Implementation of a Port-graph Model for Finance

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In this paper we examine the process involved in the design and implementation of a port-graph model to be used for the analysis of an agent-based *rational negligence* model. Rational negligence describes the phenomenon that occurred during the financial crisis of 2008 whereby investors chose to trade asset-backed securities without performing independent evaluations of the underlying assets. This has contributed to motivating the search for more effective and transparent tools in the modelling of the capital markets.

This paper shall contain the details of a proposal for the use of a visual declarative language, based on strategic port-graph rewriting, as a visual modelling tool to analyse an asset-backed securitisation market.

1 Introduction

In this paper we examine the process involved in the design and implementation of a port-graph model to be used for the analysis of an agent-based *rational negligence* model. Rational negligence describes the phenomenon that occurred during the financial crisis of 2008 whereby investors chose to trade assetbacked securities without performing independent evaluations of the underlying assets. This proposal is motivated by the interest an analyst or policy-maker might have in analysing whether or not the purchase of a particular class of asset-backed security ought, going forward, to be subjected to a full due-diligence.

By replacing more traditional Dynamic Stochastic General Equilibrium (DSGE) models [6] with heterogenous, proactive, agent-based models able to produce more realistic representations, a more efficient system is produced. Such a system is able to support rapid prototyping, run system simulations, and, thanks to its formal semantics, also reason about system properties. Turning to a declarative port-graph transformation system facilitates the analysis of processes of interest. This is because it is able to model the dynamic behaviour of complex systems given that its declarative nature and visual aspects produces a shorter distance between mental picture and implementation. Such a tool is not only able to convert a black box model into a white box but also provide an extensively flexible platform as shall be seen later. There are non-visual elements, such as an inbuilt strategy language, but details of the states produced by the resulting strategy programs are highlighted in a visual trace/derivation tree.

In addition, over extant approaches, the rewrite rules that drive graph transformation systems are an intuitive and natural way of expressing dynamic, structural changes which are generally more difficult to model using traditional simulation approaches where the structure of the model is usually fixed [5]. Over many of the declarative languages used within agent-based simulation tools, this approach, given its visual declarative nature, provides more conceptualization support. The use of the port-graph highlights the heterogeneous agent-based nature of the model. Given support for attributes at component-level more realistic simulations can be produced. As a graph transformation system underlying, port-graphs are in addition particularly useful in the development of more concise graph models and in the production

© N. Ene This work is licensed under the Creative Commons Attribution License. of more concise model results given support of both topology and data at the same time. The portgraph makes comprehensive modeling much easier to manage and provides extensive conceptualization support.

Contributions. We outline the typical specification of a graph program that can be used to represent the workings of a small Rational Negligence agent-based system. We include broad implementation details highlighting design choices and alternatives that could otherwise have guided our approach. We also carry out a thorough validation of the model with respect to its equational specification producing a base case that provides an effective platform for incrementally increasing the complexity and scope of the model. In a previous work; [2], we have defined a hierarchical port-graph extension useful in producing a more integrated model.

Overview. This paper is organised as follows: We briefly examine the general structure of a simple Port-graph Transformation System and equational semantics of the chosen Rational Negligence model in Section 2. Details of the actual implementation can be found in Section 3 and Section 4 examines the results of key tests and checks on the system. We finally conclude and briefly outline future plans in Section 5.

2 Background

2.1 Port-graph Transformation Systems

A port graph is a graph where nodes have explicit connection points, called ports, and edges are attached to ports. Nodes, ports and edges are labelled by a set of attributes describing properties such as colour, shape, etc. Port graphs are transformed by applying port graph rewrite rules. We refer to [4] for a formal definition of labelled port graphs, where labels are records, i.e., lists of pairs attribute-value. The values can be concrete (numbers, Booleans, etc.) or abstract (expressions in a term algebra, which may contain variables). More precisely:

Definition 1 (Signature) A port graph signature ∇ consists of the following pairwise disjoint sets: $\nabla_{\mathscr{A}}$, a set of attributes; $\mathscr{X}_{\mathscr{A}}$, a set of attribute variables; $\nabla_{\mathscr{Y}}$, a set of values; $\mathscr{X}_{\mathscr{Y}}$, a set of value variables.

Definition 2 (Attributed Port Graphs) Let $\mathcal{N}, \mathcal{P}, \mathcal{E}$ be pairwise disjoint sets. A port graph over a signature ∇ is a tuple $G = (V, P, E, D)_{\mathscr{F}}$ where $V \subseteq \mathcal{N}$ V is a finite set of nodes $(n, n_1, ..., range over nodes)$; $P \subseteq \mathcal{P}$ P is a finite set of ports $(p, p_1, ..., range over ports)$; $E \subseteq \mathcal{E}$ E is a finite set of edges between ports $(e, e_1, ..., range over edges; edges are undirected and two ports may be connected by more than one edge); D is a set of records over <math>\nabla$, and \mathscr{F} is a set of functions Connect : $E \to P \times P$, Attach: $P \to V$ and $\mathscr{L}: V \cup P \cup E \to D$ such that

- for each edge $e \in E$, Connect(e) = (p1, p2) the two ports connected by e
- for each port $p \in P$, Attach(p) = n, the node to which p belongs
- $\mathscr{L}: V \cup P \cup E \rightarrow D$ is the labelling function that returns the record associated to a component

Each $n \in V$ maintains an attribute called Interface whose value is the list of names of the ports attached to n, that is, $\mathcal{L}(n)$.Interface = { $\mathcal{L}(p_i)$.Name | Attach $(p_i) = n$ }, $\mathcal{L}(n)$.Interface = [$\mathcal{L}(p_i)$.Name | Attach $(p_i) = n$], satisfying the following constraint: $\mathcal{L}(n_1)$.Name = $\mathcal{L}(n_2)$.Name $\Rightarrow \mathcal{L}(n_1)$.Interface = $\mathcal{L}(n_2)$.Interface. **Definition 3 (Port-graph Rewrite Rule)** A port graph rewrite rule $L \Rightarrow_C R$ is a port graph consisting of two subgraphs L and R together with a node (called arrow node) that captures the correspondence between the ports of L and the ports of R, and includes a condition C that will be checked at matching time. More precisely, each of the ports in the arrow node has an attribute Type, which can have three different values: bridge, wire and blackhole, values that indicate how a rewriting step using this rule should affect the edges that connect the redex to the rest of the graph and that satisfy the following conditions:

- 1. A port of type bridge must have edges connecting it to L and to R (one edge to L and one or more to R).
- 2. A port of type blackhole must have edges connecting it only to L (at least one edge).
- 3. A port of type wire must have exactly two edges connecting to L and no edge connecting to R.

Let *X* and *Y* be two port graphs over the same signature ∇ . A port graph morphism $g: X \to Y$ maps nodes, ports and edges of *X* to those of *Y* such that the attachment of ports and the edge connections are preserved, and all attributes are preserved except for variables in *X*, which must be instantiated in *Y* (attribute instantiation can take place within the *algorithm tab* associated with the rule). Intuitively, the morphism identifies a subgraph of *Y* that is equal to *X* except at positions where *X* has variables (at those positions *Y* could have any value).

Definition 4 (Port Graph Morphism) *More formally given two port graphs* $X = (V_X, P_X, E_X, D_X)_{\mathscr{F}_X}$ and $Y = (V_Y, P_Y, E_Y, D_Y)_{\mathscr{F}_Y}$ over the same signature ∇ , a morphism f from X to Y, denoted $f : X \to Y$, and with a definition domain Dom(f), is a family injective functions $\langle f_V : V_X \to V_Y, f_p : P_X \to P_Y, f_E :$ $E_X \to E_Y, f_D : D_X \to D_Y \rangle$ such that:

- 1. f_V, f_P, f_E are injective i.e. distinct components are not identified
- 2. $\forall e \in E_X : if Connect_X(e) = (p_1, p_2) then (f_P(p_1), f_P(p_2)) = Connect_Y(f_E(e))$
- 3. $\forall n \in V_X$: if $Attach_X(p) = n$ for some p then $f_V(n) = Attach_Y(f_P(p))$
- 4. $\forall n \in Dom(f), f_D(\mathscr{L}_X(n)) = \mathscr{L}_Y(f_V((n)))$ $\forall p \in Dom(f), f_D(\mathscr{L}_X(p)) = \mathscr{L}_Y(f_P((p)))$ $\forall e \in Dom(f), f_D(\mathscr{L}_X(e)) = \mathscr{L}_Y(f_E((e)))$

Note that f_D can also instantiate variables

We denote by g(X) the subgraph of Y consisting of the set of nodes, ports and edges that are images of nodes, ports and edges in X.

Let *G* be a port graph. A *rewrite step* $G \Rightarrow H$ via the port graph rewrite rule $L \Rightarrow_C R$ is obtained by replacing in *G* a subgraph g(L) by g(R), where *g* is a morphism from *L* to *G* satisfying *C*. More precisely:

Definition 5 (Match) [4] Let $L \Rightarrow R$ be a port graph rewrite rule and G a port graph. We say a match g(L) of the left-hand side (i.e., a redex) is found if: there is a port graph morphism g from L to G (hence g(L) is a subgraph of G), C holds, and for each port in L that is not connected to the arrow node, its corresponding port in g(L) must not be an extremity in the set of edges of G - g(L). This last point ensures that ports in L that are not connected to the arrow node are mapped to ports in g(L) that have no edges connecting them with ports outside the redex, thus ensuring that there will be no dangling edges when g(L) is replaced by g(R).

For a given graph, several outcomes on application of a rule may be possible (due to the intrinsic non-determinism of rewriting). Strategies in rewriting systems are a means of controlling the creation of rewriting steps and improving rewriting opportunities. A sequence of rewriting steps is called a *deriva*tion. A derivation tree is a collection of derivations with a common root. Intuitively, the derivation tree is a representation of the possible evolutions of the system starting from a given initial state (each derivation provides a trace, which can be used to analyse and reason about the behaviour of system). In PORGY [4], the strategy language allows us to control the way derivations are generated. The strategy expression setPos(crtGraph) sets the position graph as the full current graph. If T is a rule, then the strategy one(T) randomly selects one possible occurrence of a match of rule T in the current graph G, which should superpose the position subgraph P but not superpose the banned subgraph Q. This strategy fails if the rule cannot be applied. Id and Fail denote success and failure, respectively. The strategy expression match(T) is used to check if the rule T can be applied (i.e., if there is a match for the left hand side of the rule in the current graph) but does not apply the rule. (S) orelse(S') tries strategy S and if it fails then tries to apply S'. If both strategies fail then the whole statement fails. $ppick(T_1, \ldots, T_n, \Pi)$ selects one of the transformations T_1, \ldots, T_n according to the given probability distribution Π . while (S)[(n)]do(S')executes strategy S' (not exceeding n iterations if the optional parameter n is specified) while S succeeds. repeat(S)[max n] repeatedly executes a strategy S, not exceeding n times. It can never fail (when S fails, it returns Id).

2.2 The Rational Negligence Model.

As defined in [7] "Securitisation is the process of converting cash flows arising from underlying assets or debts/receivables (typically illiquid such as corporate loans, mortgages, car loans and credit cards receivables) due to the originator into a smoothed liquid marketable repayment stream" and this ensures that the originator can raise asset-backed finance through loans or the issuance of debt securities also known as assets. An originator is any financial intermediary with a portfolio of assets on its balance sheet. In a securitisation, assets are selected, pooled and transferred to a tax neutral, liquidation-efficient (i.e bankruptcy avoiding), Special Purpose Vehicle (SPV), who funds them by issuing securities.

In the core rational negligence model [1], the profit \mathscr{U}_w expected by an agent (e.g., a bank) w from trading an asset depends on whether or not w follows the *negligence rule*, i.e., the rule of not performing independent risk assessment. Let z be a binary variable indicating whether or not the agent is following the negligence rule, then $\mathscr{U}_w(z)$ can be characterised by the following equations, where p is the probability of asset toxicity, Z is the average of all z's in the domain, c is the cost of purchasing an asset (note that the payoff from successfully reselling the asset is normalised to unity), x_w is the cost of performing a complete risk analysis, k is the number of trading partners of the seller bank and \mathscr{N}_i is the set of agents.

• Expected profit for w when following the negligence rule, i.e., when z(w) = 1, if w buys an asset and then tries to sell it to w':

$$\mathscr{U}_{w}(1) = {}^{def} - p(1 - z(w'))c + [1 - p(1 - z(w'))](1 - c) \approx 1 - p(1 - Z) - c$$

This is because if the asset is toxic then w will loose c if w' checks, and will have a profit of 1 - c if w' does not check. Of course w does not know a priori whether w' will or not follow the rule, but it can estimate z(w') as the average of all the values of z in the system, Z. Note that when p = 0 the profit is 1 - c as expected

• Similarly, the expected profit for w when the rule is not followed, i.e., z(w) = 0, is defined by:

$$\mathscr{U}_{w}(0) = (1-p)(1-c) - x_{w}$$

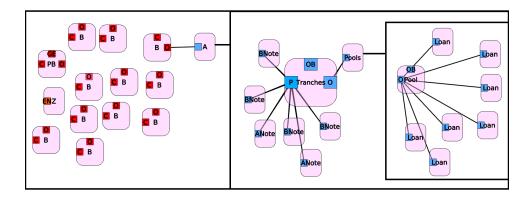


Figure 1: All Tiers Flattened and Condensed

This is because if the asset is toxic, then *w* will not buy it (losing only x_w), but if it is not toxic then it will resell it with a profit of $1 - c - x_w$.

So the best response of agent *w* to a buying request is determined by analysing the projected profitability of assuming a negligent approach over a diligent approach:

$$\mathscr{U}(1) - \mathscr{U}(0) = p(Z - c) + x_w = p\left(\frac{1}{k}\sum_{j \in \mathscr{N}i} z_j - c\right) + x_w$$

Following [1], we implement a model that mimicks the transactions that follow the trading of one asset since this is sufficient to perform validations against equivalent DSGE analyses. The goal of the model is to study the evolution of the system till *fixed point* or *stable state* is reached i.e., in this case, a state such that all potential buyers in the universe of discourse no longer alternate between diligent and non-diligent behaviour in their handling of the purchase of a particular asset. Details of the set of port-graph rules that encapsulate this computation can be found in Tables 1 and 2. The *beginanalysis* rule in Table 1 performs the profitability check.

3 Implementation

We represent the full ABS (Asset-backed Securitization) universe hierarchically as several initial graphs. Port graph rewriting rules and strategies are used to control the step-wise evolution of the graphs and to create a derivation tree that can be used for plotting and analysing parameter values. The asset trading model sits at the top level of the model hierarchy and it is non-deterministic in nature. Below this system, also able to handle asset pricing and valuation issues, lie several deterministic subsystems that model origination, structuring of the deal, SPV transfers and profitability of the sale.

We chose to model asset-transfer transactions using a combination of global and local data, and a global state, the Z node (an indicator of market behaviour obtained as the average value of each individual bank's approach, represented by the local lower-case attribute z and not to be confused with the global value Z). A *Change* indicator node is used within a rule to detect whether the market has reached a stable state. Tables 3 and 4 contain a description of the nodes used. Alternative designs are possible, highlighting the flexibility of the approach: For example, local copies of the z-attribute could be used to propagate negligent/diligent behaviour using propagation algorithms borrowed from social network models whereby information is transmitted, or in this case received, based on the actions of neighbours

- 1 #AllTrade#;
- 2 while(match(change))do(
- 3 one(change);
- 4 #AllTrade#)

Strategy 1: FixedPointSearch

- 1 setPos(crtGraph);
- 2 repeat(one(requesttobuy);
- 3 one(beginanalysis);
- 4 (one(deviationresult);one(deviationdecision)) orelse
- 5 (one(followresult);one(followdecision))
- 6 setPos(crtGraph);
- 7 one(updatez))(k)

Strategy 2: AllTrade

or neighbours-of-neighbours or other clusters [9]. The details of this alternative model shall be future work.

Tables 3 and 4 describe the nodes in the system and their ports respectively.

After the creation of a comprehensive set of rules, reduction strategies were created that defined the sub-graphs to be selected for evaluation and which rules should be applied to the starting state of the model and it is from this point that the derivation tree begins to undergo construction as the execution strategy calls on rules that create step-wise transformations. Specifically, the asset transfer processes are governed by the strategies *AllTrade* and *FixedPointSearch* (see Strategies 1 and 2 below), using 8 rewrite rules summarised in Tables 1 and 2.

Also highlighting flexibility is the fact that a variant of strategy *AllTrade* can simply replace the orelse operator by a ppick operator, and then begin to model probabilistic choice of logit type between following or deviating from the negligence rule. The probability distribution used in this case implements the stochastic "trembles" described in [3] and can be written within our strategy environment as follows:

ppick(followResult, deviationResult, udfLogitModel)

where udfLogitModel is a function that reads the profitability of being negligent or diligent (attributes U1 and U0 in the node Theta of the graph produced by the relevant rule) and returns the following values as a list: $\mathscr{Q}_{U(z-1)}$

$$\frac{\exp^{\mathscr{B}U_i(z=1)}}{\exp^{\mathscr{B}U_i(z=1)} + \exp^{\mathscr{B}U_i(z=0)}} \quad and \quad 1 - \left(\frac{\exp^{\mathscr{B}U_i(z=1)}}{\exp^{\mathscr{B}U_i(z=1)} + \exp^{\mathscr{B}U_i(z=0)}}\right)$$

where *i* is the current agent number and \mathscr{B} is the intensity of choice parameter that controls the ease by which fixed point is reached (as specified in [3]).

4 Testing

A successful base case validation has seen test results (see Figure 2 where average Z count value is plotted versus depth of the simulation) line up qualitatively with results from the more traditional ABM simulation given in [1]. In particular, for high values of p (that is, high probability of toxicity), we observe the expected result when the initial state contains a mixture of negligent and diligent agents: a sharp drop in Z, corresponding to a sharp switch in average approach (i.e., more banks decide to perform

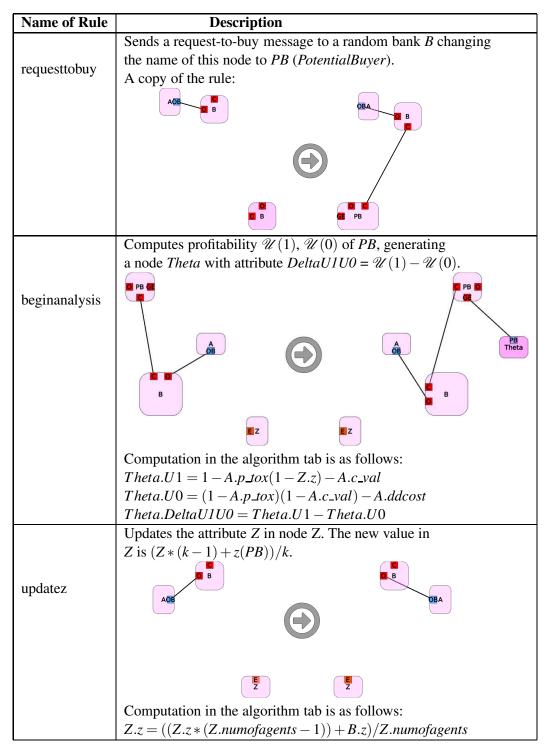


Table 1: Rewrite Rules

Name of Rule	Description		
followresult	Applies if $DeltaU1U0 \ge 0$.		
	As additional visualisation support, it generates a <i>follow</i>		
	node if more profitable to not do a full risk analysis.		
lonowiesuk	Theta Theta		
	follow		
	Arrow-node Application Condition:		
	If Theta.DeltaU1U0 ≥ 0		
deviationresult	Applies if <i>DeltaU1U0</i> < 0		
	As additional visualisation support, it generates a <i>deviation</i>		
	node if more profitable to do a full risk analysis		
	(Similar to <i>followresult</i>).		
	Transfers asset and prepares for a new transaction (i.e. cleans		
followdecision	up after the decision negligence rule), updating bank's attribute z ,		
	updating the <i>Change</i> counter if necessary.		
	Theta follow Chiange		
deviationdecision	Transfers asset and prepares for a new transaction (i.e. cleans up after the decision to deviate from the negligence rule), updating bank's attribute <i>z</i> , updating the <i>Change</i> counter if necessary (Similar to <i>followdecision</i>).		
	Sets the <i>Change</i> counter back to 0 if greater than 0.		
change	Change Change		
	Computation in the algorithm tab is as follows:		
	Change.change = 0		
	Change.sumofchange = 0		

Table 2: Rewrite Rules

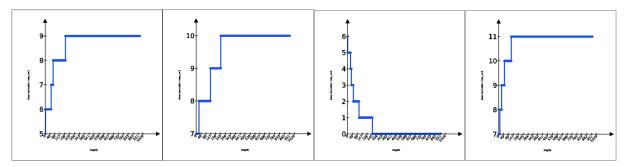
Entity	Attribute	Description
	Payoff (payoff)	Returns from re-selling an asset.
Bank/Potential		
Buyer (B/PB)	z	Indicates whether or not,
		as a rule, the institution
		performs independent risk analyses
	Bank ID (b_id)	Bank identifier
	Current Value (c_val)	Cost of purchasing an asset
	Probability	An asset is toxic if the borrowers
	of Toxicity (p_tox)	of the underlying loans are
Asset		likely to default or are in default
	Actualised Toxicity	Current toxicity level
	(a_tox)	
	Perception (pe)	External rating of the asset
		by rating agencies
	Due Diligence Cost	Full cost of an independent
	(ddcost)	risk assessment
Change	change	Change in bank approach
Change	Sum of change	Sums all changes in a current cycle
	(sumofchange)	
Z	Z	Represents the global average z
	Number of Iterations	Counter that keeps track of
	(numofiterations)	AllTrade iterations
	Number of Agents	Variable that keeps track of
	(numofagents)	number of banks
Theta	U1	Profitability of being negligent
	UO	Profitability of being diligent.
	DeltaU1U0	Difference between U1 and U0.

Table 3: Nodes and Attributes

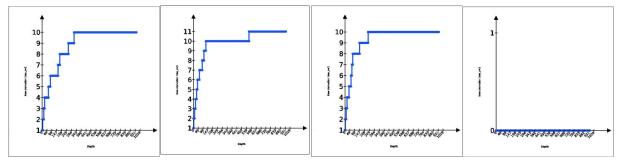
Entity	Ports	Description
Bank	O (Owns)	Edges attached to this port highlight
		assets owned by the bank
	C (Contacts)	Communication channel with another bank
Asset	OB (Owned_by)	Connects the asset to its current owner
Z	EN (Environment)	Global entity that tracks current average sentiment
PotentialBuyer	O (Owns)	Links to assets owned by the bank
	C (Contacts)	Communication channel with another bank
	GE (Generates)	Declares a relationship with an analysis node
Change	CH (change)	Counter that keeps track of behaviour changes
Theta	PB (Produced_by)	Entity that produces this computation helper

Table 4: Ports in each kind of node

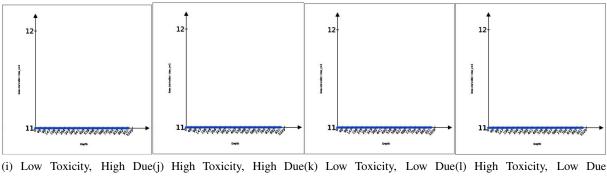
independent analysis), which in turn will generate stability. An illustration of this can be seen in Figure 2c and notice that given high due diligence costs Figures 2b and 2e highlight a negligent approach whereas Figures 2c and 2h reflect the favouring of a diligent approach. However, even for high toxicity, if the initial state is a set of negligent agents, the model reaches equilibrium without switching approach as seen in Figure 2l.



(a) Low Toxicity, High Due(b) High Toxicity, High Due(c) High Toxicity, Low Due(d) Low Toxicity, Low Due Diligence Cost, Mixture of Dili-Diligence Cost, Mixture of Diligence Cost, Mixture of Diligence Cost, Mixture of gent and Negligent Banks Diligent and Negligent Banks Diligent and Negligent Banks



(e) High Toxicity, High Due(f) Low Toxicity, High Due(g) Low Toxicity, Low Due(h) High Toxicity, Low Due Diligence Cost, Diligent Banks Diligence Cost, Diligence Cost,



(i) Low Toxicity, High Due(j) High Toxicity, High Due(k) Low Toxicity, Low Due(l) High Toxicity, Low DueDiligence Cost, NegligentDiligence Cost, NegligentDiligenc

Figure 2: *Experiment Results.* (y-axis: Count of the number of negligent banks. The intersection of x and y axes in the case of a starting universe of purely diligent banks corresponds to the co-ordinates (0,0) as opposed to (11,0) in the case where we begin with negligent banks. Curves tending upwards reflect a negligent equilibrium result)

5 Some Related Work and Conclusion

Fundamentally, general purpose agent-based simulation tools and platforms¹ like JAS, Netlogo, Agent-Builder, Swarm, MASON, Repast, SeSAm, GAMA and INGENIAS Development Kit, support an imperative object-oriented approach to model development, facilitating the modular approach to coding. EMERALD and JADE middleware integrate a declarative approach but without any visualization support. Other tools and languages like Stratego, Maude and ELAN [8] support a pure term-writing approach which in the case of Maude is augmented by probabilistic features. What can we do in our chosen approach that we could not have achieved using existing solutions cited? The visual, declarative nature of graph transformation systems are welcome in the cases where users seek to primarily focus on describing what the system should accomplish in terms of problem domain versus the how, and maintain strong conceptualization support that can subsume the details of spatial and topological constraints.

We have shown that strategic port-graph rewriting provides a basis for the design and implementation of a multi-level graph model able to capture the inner workings of the sub-prime secondary securitisation market in a manner that reflects the aforementioned rational negligence phenomenon and that provides optional operational support.

We observed that a declarative approach is much easier to program and maintain, and the incremental manner in which development was approached (e.g. coarse-grained rules tested before finer optimizations), in addition to the modular nature of development, eliminated many coding bugs. Interacting with the system was not convoluted in anyway and being able to view the states generated by each rule within the resulting derivation tree was useful in addition to being able to view nodes within the tree that maintained specific properties.

In future, we hope to further develop the hierarchical model to be able to capture all details of the full securitisation life-cycle and cater for more dynamically to changing parameters.

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