then also refer to these formulas and become (again, after appropriate simplification) essentially as readable as the specifications and invariants. The distribution of the RISC ProgramExplorer comes with a couple of examples (e.g. operations on arrays like sorting, linear search, binary search, a class implementing the datatype "stack", and others) that may serve as a starting point for further applications.

6 State Relations and Verification Conditions

One of the advantages of the translation of a program into its "semantic essence", i.e., into a formula denoting its state relation, is that by this translation the derivation and interpretation of the individual verification tasks becomes very transparent. In the following we explain the logical interpretation of the various verification tasks generated by the RISC ProgramExplorer from every method and their relationship to the method's semantic essence.

Postcondition For a method with precondition p, postcondition q and state relation r of the method body, the goal of this task is to prove

$$p \wedge r \Rightarrow q$$

which is logically equivalent to $r \Rightarrow (p \Rightarrow q)$. Here p and q are taken from the method contract (`requires` p `ensures` q) while r is shown in the "Semantics" view of the method body (section "Transition Relation").

This task shows the *partial correctness* of the method, provided that all preconditions and loop-related verification conditions (such as the preservation of the loop invariant) are indeed correct (which is shown by other tasks explained below). We therefore can detect in this stage that the invariant of a loop in the body of the method is too weak to show the partial correctness of the method, even before the proof of the correctness of the invariant is actually attempted.

Termination The goal of the proof that the "method body terminates" is of form

$$p \wedge \neg d \Rightarrow t$$

where p represents the method's precondition, d represents the `diverges` condition in the method's contract (an optional condition under which the method is allowed to run forever) and t is the method body's termination condition as depicted in the method's "Semantics" view (Section "Termination Condition").

This task shows (together with task "postcondition") the *total correctness* of the method, provided that the body of every loop in the method terminates and that the associated termination term is well-formed and decreased (which is shown by other tasks explained below). We can detect in this task that a termination term has initially a wrong (negative integer) value or that some method is called in a state in which the negation of its `diverges` condition does not hold.

Preconditions For every command with pre-state knowledge k and precondition c, a task is generated to show that the pre-condition is met, i.e. to prove a goal of form

$$k \Rightarrow c$$

where both k and c are displayed in the *Semantics* view of the method (sections "Pre-State Knowledge" and "Precondition"). We have chosen this "top-down" generation of preconditions over the "bottom-up" calculation of the calculus presented in Section 3 in order to foster a closer and more illustrative relationship between a precondition and its statement respectively the associated pre-state knowledge.

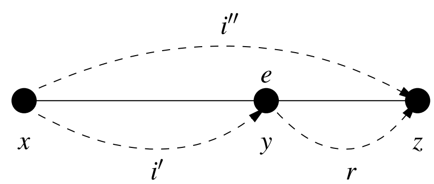
We can thus detect in this task that a command is executed in a state in which the consequence of the execution may not be properly described by the command's state relation.

Loops For a while loop with an invariant i and a body with state relation r, there are four tasks generated (compare with the corresponding rules in Section 3).

The first task is to verify the correctness of the invariant amounts to proving a formula of form

$$i' \wedge e \wedge r \Rightarrow i''$$

Here i' represents a variant of i that expresses the relationship between the initial state x of the loop and the state y before the current loop iteration, e expresses the fact that the loop condition holds at state y, r expresses the relation between the states y and z before and after the current loop iteration, and i'' represents a variant of i that expresses the relationship between states x and z:



The other three tasks are related to showing the termination of the loop. The task to show that the loop "body terminates" is essentially to prove a goal

$$i' \wedge e \Rightarrow b$$

where i' and e are as indicated above and b is the termination condition derived from the loop body. The task to show that the loop "measure is well-formed" is essentially to prove a goal

$$i' \wedge e \wedge r \Rightarrow 0 \leq t'$$

where i', e, and r are as indicated above and t' represents the value of the termination term after the iteration of the loop. The task to show that the loop "measure is decreased" is essentially to prove a goal

$$i' \wedge e \wedge r \Rightarrow t' < t$$

where i', e, r, and t' are indicated as above and t represents the value of the termination term before the iteration of the loop.

Specification Validation An open question in every formal verification is whether the specification indeed expresses the informal requirements that the programmer wants to impose on the program. A typical beginner's error is that a specification (due to some error in the logical formulation, e.g., some wrong logical connective) admits *every* possible output from an implementation. Given a specification with input variable x and output variable y, precondition p and postcondition q, the verification condition

$$\forall x : p(x) \Rightarrow \exists y : \neg q(x, y)$$

rules such a “trivial” specification out. Likewise, the verification condition

$$\forall x : p(x) \Rightarrow \exists y : q(x,y)$$

ensures that the specification allows *some* output, i.e. that it is actually “satisfiable” (implementable by a method). Furthermore, we may show that for some concrete input i some desired output o is indeed legal by proving

$$p(i) \wedge q(i,o)$$

respectively we may show that some undesired output o' is illegal

$$p(i) \wedge \neg q(i,o')$$

An extensive validation of every specification is recommended before any of the previously described verification tasks is attempted.

The first two kinds of conditions are are generated by the RISC ProgramExplorer for every method specification; p and q are derived from the method contract and the roles of x and y are taken by variables that represent a method body’s pre- and post-state. The other two kinds of conditions will also be generated in a future version of the software.

7 General Workflow

The RISC ProgramExplorer has been designed to support the following workflow that leads in a systematic way from an informal problem statement to a formal problem specification and subsequently to a problem solution that is verified to be correct with respect to the specification:

1. **Theory Development:** Considering the particular domain of the problem at hand, we formalize in the RISC ProgramExplorer a corresponding mathematical theory by defining or axiomatizing constants, functions, and predicates that are suitable to express the concepts that are of interest in that domain. We also formulate prospective theorems that may become useful as knowledge about these concepts in the subsequent verification tasks. We may prove these theorems immediately or delegate the proofs to a later stage.
2. **Method Specifications:** Considering the particular computational problem in the domain, we describe the solution to the problem by a method signature (method declaration with empty body) and a specification of the method with the help of the concepts introduced in the previously developed theory. We validate the specification by showing that it is non-trivial, satisfiable, and holds for certain legal input/output pairs (respectively does not hold for certain illegal pairs).
3. **Method Design:** We sketch a method solution by providing a skeleton of the method body; the body may contain loop skeletons and calls of new auxiliary methods that are specified by loop invariants, termination terms, respectively method contracts. The loop respectively method bodies can be implemented at a later stage (a loop body must however indicate by dummy assignments which variables are to be changed by an iteration).
4. **Semantic Analysis:** Based on the sketch of the method body (and the specification of the method, the loops executed, and other methods called), we investigate the semantic essence of the method. Do the derived state relations represent the envisioned behavior? Are the command’s preconditions and termination conditions implied by the respective pre-state knowledge of the method? If some problem is detected, we revise the method specifications, loop invariants, etc.

5. **Verification:** We attempt to verify the method's postcondition (partial correctness). If the proof fails, it may be necessary to revise the implementation, specifications, loop invariants. It may be also necessary to extend the theories by introducing additional theorems that provide knowledge that is required in the verification.

   Once the proof succeeds, we attempt to verify the method's termination in the same manner (which may lead to a revision of the termination terms).

   Once these "major" verification tasks have been successfully completed, we may turn to the "minor" tasks such as proving the statement preconditions and the loop-related tasks (correctness of invariants and termination terms). If failures are detected, again the implementation, specifications, loop invariants, termination terms, or theories may need to be revised.

6. **Refinement:** We refine the still open bodies of loops and auxiliary methods; we analyze, and verify them, as described above.

7. **Theorems:** We prove the still open theorems that were used in the verification process (respectively were newly introduced in the course of this process).

In this methodology (which is iteratively performed if problems/errors are detected at a certain stage), the actual program verification is a core step, but not the only one. Most important, the semantic analysis *precedes* the verification; programmers should thus get insight into the program methods and their specification *before* they dive into the depths of their formal verification. The verification itself is also organized in a layered fashion where e.g. a proof of partial correctness is performed before the correctness of the loop invariants or of the contracts of auxiliary methods has been established; the suitability of the invariants respectively contracts for the task at hand can thus be tested at an early stage. It should be also noted that the process integrates the principle of *refinement* (respectively "correct by construction") by deferring the implementation of loop respectively method bodies to later stages of the development.

8 Conclusions

The approach to program reasoning presented in this paper and its implementation in the RISC Program-Explorer were motivated by a personally experienced lack of transparency in existing tools which made it hard for the author to get deeper insight into a program from the automatically generated verification conditions and the (failed) attempts to prove them. Our goal was to make not only the derivation transparent but to base this derivation on a semantic model that can be presented to the user and is open for further investigation (prior to any actual verification attempt). For this purpose, we have constructed a denotational semantics of a program that is based on the model of a program as a state relation; this relation can be described by a classical logical formula and can serve in a quite direct way as the basis of a verification of the program. Since the relations are derived from the specifications of the methods called and the loops executed (not from the bodies of the methods respectively loops), the approach is inherently modular and gives rise to a step-wise refinement of program specifications to implementations.

In a certain sense, the presented approach can be considered as the translation of an imperative model of programming to a declarative one. In the imperative model, the focus is on a sequence of variable updates that gradually transforms a store from a given initial situation to a desired terminal situation; in the declarative model, the focus is on the relationship between the given and the desired situation. In our experience, most students think of computational problems mainly in an imperative view and are insecure with the declarative view (which is however the basis of formal reasoning), i.e., they tend to think in terms of "how" rather than in terms of "what". The difference is predominant in the typical programs that express repetitive computations in the form of loops (if a program is written in a functional style where recursion is applied for this purpose, the difference between both views becomes blurred). The presented translation is designed to help people that are trained in the imperative view to become also proficient with the declarative one.

Consequently the RISC ProgramExplorer was developed to provide a close integration between programs, theories, specifications, and semantic models in order to emphasize the co-development of the declarative view of a problem (its specification) and the operational view (its implementation) and to exhibit the relationship between them. To support actual verifications, the RISC ProofNavigator was designed as a compromise between a certain level of automation (which is necessary to perform proofs successfully) and a comfortable user interaction (which is necessary to direct the prover into the right direction and, in particular, to learn from failed proof attempts). A particular challenge was the appropriate simplification of the automatically derived transition formulas to a form that a human would consider as the most "natural" one. First experiments seem to indicate that by appropriate simplification of these formulas (flattening quantifier structures, eliminating variables, etc.), also the consequent verifications become technically simpler; we consider this as an interesting problem that needs further research.

The RISC ProofNavigator has been applied since 2005 in a regular course on "Formal Methods in Software Development" for the proof of (manually derived) verification conditions. A detailed presentation of our experience is beyond the scope of this paper (see also [24]; in a nutshell, we found that most students become able, after a comparatively short repetition of the basics of logic and proving (some prior background in logic is assumed) and a corresponding introduction to the system and its user interface, to perform verifications of correct programs with given correct specifications and program annotations (loop invariants and termination terms). However, many tend to have big problems if programs, specifications, and/or annotations contain errors (the less bright/motivated ones then give quickly up or perform seemingly random proving commands).

The RISC ProgramExplorer and the methodology it supports are being tested for the first time in the current iteration of a course on "Formal Methods in Software Development" that has started at the Johannes Kepler University Linz in October 2011; this course is mandatory for students of the master programmes "Software Engineering" and "Computer Mathematics". In previous iterations, we have experienced that many students did not really get deeper insight into e.g. the expressiveness of loop invariants and their role and suitability with respect to proving the partial correctness of a method. We hope that by the new tool and the corresponding methodology, this insight will be substantially deepened. A critical point will certainly be the ability to deal with the various views on a program and to relate them to each other. Our experience will show to which extent our idea will be successful and also give feedback for the further evolution of our software and for the development of an accompanying didactic approach.

References

[1] R.-J. Back & J. von Wright (1998): *Refinement Calculus: A Systematic Introduction*. Springer, New York.

[2] Ralph-Johan Back (2009): *Invariant Based Programming: Basic Approach and Teaching Experiences*. *Formal Aspects of Computing* 21(3), pp. 227–244, doi:10.1007/s00165-008-0070-y.

[3] Clark Barrett & Sergey Berezin (2004): *CVC Lite: A New Implementation of the Cooperating Validity Checker*. In: *Computer Aided Verification: 16th International Conference, CAV 2004, Boston, MA, USA, July 13–17, 2004*, *LNCS* 3114, Springer, pp. 515–518, doi:10.1007/978-3-540-27813-9_49.

[4] Bernhard Beckert, Reiner Hähnle & Peter H. Schmitt, editors (2007): *Verification of Object-Oriented Software: The KeY Approach*. *LNCS* 4334, Springer, doi:10.1007/978-3-540-69061-0.

[5] P. Boca, J. P. Bowen & D. A. Duce, editors (2006): *Teaching Formal Methods: Practice and Experience*. Electronic Workshops in Computing (eWIC), British Computer Society, London, UK, December 16. http://www.bcs.org/category/8726.

[6] Raymond T. Boute (2006): *Calculational Semantics: Deriving Programming Theories from Equations by Functional Predicate Calculus*. *ACM Transactions on Programming Languages and Systems* 28(4), pp. 747–793, doi:10.1145/1146809.1146814.

[7] Lilian Burdy, Yoonsik Cheon, David R. Cok, Michael D. Ernst, Joseph R. Kiniry, Gary T. Leavens, K. Rustan M. Leino & Erik Poll (2005): *An Overview of JML Tools and Applications*. *International Journal on Software Tools for Technology Transfer* 7, pp. 212–232, doi:10.1007/s10009-004-0167-4.

[8] C. Neville Dean & Raymond T. Boute, editors (2004): *Teaching Formal Methods, CoLogNET/FME Symposium, TFM 2004, Ghent, Belgium, November 18–19, 2004*. *LNCS* 3294, Springer, doi:10.1007/b102075.

[9] Edsger W. Dijkstra (1968): *A Constructive Approach to the Problem of Program Correctness*. *BIT Numerical Mathematics* 8(3), pp. 174–186, doi:10.1007/BF01933419.

[10] Ingo Feinerer & Gernot Salzer (2009): *A Comparison of Tools for Teaching Formal Software Verification*. *Formal Aspects of Computing* 21(3), pp. 293–301, doi:10.1007/s00165-008-0084-5.

[11] Jean-Christophe Filliâtre & Claude Marché (2007): *The Why/Krakatoa/Caduceus Platform for Deductive Program Verification*. In: *Computer Aided Verification, 19th International Conference, CAV 2007, Berlin, Germany, July 3-7, 2007*, *LNCS* 4590, Springer, pp. 173–177, doi:10.1007/978-3-540-73368-3_21.

[12] Jeremy Gibbons & José Nuno Oliveira, editors (2009): *Teaching Formal Methods, Second International Conference, TFM 2009, Eindhoven, The Netherlands, November 2–6, 2009*. *Lecture Notes in Computer Science* 5846, Springer, doi:10.1007/978-3-642-04912-5.

[13] Mike Gordon (2010): *Specification and Verification I*. University of Cambridge, UK, Lecture Notes, http://www.cl.cam.ac.uk/~mjcg/Lectures/SpecVer1/SpecVer1.html.

[14] Eric C.R. Hehner (2006): *A Practical Theory of Programming*. Springer, New York. http://www.cs.utoronto.ca/~hehner/aPToP.

[15] C.A.R. Hoare & He Jifeng (1998): *Unifying Theories of Programming*. Prentice Hall, London, UK. http://www.unifyingtheories.org.

[16] Leslie Lamport (2002): *Specifying Systems: The TLA+ Language and Tools for Hardware and Software Engineers*. Addison-Wesley. http://research.microsoft.com/users/lamport/tla/book.html.

[17] Carroll Morgan (1998): *Programming from Specifications*, 2nd edition. Prentice Hall, London, UK. http://www.cs.ox.ac.uk/publications/books/PfS.

[18] J. M. Morris (1987): *A Theoretical Basis for Stepwise Refinement and the Programming Calculus*. *Science of Computer Programming* 9(3), pp. 287–306, doi:10.1016/0167-6423(87)90011-6.

[19] José Nuno Oliveira (2004): *A Survey of Formal Methods Courses in European Higher Education*. In: *[8]*, pp. 235–248, doi:10.1007/978-3-540-30472-2_16.

[20] S. Owre, J. M. Rushby & N. Shankar (1992): *PVS: A Prototype Verification System*. In Deepak Kapur, editor: *11th International Conference on Automated Deduction (CADE)*, *Lecture Notes in Artificial Intelligence* 607, Springer, Saratoga, NY, June 14–18, pp. 748–752, doi:10.1007/3-540-55602-8_217.

[21] David A. Schmidt (1986): *Denotational Semantics – A Methodology for Language Development*. Allyn and Bacon, Boston, MA. http://people.cis.ksu.edu/~schmidt/text/densem.html.

[22] Wolfgang Schreiner (2008): *A Program Calculus*. Technical Report, RISC, Johannes Kepler University, Linz, Austria. http://www.risc.jku.at/people/schreine/papers/ProgramCalculus2008.pdf.

[23] Wolfgang Schreiner (2008): *Understanding Programs*. Technical Report, Research Institute for Symbolic Computation (RISC), Johannes Kepler University, Linz, Austria. http://www.risc.uni-linz.ac.at/people/schreine/papers/Understanding2008.pdf.

[24] Wolfgang Schreiner (2009): *The RISC ProofNavigator: A Proving Assistant for Program Verification in the Classroom*. *Formal Aspects of Computing* 21(3), pp. 277–291, doi:10.1007/s00165-008-0069-4.