

Rapid Recovery for Systems with Scarce Faults*

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Our goal is to achieve a high degree of fault tolerance through the control of a safety critical systems. This reduces to solving a game between a malicious environment that injects failures and a controller who tries to establish a correct behavior. We suggest a new control objective for such systems that offers a better balance between complexity and precision: we seek systems that are *k-resilient*. In order to be *k-resilient*, a system needs to be able to rapidly recover from a small number, up to k , of local faults infinitely many times, provided that blocks of up to k faults are separated by short recovery periods in which no fault occurs. *k-resilience* is a simple but powerful abstraction from the precise distribution of local faults, but much more refined than the traditional objective to maximize the number of local faults. We argue why we believe this to be the right level of abstraction for safety critical systems when local faults are few and far between. We show that the computational complexity of constructing optimal control with respect to resilience is low and demonstrate the feasibility through an implementation and experimental results.

1 Introduction

We study the problem of achieving fault tolerance by construction (i.e., by synthesis [8, 28, 31]). This is a challenging problem for two simple reasons. One reason is that the size of the relevant systems is too large for exhaustive analysis. A second difficulty is the need to preserve the behavior of the system under failures and the proposed recovery mechanism, related to (i.e., included in) the non-failing behavior of the *original system*. This makes the problem much harder than checking whether or not a system satisfies a given set of simple temporal properties.

Our goal is to develop a technique for synthesizing a controller that is resilient to an *unbounded* number of failures. With this problem in mind, we took our inspiration from methods for resilient avionic systems [34], where fault tolerance is designed to recover from a *bounded* number of failures. The number of failures a system needs to tolerate is inferred from the given maximal duration of a flight and the mean time between failures of the individual components. Designing an optimal control with respect to this objective reduces to finding the maximal number of failures a system can tolerate without exhibiting an error. Maximizing the number of tolerable failure is an abstraction from the control objective of minimizing the chance of erroneous system behavior. This abstraction is justified by the simplicity of the control objective, but we argue that a finer abstraction provides for better control while maintaining a simple objective.

We develop a technique for synthesizing a controller that is resilient to an *unbounded* number of failures. Instead, we assume a bound on the number of *dense failures*, which may occur before the

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system is fully recovered. After full recovery, the system is allowed again the same number of failures. Now, if the mean time between failures is huge compared to the time the system needs to fully recover, then the expected time for system breakdown grows immensely. We assume an abstract representation of the system and recovery building blocks such as redundancy and majority checking among components.

Comparison. To demonstrate the difference between the objective to tolerate k errors and k dense errors, we exemplify the quality guarantees one obtains for a system (e.g., an aeroplane) with an operating time of 20 hours and a mean time between exponentially distributed local failures (MTBF) of 10 hours, assuming a repair time of 36 seconds.

	1	2	3	4	5	6	...
k failures	59.4%	33.3%	14.3%	5.3%	1.7%	0.5%	...
k dense failures	0.2%	$2 \cdot 10^{-4}\%$	$2 \cdot 10^{-7}\%$	$2 \cdot 10^{-10}\%$	$2 \cdot 10^{-13}\%$	$2 \cdot 10^{-16}\%$...

The figures for k failures are simply the values for the Poisson distribution with expected value 2 (20 hours flight and 10 hours MTBF). To explain the figures for k dense errors, consider the density of 2 dense errors occurring in close succession. If a failure occurs, the chance that the next failure occurs within the repair time (36 seconds) is approximately $\frac{1}{1000}$. Only failures for which this is the case could cause an error. The mean time between blocks of two dense failures is therefore not ten hours, but ten thousand hours. Likewise, it increases to ten million and ten billion hours for blocks of three and four dense failures, respectively. Maximizing the number of dense failures that are permitted is therefore a natural design goal.

Our proposed correctness criterion is formalized through a game between the system and an environment that induces the failures; the system is trying to recover, while the environment can inject further failures. We provide a technique, based on finding a game strategy, that allows checking whether or not we can control our system to meet this predefined resilience level. It also allows finding the maximal resilience level of the system. Moreover, the sought strategy provides the control that can be added to the system to achieve this resilience level. This is a more ambitious control problem than just meeting predefined bounds, allowing us to tap the full potential of a system.

Related Work. Dijkstra’s self-stabilization criterion [1, 12] suggests to build systems that eventually recover to a ‘good state’, from where the program commences normally. Instead of *constructing* a system to satisfy such a goal, one may want to apply control theory to *restrict* the execution of an existing system to achieve an additional goal. Our control objective is a recovery mechanism for up to k failures. After recovery, the system has to tolerate up to k failures again, and so forth. In this work, we suggest a mechanism to synthesize a recovery mechanism for a given fault model and recovery primitives.

Synthesis goes back to Church’s solvability problem [8]. It inspired Rabin’s work on finite automata over infinite structures [31] and Büchi and Landweber’s works on finite games of infinite duration [6, 7]. These techniques have been developed further in open synthesis [28, 29, 23, 35] and control theory [3, 14, 18, 34].

Traditionally, fault tolerance refers to various basic fault models [3], such as a limited number of errors [21]. These traditional fault models are subsumed by more general synthesis or control objectives [2, 3, 32, 36]; as simple objectives with practical relevance, they have triggered the development of specialized tools [14, 18].

In [13], an interesting notion of robustness based on Hamming and Lewenstein distance related to the number of past states is defined. It establishes a connection between these distances with a notion of synchronization that characterizes the ability of the system to reset for combinatorial systems. In [5],

‘ratio games’ are discussed, where the objective is to minimize the ratio between failures induced by the environment and system errors caused by them.

Maximizing the number of dense failures can, just like maximizing the overall number of failures, be viewed as an abstraction of the objective to minimize the likelihood of an error. In principle, one could seek optimal control against a Markovian failure model rather than approaching an abstraction. But approximating optimal control seems to be expensive [16] and results in more complex control strategies. Two further problems are that the precise probabilistic behavior of the environment is usually unknown, and that probabilistic analysis does not mix well with abstraction.

2 Preliminaries

We propose a method for achieving fault tolerance under an architecture that allows generic building blocks such as redundancy and majority checks [30], while making some realistic restricting assumptions on the behavior of the faults. The original system can be described using a transition system or an automaton. We impose a failure model that transforms the system by allowing new *fault transitions*. We include a recovery mechanism that transforms the resulting faulty version of the system by allowing new *recovery transitions*.

We assume that the transitions of the recovery mechanism cannot block the added fault transitions. They refer to standard recovery building blocks, e.g., providing redundancy and performing majority checks. We do not study or try to design these building blocks themselves, but resolve the choices that need to be made in the presence of alternative recoveries, e.g., which of them should be initiated. The recovery mechanism itself does neither take the number of the faults into account, nor their density.

Our task is to prune the choices available to the recovery mechanism, based on assumptions made on the behavior of the fault model. One can consider this as a game between two players: the recovery mechanism that needs to make the right decision when choosing from several recovery transitions, against the failure model that can inject some (restricted) amount of fault transitions into the system.

The ability to choose makes the moves of the recovery mechanism *controllable*, while the moves represented by the failure model are *uncontrollable*. Although the moves of the antagonistic failure mechanism are uncontrollable, we may make some reasonable assumptions about them, e.g., limiting the number of failures occurring in close succession. Similar restrictions are used in practice: in avionics, for example, one can provide some measurable bounds for such parameters per aircraft, based on capabilities such as flight distance and speed, and then add some safety factor.

Performing a game strategy check on the level of the system combined with the fault model and recovery mechanism is unrealistic. In fact, it is also unnecessary: standard building blocks, e.g., redundancy and majority-checking for fault tolerance, provide a simple abstraction of it into a finite state system. It includes in particular some *error* states that are unrecoverable, represented as sinks in the state graph of the system. The abstraction is often independent of the actual details of the original system.

A game theoretical search can then be applied to the abstract version. The goal of the protagonist is to avoid the error states; the fault tolerance mechanism must ensure this goal in the presence of uncontrolled (malicious) failures imposed by the failure mechanism. The game takes the restrictions on the faults into account. A win of the protagonist (the recovery mechanism) provides a strategy, which can immediately be translated into a controller that can be used to restrict the behavior of the recovery transitions. Playing the game on the actual system rather than the abstract one according to this strategy guarantees the same goal for the actual system with the fault model and recovery mechanism. We can not only apply such a strategy search in order to check the resilience level: the strategy obtained provides a control for the

recovery mechanism that achieves this resilience level. Note that the controller can only block choices of the recovery mechanism and not the transitions of the original system (or the fault model).

The modeling of the transformation involves the fault model and the recovery mechanism. The abstractions used to reason on a small model are rather standard model-checking techniques, hence would not be formally presented here. In the following section, we reason formally on the model that represents the abstracted system including the fault model and the recovery mechanism.

Running Example: Controlling Avionic Systems. Consider an example of a system that includes n processors, each can follow the instructions of the original system M , or be engaged in memory recovery. There are also m copies of the memory. When a copy of the memory fails, a processor can be assigned to recover it. Majority check can be used to detect that a processor is faulty or that memory is defected (usually, both would happen at the same time). For recovery, we can set a free processor to recover some memory, or make a processor follow the code of the majority of processors. The problem of the controller is to decide whether to make a processor follow the execution of the majority, or to assign it to recover faulty memory. We assume the following uncontrollable fault and controllable recovery transitions:

Uncontrollable failure events:

- A processor performing the code of M fails.
- Memory fault occurs.

Controllable recovery events:

- Remove a process from following the main system M and being subjected to majority check.
- Assign a free processor to recover a memory that is known to be faulty.
- Assign a free processor to follow the code of the majority processors.

If there are no processes to perform any of the above controllable recovery events, then the system enters immediately an unrecoverable failure state.

The recovery mechanism in the above is quite typical in the design of fault-tolerant systems [30]. As explained, a practical recovery mechanism usually does not rely on the detailed structure of M . Instead, fault-detection techniques such as parity checks, voting (for majority checks), etc., are usually employed. They are generic and therefore independent of the structure of M . Thus, an abstraction of M can be used in verifying the resilience of the recovery mechanism to dense faults. Such an observation can greatly reduce the cost of verifying the recovery mechanism.

We assume that the events of the recovery mechanism and fault detection are reliable. The issue of how to make these events reliable is orthogonal to the problem we are solving in this paper.

3 Resilience to k -dense Failures

We propose as a correctness criterion that can refer to a small abstract model \mathcal{T} , which contains the recovery and fault actions as its controllable and uncontrollable transitions, respectively. Such a *transition system with failures* $\mathcal{T} = (S, \mathfrak{t}, \tau_c, \tau_u, F)$ has

- states S , with an initial state $\mathfrak{t} \in S$ and error (i.e., *final* or *sink*) states $F \subseteq S$,
- *controlled* (recovery) transitions $\tau_c \subseteq (S \setminus F) \times (S \setminus F)$ and
- *uncontrolled* (failure) transitions $\tau_u \subseteq (S \setminus F) \times S$, where $\tau = \tau_c \cup \tau_u$.

We denote with $\text{suc}_c(s) = \{t \in S \mid (s,t) \in \tau_c\}$ and $\text{suc}_u(s) = \{t \in S \mid (s,t) \in \tau_u\}$ the controlled and uncontrolled successors of a state s , respectively. The size of \mathcal{T} , denoted $|\mathcal{T}|$, is $\max\{|S|, |\tau|\}$. Every (non-error) state in $S \setminus F$ has at least one controlled successor.

A state s is k -resilient if the system can be controlled such that groups of up to k dense failures can be tolerated infinitely many times, provided that the system is given enough time to recover. We describe informally how k -resilience is tested using a two-stage game over \mathcal{T} between the protagonist, i.e., the recovery mechanism, and the antagonist, i.e., the failure mechanism, who jointly move a pebble over the transition system. The construction of this set is described in Construction 3.3.

Let $G \subseteq S \setminus F$ be a subset of the non-error states. These states are intuitively the “good region” that the protagonist player wishes to remain in, and, if left, wishes to return to. In order to get a good intuition for the objective of this game, it is useful to consider its behavior when using the set of k -resilient states as G . The game is, however, defined for any set $G \subseteq S \setminus F$; we assume for the moment that this subset is given, and will later show how to calculate it.

The pebble is initially placed on a state $s \in G$. In every move, the protagonist starts with selecting a controlled transition originating from the state on which the pebble is. The antagonist can either agree on executing this transition (moving the pebble along it) or play a failure by selecting an uncontrollable transition, which is then executed.

The first part of the game is a safety part, which ends when the antagonist selects his first uncontrollable transition. With this move, the game proceeds to a reachability part. In the safety part of the game, the protagonist must remain in G ; she loses (and the antagonists wins) immediately if G is left. In the second phase, the protagonist becomes a reachability player with the goal to recover to G . She wins immediately when she has reached G again. The antagonist is further restricted in that he can, overall, play no more than k failures. Of course, if a state in F is reached, the antagonist wins immediately, as these states are sinks and are never in G . In *infinite* plays, the protagonist wins if the play stays for ever in the safety phase; otherwise the antagonist wins. The set of states from which the protagonist wins this game is the set of k -resilient states, denoted $\text{res}_k(G)$.

A state s is called k -sfrch (where sfrch refers to the combined safety/reachability objective) with respect to a set $G \subseteq S \setminus F$ of non-error states, denoted $s \in \text{sfrch}_k(G)$, if there is a strategy for the protagonist to win this game. This can be defined as follows:

Definition 3.1 Let $\text{sfrch}_k : 2^{S \setminus F} \rightarrow 2^{S \setminus F}$ be the function such that $\text{sfrch}_k(G)$ is the subset of G , from which the protagonist wins in the above game.

As an example for k -sfrch-ty, consider the transition system with four states, including a single error state (state 4, marked by a double line) shown in Figure 1. The controlled transitions are depicted as black full arrows, the uncontrollable (or fault) transitions are depicted as **dashed red arrows**. For $G = S \setminus F = \{1, 2, 3\}$, all states in G are in $\text{sfrch}_0(G)$. For all $k \geq 1$, we have $\text{sfrch}_k(G) = \{1, 2\}$: the protagonist can simply stay in $\{1, 2\}$ during the safety phase of the game, and once the antagonist plays a failure transition, the game progresses into the reachability phase, where the reachability objective is satisfied immediately. This outlines the difference between k -sfrch-ty and the linear time property of being able to tolerate k failures, which would be satisfied by state 1 and 2 only for $k \leq 2$ and $k \leq 1$, respectively.

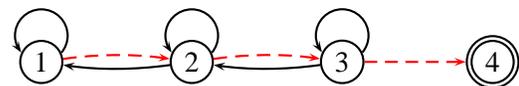


Figure 1: An example for calculating sfrch_k

This difference raises the question if the rules of our game are depriving the antagonist of some of the k failures he should intuitively have. The answer is that this is not the case if we use the k -resilient

states (or, more generally, any fixed point of sfrch_k) as G . In this case, we would win again from the state we reached; instead of depriving the antagonist, one could say that we reset the number of failures he can play to k .

For a state to be in $\text{sfrch}_k(G)$, the system has a strategy to recover to G , given that a long enough execution commenced without another failure happening. We say that two successive failures are in the same *group of dense failures* if the sequence of states separating them was not long enough for recovery in the respective safety/reachability game. Vice versa, if two successive failures are far enough apart such that the protagonist can guarantee recovery in this game, then they do not belong to the same group. In order to define k -resilience, we need to find a set of states G such that recovering to G by the protagonist (the fault tolerance mechanism) is always possible, provided that at most k failures occurred. To obtain this, observe that nesting sfrch_k once, i.e., $\text{sfrch}_k(\text{sfrch}_k(\cdot))$, corresponds to tolerating up to two sets of up to k close errors, and so forth. Thus, k -resilience is simply the greatest fixed point of the operator sfrch_k from Definition 3.1: a state is k -resilient if it is in $\text{sfrch}_k(\text{sfrch}_k(\text{sfrch}_k(\dots \text{sfrch}_k(G)))$, using sufficiently deep nesting that a fixed point is reached. For the control strategy, it suffices to use the control strategy from the outermost sfrch_k .

Lemma 3.2 *sfrch_k has a greatest fixed point.*

Proof: Follows from the facts that the function sfrch_k is monotonic ($G \subseteq G'$ implies $\text{sfrch}_k(G) \subseteq \text{sfrch}_k(G')$) because a winning strategy for the protagonist for G is also a winning strategy for G' for all states in $\text{sfrch}_k(G)$ and operates on a finite domain. \square

As an example, consider again the transition systems with four states. For $G = \{1\}$ ($\{1\} = \text{res}_2(\{1,2,3\})$), the only state in G , state 1, is 2-resilient: it can recover with the recovery strategy to always go to the left.

Construction 3.3 *$\text{res}_k(G_0)$, for $G_0 = S \setminus F$, can be constructed by choosing $G_{i+1} = \text{sfrch}_k(G_i)$ and fixing $\text{res}_k(G_0) = G_\infty$ to be the greatest fixed point of this construction.*

Note that this fixed point is what we are really interested in, while sfrch_k is a technical construction. If this greatest fixed point $G = \text{sfrch}_k(G)$ is non-empty, the protagonist's strategy for the fixed point G (guaranteeing eventual recovery to a state in G within no more than k failures, i.e., k -resilience) can be used to control the recovery mechanism, constraining its transitions to follow its winning strategy.

The natural control problem is to find optimal control that starts in the initial state ι of the transition system with failures.

Definition 3.4 *For a non-error state $s \in S \setminus F$, the resilience level k_{\max} is the maximal k such that s is k -resilient.*

To illustrate this point on our running example, suppose we have $2k+1$ copies of the system, with the ability to perform majority checks and identify the bad processes. According to the first, naïve solution, there is nothing to do after k failures. The majority checks is still capable to maintain the correctness of the combined behavior to follow the design of the original system M . However, there is no expectation that the system will be able to recover at any point in the future into a situation where it is again k -resilient; it *will* fail at the next round of faults. Our dense fault tolerance criterion requires that, given no more failures for enough time to allow recovery, the system will eventually recover to k -resilience again.

3.1 Construction of sfrch_k

Let $\text{sfrch}_0(G) = \bigcup \{G' \subseteq G \mid \forall g \in G' \exists s \in G'. (g, s) \in \tau_c\}$. This is the usual safety kernel of G consisting of states from which there is an infinite controlled sequence. It can be computed by the usual greatest fixed point construction.

Lemma 3.5 $\text{sfrch}_0(G)$ can be constructed, together with a suitable memoryless control strategy, in time linear in (S, τ_c) , and testing if a state is in $\text{sfrch}_0(G)$ is NL-complete.

Proof: Linear time is obvious for standard constructions of (least and) greatest fixed points, and for the memoryless control strategy it suffices to stay in $\text{sfrch}_0(G)$.

NL completeness can be shown by reduction to and from the repeated ST-reachability [27] (the question whether there is a path from a state S to a state T and from T to itself in a directed graph). \square

An intermediate step for the construction of k -sfrch states is an attractor construction that stays, through controlled moves, in a subset $L \subseteq S \setminus F$ of non-error states. As only controlled moves are allowed, this is merely a backwards reachability cone.

Definition 3.6 The controlled limited attractor set of a set G for a limited region $L \subseteq S$, denoted $\text{cone}_L(G)$ is the set $\text{cone}_L(G) = \bigcap \{A \supseteq G \mid \forall s \in L. (\text{suc}_c(s) \cap A \neq \emptyset) \text{ implies } s \in A\}$ for which there is a strategy to move to G without leaving L .

The controlled limited attractor set $\text{cone}_L(G)$ can be constructed using simple backwards reachability for G of controlled transition through states of L .

Lemma 3.7 $A = \text{cone}_L(G)$ and (a memoryless) attractor strategy for the states in $A \setminus G$ towards G can be constructed in time linear in the size of (S, τ_c) . Determining whether a state is in A is NL-complete (see [27]). \square

The controlled limited attractor set is used in the construction of $\text{sfrch}_k(G)$. We further construct a descending chain $A_0 \supseteq A_1 \supseteq \dots \supseteq A_{k-1}$ of limited attractors A_i . From A_i we have an attractor strategy towards G for the protagonist, which can tolerate up to i further failures. The respective A_i are attractors that avoid error states and, for $i > 1$, any uncontrolled transition leads to A_{i-1} .

Definition 3.8 A state $s \in S$ is fragile for a set $B \subseteq S$ if at least one of its uncontrolled successors is in B . The set of fragile states for B is $\text{frag}(B) = \{s \in S \mid \exists b \in B. (s, b) \in \tau_u\}$.

The set $\text{frag}(B)$ is easy to construct. The limited regions L_i of states allowed when approaching G also form a descending chain $L_0 \supseteq L_1 \supseteq \dots \supseteq L_k$. Using these building blocks, we can compute the k -sfrch states:

Construction 3.9 Starting with $L_0 = S \setminus F$ (if no further failures are allowed for, only error states are disallowed on the way to G), we define the A_i 's and L_i 's recursively by

- $A_i = \text{cone}_{L_i}(G)$ and
- $L_{i+1} = L_0 \setminus \text{frag}(S \setminus A_i)$,

and choose $\text{sfrch}_k(G) = \text{sfrch}_0(G \cap L_k)$.

Explanation. The states in L_{i+1} are the non failure states from which all (i.e., fault) uncontrolled transitions lead to a state in A_i . The sets A_i contain the states from which there is a controlled path to G that progresses through L_i ; all uncontrolled transitions originating from any state of this path lead to A_{i-1} . A_0 is therefore just the set of states from which there is a controlled path to G .

From all states in A_{k-1} , the protagonist therefore has a winning strategy in the second phase of the game described earlier: if the antagonist can play at most $k - 1$ failures, then the protagonist can make sure that G is reached.

Finding a control strategy for k -sfrch control within $\text{sfrch}_k(G)$ is simple: as long as we remain in $\text{sfrch}_k(G) = \text{sfrch}_0(G \cap L_k)$, we can choose any control action that does not leave $\text{sfrch}_k(G)$. Once $\text{sfrch}_k(G)$ is left through an uncontrolled transition to A_{k-1}, A_{k-2}, \dots , we determine the maximal i for which it holds that we are in A_i and follow the attractor strategy of $\text{cone}_{L_i}(G)$ towards G .

3.2 Complexity

All individual steps in the construction (intersection, difference, predecessor, and attractor) are linear in the size of the transition system with failures, and there are $O(k)$ of these operations in the construction. This provides a bi-linear (linear in k and $|\mathcal{T}|$) algorithm for the construction of sfrch_k and a strategy for the protagonist:

Lemma 3.10 *A memoryless control strategy for the states in $\text{sfrch}_k(G)$ can be constructed in time linear in both k and the size $|\mathcal{T}|$ of the transition system with failures \mathcal{T} . \square*

The complexity of determining whether or not a state s is in $\text{sfrch}_k(G)$ depends on whether or not we consider k to be a fixed parameter. Considering k to be bounded (or fixed) is natural in our context, because k is bounded by the redundancy.

Lemma 3.11 *For a fixed parameter k , testing if a state s of a transition system $\mathcal{T} = (S, \iota, \tau_c, \tau_u, F)$ is in $\text{sfrch}_k(G)$ is NL-complete. \square*

The proof is based on an inductive argument that uses the closure of NL under complementation [19] in the induction step. The details are moved to an appendix.

If k is considered an input, then reachability in AND-OR graphs can easily be encoded in LOGSPACE: It suffices to use the nodes of an AND-OR graph as the states, the outgoing edges of AND and OR nodes as controllable and uncontrollable edges, respectively, and then add a self loop for each OR node to the set of controllable edges. Choosing k to be the number of nodes of the AND-OR graph and F to be the target nodes of the AND-OR graph, a state in the AND-OR graph is not reachable from a designated root node, if the respective state in the resulting transition system is k -sfrch.

Given that reachability in AND-OR graphs is PTIME-complete [20], this provides:

Lemma 3.12 *If k is considered an input parameter, then testing if a state s of a transition system with failures $\mathcal{T} = (S, \iota, \tau_c, \tau_u, F)$ is in $\text{sfrch}_k(G)$ is PTIME-complete. \square*

The construction of $\text{res}_k(G)$ uses the repeated execution of $\text{sfrch}_k(\cdot)$. The execution of $\text{sfrch}_k(\cdot)$ needs to be repeated at most $O(|G|)$ times until a fixed point is reached, and each execution requires at most $O(k \cdot |\mathcal{T}|)$ steps by Lemma 3.10.

For the control strategy, we can simply use the control strategy from $\text{sfrch}_k(G_\infty)$ from the fixed point $G_\infty = \text{res}_k(G)$. This control strategy is memoryless (cf. Lemma 3.10).

Lemma 3.13 *$\text{res}_k(G)$ and a memoryless k -resilient control strategy for $\text{res}_k(G)$ can be constructed in $O(k \cdot |G| \cdot |\mathcal{T}|)$ time. \square*

The complexity class is (almost) independent of the parameter k :

Lemma 3.14 *The problem of checking whether or not a state s is k -resilient for a set G is PTIME-complete for all $k > 0$ and NL-complete for $k = 0$.*

Proof: For inclusion in PTIME, see the previous lemma. (Note that we can assume $k \leq |S|$ without loss of generality.) For hardness in the $k > 0$ case, we can use the same reduction from the reachability problem in AND-OR graphs as for k -sfrch-ty.

For $k = 0$, $\text{sfrch}_0(G) = \text{sfrch}_0(\text{sfrch}_0(G))$ implies $\text{res}_0(G) = \text{sfrch}_0(G)$. The problem of checking if a state is in $\text{res}_0(G)$ is therefore NL-complete by Lemma 3.5. \square

Finding the resilience level k_{\max} for the initial state ι requires at most $O(\log k_{\max})$ many constructions of $\text{res}_i(G)$. We start with $i = 1$, double the parameter until k_{\max} is exceeded, and then use logarithmic search to find k_{\max} .

Corollary 3.15 *For the initial state ι , we can determine the resilience level $k_{\max} = \max\{n \in \mathbb{N}_0 \mid \iota \in \text{res}_n(S \setminus F)\}$ of ι , $\text{res}_{k_{\max}}(S \setminus F)$, and a memoryless k_{\max} -resilient control strategy for $\text{res}_{k_{\max}}(S \setminus F)$ in $O(|S| \cdot |T| \cdot k_{\max} \log k_{\max})$ time. \square*

3.3 Handling Recovery Delay

To keep the representation concise, we distinguish three types of transitions instead of two: failure, control, and repair transitions. Repair transitions are denoted $\tau_r \subseteq (S \setminus F) \times (S \setminus F)$ and $\tau = \tau_c \cup \tau_r \cup \tau_u$ is the new set of transitions. Repair transitions intuitively refer to good events; in our running example, e.g., the completion of a recovery.

Repair transitions cannot be added to the fault tolerance mechanism, unless we keep track of *when* they are scheduled to happen, which led to significant blow-up of the state space. But they do not represent failures; to the contrary, they represent improvements that help the fault tolerance player and should therefore not be considered in the quota allowed for the failure mechanism to win (towards an unrecoverable situation).

To illustrate this, consider our running example from Section 2, where some component recovery of a memory unit is *initiated* by the fault tolerance mechanism, assigning a process for that recovery. Such a component recovery cannot be modeled as immediate (atomic), because such a model would mask the competition on the free processes, leading to the trivial and unrealistic optimal strategies based on instant component recovery from every fault. Thus, transitions which represent that a component recovery *terminates* should not be under the control of the fault tolerance mechanism. On the other hand, these transitions help the fault tolerance mechanism, hence they must not come on the expense of the k allowed attempts of the failure mechanism.

We change the game from the definition of sfrch_k states as follows. In a state s , the protagonist chooses a set of states contained in $\text{suc}_c(s) \cup \text{suc}_r(s)$ that must contain all states $\text{suc}_r(s) = \{t \in S \mid (s, t) \in \tau_r\}$ reachable through a recovery transition. (As recovery is an abstraction, she cannot prevent it from happening.) The antagonist then either chooses among these states (this choice is, of course, not counted against his k failure moves) or overwrites this selection by a failure transition. Similar to strong fairness, we require that, if repair transitions are offered infinitely often, then they must be taken infinitely often.

The effect of adding the recovery transitions in this way is on the construction of the controlled limited attractor $\text{cone}_L(G)$. We now get:

$$\text{cone}_L(G) = \bigcap \{A \supseteq G \mid (\forall s \in L. (\text{suc}_c(s) \cup \text{suc}_r(s)) \cap A \neq \emptyset \text{ and } \text{suc}_r(s) \subseteq A) \text{ implies } s \in A\}$$

is the smallest set $A \supseteq G$ such that $s \in A$, if

1. there is a controlled *or* a recovery transition from s to A , and
2. all recovery transitions originating from s lead to A .

Note that we assume here that, as part of its strategy, the recovery mechanism can decide *not* to play a move, in the light of the possibility that, when she waits long enough, the repair agent will improve the situation, progressing towards G .

Complexity. When used on graphs (as in Subsection 3.1), then determining if a state is in the controlled limited attractor is NL-complete (cf. Lemma 3.7). This situation changes when we use the controlled limited attractor on games: the problem becomes PTIME-complete. From a more applied point of view, however, nothing changes: the attractor construction still needs to visit each transition of τ_u and each transition in τ_r once.

Lemma 3.16 $\text{cone}_L(G)$ can be constructed in time $O(|\mathcal{T}|)$, and checking membership of a state in $\text{cone}_L(G)$ is PTIME-complete.

Proof: The running time is implied by the construction, while hardness can be shown by the same reduction to AND-OR graphs as in the hardness proof of Lemma 3.12, replacing fault transitions by repair transitions. \square

The maintained bound on the running time for computing the controlled limited attractor in games implies that the bounds of sfrch_k and res_k are maintained as well.

Corollary 3.17 $\text{sfrch}_k(G)$ resp. $\text{res}_k(G)$ can be constructed in time $O(k \cdot |\mathcal{T}|)$ resp. $O(k \cdot |S| \cdot |\mathcal{T}|)$.

For the initial state \mathfrak{t} , we can determine the resilience level $k_{\max} = \max\{n \in \mathbb{N}_0 \mid \mathfrak{t} \in \text{res}_n(S \setminus F)\}$ of \mathfrak{t} , $\text{res}_{k_{\max}}(S \setminus F)$, and a memoryless k_{\max} -resilient control strategy for $\text{res}_{k_{\max}}(S \setminus F)$ in $O(|S| \cdot |\mathcal{T}| \cdot k_{\max} \log k_{\max})$ time. \square

As an alternative definition, we could force the control player to make a move instead of allowing her to abstain. In this alternative version, the evaluation of the attractor game is similar to the evaluation of a reachability game with strong fairness. Consequently the complexities would grow by a factor of $|S|$.

Theorem 3.18 In the alternative setting, $\text{sfrch}_k(G)$ and $\text{res}_k(G)$ can be constructed in time $O(k \cdot |S| \cdot |\mathcal{T}|)$ and $O(k \cdot |S|^2 \cdot |\mathcal{T}|)$, respectively.

For the initial state \mathfrak{t} , we can determine the resilience level $k_{\max} = \max\{n \in \mathbb{N}_0 \mid \mathfrak{t} \in \text{res}_n(S \setminus F)\}$ of \mathfrak{t} , $\text{res}_{k_{\max}}(S \setminus F)$, and a memoryless k_{\max} -resilient control strategy for $\text{res}_{k_{\max}}(S \setminus F)$ in $O(|S|^2 \cdot |\mathcal{T}| \cdot k_{\max} \log k_{\max})$ time. \square

The proof and the details of the construction are moved to the appendix.

4 Tool Implementation and Experiments

We adopt *CEFSM* (communicating extended finite-state machine) [4] as a convenient language for the description of abstract models of our state transition systems. A CEFSM consists of several finite-state machines extended with shared variables for the modeling of shared memory and with synchronizations for the modeling of message-passing in distributed systems. This is justifiable since the fault-tolerant algorithms may themselves be subject to restrictions in concurrent or distributed computation. Indeed, we found CEFSM very expressive in modeling the benchmarks from the literature [9, 33].

Implementation. In the following, we report our implementation and experiment with our constructions. Our implementation is based on symbolic on-the-fly model-checking techniques and built on the simulation/model-checking library of REDLIB in <http://sourceforge.net/projects/redlib/> for fast implementation. Our implementation and benchmarks can also be found in the same page.

The translation from our CEFSMs to state transition systems, for example finite Kripke structures, is standard in the literature. All state spaces, conditions, preconditions, post-conditions, fixed points, etc. are represented as logic formulas. The logic formulas are then implemented with multi-value decision diagrams (MDD) [26]. Due to the huge complexity of the transition relations, we did not explicitly construct the transition relations for precondition and post-condition calculation. Instead, the preconditions and post-conditions are constructed in a piecewise construction from the basic conditions for simple actions and transition rule triggering conditions.

Benchmarks. We use the following five parameterized benchmarks to check the performance of our techniques. Each benchmark has parameters for the number of participating modules in the model. Such parameterized models come in handy for the evaluation of the scalability of our techniques with respect to concurrency and model sizes.

1. We use the running example of avionic systems in Section 2 as our first benchmark. An important feature of this benchmark is that there is an assumed mechanism in detecting faults of the modules. Once a fault is detected, a processor can be assigned to recover the module, albeit to the cost of a reduced redundancy in the executions.
2. Voting is a common technique for fault tolerance through replication when there is no mechanism to detect faults of the modules. In its simplest form, a system can guarantee correctness, provided less than half of its modules are faulty.
3. This is a simplified version of the previous voting benchmark, where we assume that there is a blackboard for the client to check the voting result.
4. *Practical Byzantine fault-tolerance (PBFT)* algorithm: We use an abstract model of the famous algorithm by Castro and Liskov [9]. It does not assume the availability of a fault-detection mechanism but uses voting techniques to guarantee the correctness of computations when less than one third of the voters are faulty. This algorithm has impact on the design of many protocols [25, 11, 22, 17, 10] and is used in Bitcoin (<http://bitcoin.org/>), a peer-to-peer digital currency system.
5. *Fault-tolerant clock synchronization algorithm*: Clock synchronization is a central issue in distributed computing. In [33], Ramanathan, Shin, and Butler presented several fault-tolerance clock synchronization algorithms in the presence of Byzantine faults with high probability. We use a nondeterministic abstract model of the convergence averaging algorithm in their paper. The algorithm is proven correct when no more than one third of the local clocks can drift to eight time units from the median of all clock readings.

Modeling of the fault-tolerant systems. Appropriate modeling of the benchmarks is always important for the efficient verification of real-world target systems. Many unnecessary details can burden the verification algorithm and blow up the computation. On the other hand, sketchy models can then give too many false alarms and miss correct benchmarks. In this regard, we find that there is an interesting issue in the modeling of the above benchmarks. Replication and voting are commonly adopted techniques for achieving fault-tolerance and resilience. As can be seen, such fault-tolerant algorithms usually consist of several identical modules that use the same behavior templates. This observation implies that the identity of individual modules can be unimportant for some benchmarks. For such benchmarks, we can use counter abstraction [15, 24] in their models. (Details on the counter abstraction we used are provided in an appendix.) Specifically, we found that we can use counter abstraction to prove the correctness of benchmarks 1, 2, and 3. In contrast, the PBFT and the clock synchronization algorithms use counters for each module to model the responses received from its peer modules. As a result, we decided not to use counter abstraction to model these two algorithms in this work.

In the following, we use the running example of avionic systems in Section 2 to explain how we model a benchmark either as a plain CEFSM or with counter abstraction. We first present its CEFSM model template in Figure 2. The CEFSM model has n processors and m memory modules. Figures 2(a) and (b) are for the abstraction of processors and memory copies, respectively. The ovals represent local states of a processor or a memory module, while the arrows represent transitions. The transitions of a CEFSM are labeled with labels ‘U’ (for uncontrollable), ‘C’ (for controllable), or ‘R’ (for recovery). For example, when a memory module moves into a faulty state, an idle processor may issue an fd (fault-detected) event and try to repair the module by copying memory contents from normal memory modules. Such fault-detection is usually achieved with standard hardware. Note that the benchmarks are models that reflect the recovery mechanism, abstracting away the details of the original systems. A central issue

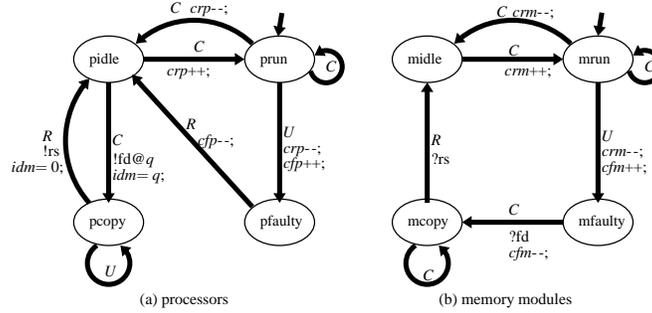


Figure 2: State graphs of processes and memory copies

Table 1: Performance data for resilience calculation

s: seconds; M: megabytes

benchmarks	concurrency	sfrch _k		res _k	
		time	memory	time	memory
avionics	6 processors & 6 memory modules	2.89s	129M	3.54s	516M
	7 processors & 7 memory modules	10.7s	216M	23.4s	808M
	8 processors & 8 memory modules	43.8s	1009M	135s	2430M
voting	1 client & 20 replicas	5.18s	229M	13.8s	236M
	1 client & 26 replicas	15.2s	334M	48.1s	348M
simple voting	1 client & 150 replicas	0.71s	159M	31.7s	219M
	1 client & 200 replicas	1.06s	161M	162s	337M
	1 client & 250 replicas	1.36s	163M	307s	499M
PBFT	1 client & 6 replicas	0.34s	72M	1.05s	193M
	1 client & 9 replicas	13.3s	564M	58.5s	1657M
clock sync	1 client & 15 servers	13.3s	547M	25.1s	648M
	1 client & 17 servers	20.7s	957M	58.3s	1127M

in the design of this recovery mechanism is then the resilience level of the controlled systems. More details are provided in an appendix.

Performance data. We report the performance data in Table 1 for the resilience algorithms described in Section 3.3 against the parameterized benchmarks in the above with various parameters. For the avionics system, the resilience level k is set to one less than half the number of processors. For the voting and simple voting benchmarks, the value of k is set to one less than half the number of replicas (voters). For the PBFT and clock synchronization algorithm, we choose k to be one less than one third of the number of replicas.

The performance data has been collected with a Virtual Machine (VM) running opensuse 11.4 x86 on Intel i7 2600k 3.8GHz CPU with 4 cores and 8G memory. The VM only uses one core and 4G memory.

The time and space used to calculate resilience is a little bit more than that to check for sfrch. The reason is that sfrch_k is a pre-requisite for calculating res_k. In our experiment, sfrch_k is usually very close to res_k and does not require much extra time in calculating res_k out of sfrch_k.

The experiments show that our techniques scale to realistic levels of redundancy. For fault-tolerant hardware, usually the numbers of replicas are small, for example, less than 10 replicas. Thus our techniques seem very promising for the verification of hardware fault-tolerance.

On the other hand, nowadays, software fault-tolerance through networked computers can create huge numbers of replicas. Our experiment shows that abstraction can be a useful techniques for the modeling and verification of software resilience.

5 Discussion

We introduced an approach for the development of a control of safety critical systems that maximizes the number of *dense* failures the system can tolerate. Our techniques are inspired by the problem of controlling systems with redundancy: in order to deflect the effect of individual failures, safety critical systems are often equipped with multiple copies of various components. If one or more components fail, such systems can still work properly as long as the correct behavior can be identified.

This inspired the two layered approach we developed in this paper. In a first layer, a complex system is simplified to the aspects relevant for its control. This abstraction layer of our approach reduces life size examples to small abstractions. In a second layer, we develop a control strategy, in which the controller identifies a k -resilient region; the controller can recover without seeing an additional error to the k -resilient region. Such a recovering strategy is memoryless. Being memoryless on a small abstraction in particular implies that the recovery is efficient.

The system can, once recovered, tolerate and recover from k further dense failures, and so forth. Consequently, our control strategy allows for recovery from an arbitrary number of failures, provided that the number of dense errors is restricted. This is the best guarantee we can hope for: our technique guarantees to find the optimal parameter k . This parameter is bound to be small (smaller than the number of redundant components). Optimizing it is computationally inexpensive, but provides strong guarantees: the likelihood of having more than k failures appear in short succession after a failure occurred are, for independent errors, exponential in k . As failures are few and far between, each level of resilience gained reduces the likelihood of errors significantly.

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