A multi-level model for self-adaptive systems

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This work introduces a general multi-level model for self-adaptive systems. A self-adaptive system is seen as composed by two levels: the lower level describing the actual behaviour of the system and the upper level accounting for the dynamically changing environmental constraints on the system. In order to keep our description as general as possible, the lower level is modelled as a state machine and the upper level as a second-order state machine whose states have associated formulas over observable variables of the lower level. Thus, each state of the second-order machine identifies the set of lower-level states satisfying the constraints. Adaptation is triggered when a second-order transition is performed; this means that the current system no longer can satisfy the current high-level constraints and, thus, it has to adapt its behaviour by reaching a state that meets the new constraints. The semantics of the multi-level system is given by a flattened transition system that can be statically checked in order to prove the correctness of the adaptation model. To this aim we formalize two concepts of weak and strong adaptability providing both a relational and a logical characterization. We report that this work gives a formal computational characterization of multi-level self-adaptive systems, evidencing the important role that (theoretical) computer science could play in the emerging science of complex systems.

1 Introduction

Self-adaptive systems are a particular kind of systems able to modify their own behaviour according to their environment and to their current configuration. They learn from the environment and develop new strategies in order to fulfil an objective, to better respond to problems, or more generally to maintain desired conditions. Self-adaptiveness is an intrinsic property of the living matter. Complex biological systems naturally exhibit auto-regulative mechanisms that continuously trigger internal changes according to external stimuli. Moreover, self-adaptation drives both the evolution and the development of living organisms.

Recently there has been an increasing interest in self-adaptive properties of software systems. In [17] the following definition is given: *"Self-adaptive software evaluates its own behaviour and changes behaviour when the evaluation indicates that it is not accomplishing what the software is intended to do, or when better functionality or performance is possible."*

As a matter of fact, software systems are increasingly resembling complex systems and they need to dynamically adapt in response to changes in their operational environment and in their requirements/goals. Two different types of adaptation are typically distinguished:

- *Structural adaptation*, which is related to architectural reconfiguration. Examples are addition, migration and removal of components, as well as reconfiguration of interaction and communication patterns.
- *Behavioural adaptation*, which is related to functional changes, e.g. changing the program code or following different trajectories in the state space.

Several efforts have been made in the formal modelling of self-adaptive software, with particular focus on verifying the correctness of the system after adaptation. Zhang et al. give a general state-based model of self-adaptive programs, where the adaptation process is seen as a transition between different non-adaptive regions in the state space of the program [23]. In order to verify the correctness of adaptation they define a new logic called A-LTL (an adapt-operator extension to LTL) and model-checking algorithms [24] for verifying adaptation requirements. In PobSAM [13, 14] actors expressed in Rebeca are governed by managers that enforce dynamic policies (described in an algebraic language) according to which actors adapt their behaviour. Different adaptation modes allow to handle events occurring during adaptation and ensuring that managers switch to a new configuration only once the system reaches a safe state. Another example is the work by Bruni et al. [6] where adaptation is defined as the run-time modification of the control data and the approach is instantiated into a formal model based on labelled transition systems. In [5], graph-rewriting techniques [18] are employed to describe different characterizations of dynamical software architectures. Meseguer and Talcott [19] characterize adaptation in a model for distributed object reflection based on rewriting logic and nesting of configurations. Theorem-proving techniques have also been used for assessing the correctness of adaptation: in [16] a proof lattice called transitional invariant lattice is built to verify that an adaptive program satisfies global invariants before and after adaptation. In particular it is proved that if it is possible to build that lattice, then adaptation is correct.

There are several other works worth mentioning, but here we do not aim at presenting an exhaustive state-of-the-art in this widening research field. We address the interested reader to the surveys [7, 21] for a general introduction to the essential aspects and challenges in the modelling of self-adaptive software systems.

1.1 A multi-level view of self-adaptation

Complex systems can be regarded as multi-level systems, where two fundamental levels can be distinguished: a **behavioural level** B accounting for the dynamical behaviour of the system; and a higher **structural level** S accounting for the global and more persistent features of the system. These two levels affect each other in two directions: *bottom-up*, e.g. when a collective global behaviour or new emergent patterns are observed; and *top-down*, e.g. when constraints, rules and policies are superimposed on the behavioural level. These two fundamental levels and their relationships are the base to scale-up to multi-level models. In a generic multi-level model, any n-th level must resemble the behavioural level, the corresponding $n+1$-level has to match with the structural level and the relationships between them will have to show the same characteristics. We discuss how this scale-up is implemented in our setting in Section 5

Multiple levels arise also when software systems are concerned. For instance, in [9] Corradini et al. identify and formally relate three different levels: the *requirement level*, dealing with high-level properties and goals; the *architectural level*, focusing on the component structure and interactions between components; and the *functional level*, accounting for the behaviour of a single component. Furthermore, Kramer and Magee [15] define a three-level architecture for self-managed systems consisting of a *component control level* that implements the functional behaviour of the system by means of interconnected components; a *change management level* responsible for changing the lower component architecture according to the current status and objectives; and a *goal management level* that modifies the lower change management plans according to high-level goals. Hierarchical finite state machines and Statecharts [11] have also been employed to describe the multiple architectural levels in self-adaptive software systems [12, 22].

In this work we introduce $S[B]$*-systems*: a general state-based model for self-adaptive systems where the lower behavioural level describes the actual dynamic behaviour of the system and the upper structural level accounts for the dynamically changing environmental constraints imposed on the lower system. The B-level is modelled as a state machine B. The upper level is also described as a state machine where each state has associated a set of constraints (logical formulas) over variables resulting from the observation of the lower-level states, so that each S-state identifies the set of B-states satisfying the constraints. Therefore, a set of dynamically changing constraints underlies a *second-order structure* S whose states are sets of B-states and, consequently, transitions relate sets of B-states.

We focus on *behavioural and top-down adaptation*: the B-level adapts itself according to the higher-level rules. In other words the upper level affects and constrains the lower level. Adaptation is expressed by firing a higher-order transition, meaning that the S-level switches to a different set of constraints and the B-level has adapted its behaviour by reaching a state that meets the new constraints. Our idea is broadly inspired by Zhang et al. [23], i.e. the state space of an adaptive program can be separated into a number of regions exhibiting a different *steady-state behaviour* (behaviour without reconfiguration). However, in our model the steady-state regions are represented in a more declarative way using constraints associated to the states of the S-level. Moreover, in $S[B]$-systems not only the behavioural level, but also the adaptation model embedded in the structural level is dynamic. Adaptation of the B-level is not necessarily instantaneous and during this phase the system is left unconstrained but an invariant condition that is required to be met during adaptation. Differently to [23], the invariants are specific for every adaptation transition making this process controllable in a finer way. The semantics of the multi-level system is given by a flattened transition system that can be statically checked in order to prove the correctness of the adaptation model. To this aim we also formalize the notion of adaptability, i.e. the ability of the behavioural level to adapt to a given structural level. We distinguish between weak and strong adaptability, providing both a relational and a logical characterization for each of them.

$S[B]$-systems has been inspired by some of the authors' recent work in the definition of a spatial bio-inspired process algebra called *Shape Calculus* [4, 3]. In that case, a process $S[B]$ is characterized by a reactive behaviour B and by a shape S that imposes a set of geometrical constraints on the interactions and on the occupancy of the process. This idea is shifted in a more general context in the $S[B]$-systems where, instead, we consider sets of structural constraints on the state space of the B-level. We want to underline that previous work and, mainly, this work have been conceived as contributions not only in the area of adaptive software system, but also in the area of modelling complex natural systems.

The notion of multiple levels that characterizes our approach for computational adaptive systems is something well-established in the science of complex systems. As pointed out by Baianu and Poli [2] *"All adaptive systems seem to require at least two layers of organization: the first layer of the rules governing the interactions of the system with its environment and with other systems, and a higher-order layer that can change such rules of interaction."* $S[B]$-systems are similarly built on two levels: the B-level describes the state-based behaviour of the system and the S-level regulates the dynamics of the lower level. In our settings, communication and interactions are not explicitly taken into account. Indeed the behavioural finite state machine can describe the semantics of a system made by several interacting components.

Another accepted fact is that higher levels in complex adaptive systems lead to higher-order structures. Here the higher S-level is described by means of a second order state machine (i.e. a state machine over the powerset of the B-states). Similar notions have been formalized by Baas [1] with the *hyperstructures* framework for multi-level and higher-order dynamical systems; and by Ehresmann and Vanbremeersch with their *memory evolutive systems* [10], a model for hierarchical autonomous systems based on category theory.

The paper is organized as follows. Section 2 introduces the formalism and the syntax of $S[B]$-systems, together with an ecological example that will be used also in the following. In Section 3 we give the operational semantics of a $S[B]$-system by means of a flattened transition system. In Section 4 we formalize the concepts of weak and strong adaptability both in a relational and in a logical form. Finally, conclusions and possible future developments of the model are discussed in Section 5.

2 A multi-level state-based model

An $S[B]$-system encapsulates both the behavioural (B) and the structural/adaptive (S) aspects of a system. The behavioural level is classically described as a finite state machine of the form $B = (Q, q_0, \rightarrow_B)$. In the following, the states $q \in Q$ will also be referred to as B-states and the transitions as B-transitions.

The structural level is modelled as a finite state machine $S = (R, r_0, \rightarrow_S, L)$ (R set of states, r_0 initial state, $\rightarrow_S$ transition relation and L state labelling function). In the following, the states $r \in R$ will be also referred to as S-states and the transitions as S-transitions. The function L labels each S-state with a set of formulas (the constraints) over an *observation* of the B-states in the form of a set of variables X. Therefore an S-state r uniquely identifies the set of B-states satisfying $L(r)$ and S gives rise to a second-order structure $(R \subseteq 2^Q, r_0, \rightarrow_S \subseteq 2^Q \times 2^Q, L)$.

In this way, behavioural adaptation is achieved by switching from an S-state imposing a set of constraints to another S-state where a (possibly) different set of constraints holds. During adaptation the behavioural level is no more regulated by the structural level, except for a condition, called *transition invariant*, that must be fulfilled by the system undergoing adaptation. We can think of this condition as a minimum requirement to which the system must comply to when it is adapting and, thus, it is not constrained by any S-state.

Note that an $S[B]$-system *dynamically* adapts and reconfigures its behaviour, thus both the behavioural level and the structural level are dynamic.

Definition 1 (**$S[B]$-system behaviour**) *The behaviour of an $S[B]$-system $S[B]$ is a tuple $B = (Q, q_0, \rightarrow_B)$, where*

- *Q is a finite set of states and $q_0 \in Q$ is the initial state; and*
- *$\rightarrow_B \subseteq Q \times Q$ is the transition relation.*

In general, we assume no reciprocal internal knowledge between the S- and the B-level. In other words, they see each other as *black-box systems*. However, in order to realize our notion of adaptiveness, there must be some information flowing bottom-up from B to S and some information flowing top-down from S to B. In particular, the bottom-up flow is modelled here as a set of variables $X = \{x_1, \ldots, x_n\}$ called *observables* of the S-level on the B-level. The values of these variables must always be *derivable* from the information contained in the B-states, which can possibly hold more "hidden" information related to internal activity. This keeps our approach black-box-oriented because the S-level has not the full knowledge of the B-level, but only some derived (e.g. aggregated, selected or calculated) information. Concerning the top-down flow, the B-system only knows whether its current state satisfies the current constraint or not. If not, we can assume that the possible target S-states and the relative invariants are outputted by the S-system and given in input to the B-system.

Definition 2 (**$S[B]$-system structure**) *The structure of an $S[B]$-system $S[B]$ is a tuple $S = (R, r_0, \rightarrow_S, L)$, where*

- *R is a finite set of states and $r_0 \in R$ is the initial state;*

- *$\rightarrow_S \subseteq R \times \Phi(X) \times R$ is a transition relation, labelled with a formula called invariant; and*
- *$L : R \rightarrow \Phi(X)$ is a function labelling each state with a formula over a set of* observables $X = \{x_1, \dots, x_n\}$.

Thus, an $S[B]$-system has associated a finite set $X = \{x_1, \dots, x_n\}$ of typed variables over finite domains $\{D_1, \dots, D_n\}$ whose values must be completely determined in each state of Q. More formally,

Definition 3 (Observation Function) *Given an $S[B]$-system $S[B]$ with a set $X = \{x_1, \dots, x_n\}$ of observables, an* observation function *$\mathscr{O}\colon Q \rightarrow \prod_{i=1}^{n} D_i$ is a total function that maps each B-state q to the tuple of variable values $(v_1, \dots, v_n) \in D_1 \times \dots \times D_n$ observed at q.*

Note that we do not require this function to be bijective. This means that some different states can give the same values to the observables. In this case, the difference is not visible to S, but it is internal to B.

We indicate with $\Phi(X)$ the set of formulas over the variables in X. We assume that constraints are specified with a first-order logic-like language.

Definition 4 (Satisfaction relation) *Let $S[B]$ be a $S[B]$-system with a set $X = \{x_1, \dots, x_n\}$ of observables and with an observation function $\mathscr{O}$. A state $q \in Q$* satisfies *a formula $\varphi \in \Phi(X)$, written $q \models \varphi$, iff φ is satisfied applying the substitution $\{^{v_1}/x_1, \dots, ^{v_n}/x_n\}$, where $\mathscr{O}(q) = (v_1, \dots, v_n)$, using the interpretation rules of the logic language.*

Let us also define an evaluation function $[[\_]] : \Phi(X) \rightarrow 2^Q$ mapping a formula $\varphi \in \Phi(X)$ to the set of B-states $Q' = \{q \in Q \mid q \models \varphi\}$, i.e. those satisfying φ.

Let us now give an intuition of the adaptation semantics. Let the active S-state be r_i and $r_i \xrightarrow{\varphi}_S r_j$. Assume that the behaviour is in a steady state (i.e. not adapting) q_i and therefore $q_i \models L(r_i)$. If there are no B-transitions $q_i \rightarrow_B q_j$ such that $q_j \models L(r_i)$ the system starts adapting to the target S-state r_j. In this phase, the B-level is no more constrained, but during adaptation the invariant φ must be met. Adaptation ends when the behaviour reaches a state q_k such that $q_k \models L(r_j)$.

The following definition determines when the structure S of a $S[B]$-system is well formed, that is: it must no contain inconsistencies w.r.t. all possible variable observations and the initial B-state must satisfy the initial S-state.

Definition 5 (Well-formed structure) *Let $S[B]$ be a $S[B]$-system. The structural level S is well-formed if the following conditions hold:*

- *for all S-states $r \in R$, $L(r)$ must be* ***satisfiable****, in the sense that there must be a variable observation under which $L(r)$ holds ($\exists q \in Q.\ q \models L(r)$) and*
- *the initial B-state must satisfy the constraints in the initial S-state, i.e. $q_0 \models L(r_0)$.*

In the remainder of the paper we assume to deal with well-formed structures without explicitly mentioning it.

2.1 An example from ecology

In this part we introduce a case study in the field of ecology and population biology: *the adaptive 1-predator 2-prey food web*. This system describes a variant of classical prey-predator dynamics where in normal conditions the predator consumes its favourite prey p_0. When the availability of p_0 is no longer sufficient for the survival of the predator, it has to adapt its diet to survive and it consequently starts

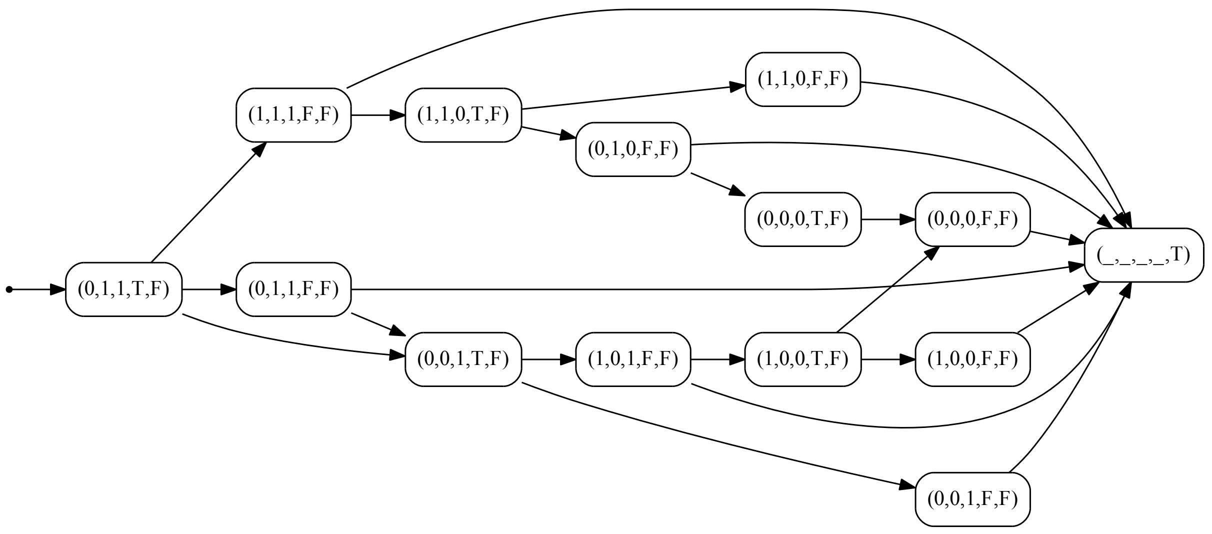


Figure 1: The behavioural state machine B for the adaptive 1-predator 2-prey food web example. Each state is characterized by a different combination of the variables $(p, a_0, a_1, eat, moved)$ (favourite prey, availability of p_0, availability of p_1, has the predator eaten?, has the predator migrated?). The initial state is $(0, 1, 1, \mathsf{true}, \mathsf{false})$. All the states where $moved = \mathsf{true}$ has been grouped for simplicity to a single state $(\_, \_, \_, \_\mathsf{true})$.

consuming another species p_1. For the sake of showing the features of our model, here we present an oversimplified version of this system that omits quantitative aspects like predation rates and growth of prey. We assume that the predator initially consumes the prey p_0 (variable $p = 0$) and that prey may be available (variable $a_i = 1,\ i = 0, 1$) or not (variable $a_i = 0,\ i = 0, 1$). The effect of consuming an available prey is to make that prey unavailable, as expected. The predator may also decide not to eat and change its diet (variable $p = 1$). A boolean variable tells whether in the current state the predator has eaten some prey (variable eat). At each step the predator can do one of the following:

- eat the currently favourite prey p_i, if available ($a_i \leftarrow a_i - 1$ and $eat \leftarrow \mathsf{true}$);
- do not eat and switch its favourite prey ($p \leftarrow |1 - p|$ and $eat \leftarrow \mathsf{false}$); or
- do not eat.

Finally, if the predator does not feed itself for two consecutive times, it migrates to a more suitable habitat (variable $moved = \mathsf{true}$) and no further actions are possible. The attentive reader may notice that under these restrictions the system will inevitably lead to a state where the predator moves to a different habitat. This is due to the fact that prey growth is not modelled here and it is always the case that the system eventually reaches a state where the predator cannot feed because of the unavailability of both prey. Each state of the behavioural level (depicted in Fig. 1) is described by a different evaluation of the involved variables:

$$(p, a_0, a_1, eat, moved) \in \{0, 1\} \times \{0, 1\} \times \{0, 1\} \times \{\mathsf{false}, \mathsf{true}\} \times \{\mathsf{false}, \mathsf{true}\}.$$

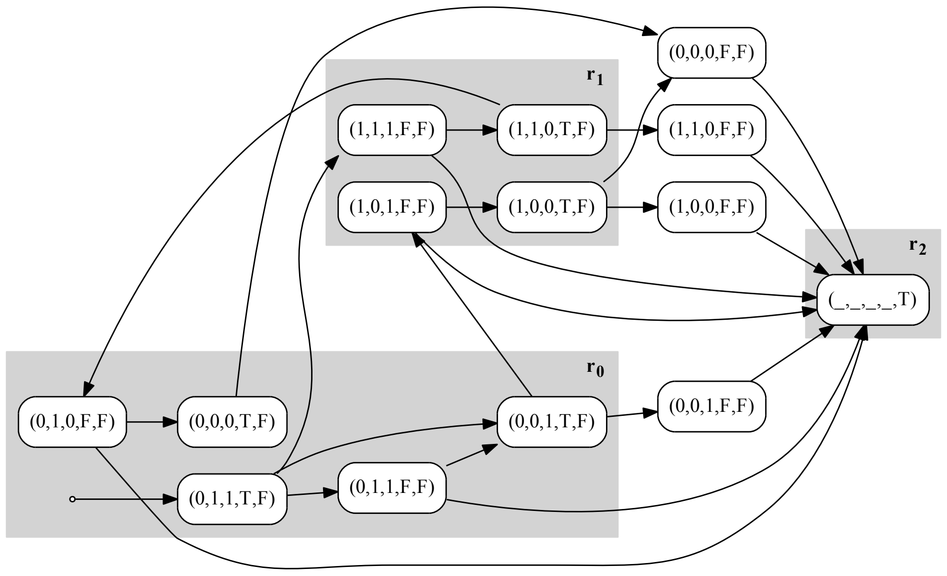


Figure 2: S-states determining stable regions in the adaptive 1-predator 2-prey system.

In this example we consider two different S-levels (represented in Fig. 3): S_0 and S_1, but with the same set of S-states. More specifically S_0 is given by:

$$\begin{aligned}
R = & \ \{r_0, r_1, r_2\}, \\
\rightarrow_S = & \ \{r_0 \xrightarrow{\neg moved} r_1, r_0 \xrightarrow{\neg eat} r_2, r_1 \xrightarrow{\neg moved} r_0, r_1 \xrightarrow{\neg eat} r_2\} \\
L(r) = & \ \{p == 0 \land (\neg eat \implies a_0 > 0) \land \neg moved\} \ \text{ if } r = r_0 \\
& \ \{p == 1 \land (\neg eat \implies a_1 > 0) \land \neg moved\} \ \text{ if } r = r_1 \\
& \ \{moved\} \ \text{ if } r = r_2.
\end{aligned}$$

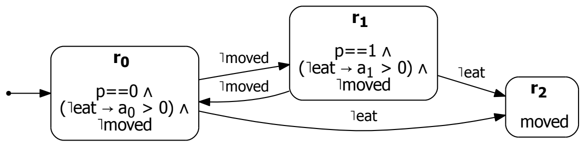
On the other hand, S_1 differs from S_0 only in the transition function, that is:

$$\rightarrow_S = \ \{r_0 \xrightarrow{p==1} r_1, r_1 \xrightarrow{\neg eat} r_2\}$$

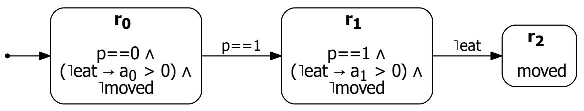
The three different S-states model three different stable regions in the prey-predator dynamics:

- r_0 : the predator consumes p_0. More precisely, the constraints require that the favourite prey must be p_0 ($p == 0$); that the predator has not moved to another habitat ($\neg moved$); and that if the predator is not currently feeding, the prey p_0 must be available so that the predator can eat in the following step ($\neg eat \implies a_0 > 0$).
- r_1 : the predator consumes p_1; the constraints are the same as r_0, but referred to prey p_1.
- r_2 : the predator has migrated.

Figure 2 shows how the structural constraints identify different stable regions in the behavioural level. The adaptation dynamics, regulated by the transitions in S_0, allow the predator to adapt from r_0 to r_1, under the invariant $\neg moved$ indicating that during adaptation the predator cannot migrate. The equivalent S-transition is defined from r_1 to r_0, so that the predator is able to return to its initially favourite prey. Both from r_0 and r_1 a S-transition to r_2 is allowed under the invariant $\neg eat$. In this way, the predator can



(a) The state machine S_0



(b) The state machine S_1

Figure 3: The two different structural levels S_0 and S_1 in the adaptive 1-predator 2-prey food web example. In each S-state r_i the constraint imposed to the behavioural level are represented. Transition labels indicate adaptation invariants. S_0 allows the predator to adapt its diet and migrate due to starvation anytime. In S_1 adaptation is guided from r_0 (consume prey p_0), to r_1 (consume prey p_1) and finally to r_2 (migration).

adapt itself and migrate to a different habitat under starvation conditions. On the other hand, the transition relation in S_1 has been defined in a simpler way, which makes the predator adapt deterministically from r_0 to r_1 and finally to r_2. In this case, the adaptation invariant from r_0 to r_1 requires that the predator has changed its diet to prey p_1.

The following section will show the operational rule for deriving the transitional semantics of the $S[B]$-system as a whole and the semantics of $S_0[B]$ and $S_1[B]$ in the adaptive 1-predator 2-prey system will be given as well.

3 Operational semantics

In this part, we give the operational semantics of an $S[B]$-system as a transition system resulting from the flattening of the behavioural and of the structural levels. We obtain a Labelled Transition System (LTS) over states of the form (q,r,ρ), where

- $q \in Q$ and $r \in R$ are the active B-state and S-state, respectively; and
- ρ keeps the target S-state that can be reached during adaptation and the invariant that must be fulfilled during this phase. Therefore ρ is either empty (no adaptation is occurring), or a singleton $\{(\varphi,r')\}$, with $\varphi \in \Phi(X)$ a formula and $r' \in R$ an S-state.

Definition 6 (Flat $S[B]$ − *system***)** *Let $S[B]$ be an $S[B]$-system. A flat $S[B]$-system is a LTS $F(S[B]) = (F, f_0, \xrightarrow{r} \cup \xrightarrow{r,\varphi,r'})$ where*

- *$F \subseteq Q \times R \times 2^{\Phi(X)\times R}$ is the set of states;*
- *$f_0 = (q_0, r_0, \emptyset)$ is the initial state;*
- *$\xrightarrow{r} \subseteq F \times F$, with $r \in R$, is a family of transition relations between non-adapting states satisfying $L(r)$; and*
- *$\xrightarrow{r,\varphi,r'} \subseteq F \times F$, with $r, r' \in R$ and $\varphi \in \Phi(X)$, is a family of transition relations between states during the adaptation determined by the S-transition $r \xrightarrow{\varphi}_S r'$. As a consequence it holds that for all r, r', φ, $\xrightarrow{r} \cap \xrightarrow{r,\varphi,r'} = \emptyset$.*

Table 1 lists the set of rules characterizing the flattened transitional semantics of an $S[B]$-system:

- Rule STEADY describes the steady (i.e. non-adapting) behaviour of the system. If the system is not adapting and the B-state q can perform a transition to a q' that satisfies the current constraints $L(r)$, then the flat system can perform a non-adapting transition $\xrightarrow{r}$ of the form $(q,r,\emptyset) \xrightarrow{r} (q',r,\emptyset)$.

$$\textsc{Steady}\frac{q \rightarrow_B q' \quad q' \models L(r)}{(q,r,\emptyset) \xrightarrow{r} (q',r,\emptyset)}$$

$$\textsc{AdaptStart}\frac{\forall q''.(q \rightarrow_B q'' \implies q'' \not\models L(r)) \quad q \rightarrow_B q' \quad r \xrightarrow{\varphi}_S r' \quad q' \models \varphi}{(q,r,\emptyset) \xrightarrow{r,\varphi,r'} (q',r,\{(\varphi,r')\})}$$

$$\textsc{Adapt}\frac{q \rightarrow_B q' \quad q' \models \varphi \quad q \not\models L(r')}{(q,r,\{(\varphi,r')\}) \xrightarrow{r,\varphi,r'} (q',r,\{(\varphi,r')\})}$$

$$\textsc{AdaptEnd}\frac{q \models L(r')}{(q,r,\{(\varphi,r')\}) \xrightarrow{r,\varphi,r'} (q,r',\emptyset)}$$

Table 1: Operational semantics of a $S[B]$-system

- Rule AdaptStart regulates the starting of an adaptation phase. Adaptation occurs when none of the next B-states satisfy the current specification ($\forall q''.(q \rightarrow_B q'' \implies q'' \not\models L(r))$, or more compactly $(q,r,\emptyset) \not\xrightarrow{r}$). In this case, for each S-transition $r \xrightarrow{\varphi}_S r'$ an adaptation towards the target state r' under the invariant φ starts and the flat system performs an adapting transition $\xrightarrow{r,\varphi,r'}$ of the form $(q,r,\emptyset) \xrightarrow{r,\varphi,r'} (q',r,\{(\varphi,r')\})$.
- Rule Adapt describes the evolution during the actual adaptation, leading to transitions of the form $(q,r,\{(\varphi,r')\}) \xrightarrow{r,\varphi,r'} (q',r,\{(\varphi,r')\})$. During adaptation the behaviour is not regulated by the specification and it must not satisfy the target constraints $L(r')$ ($q \not\models L(r')$). We also require that the invariant $\varphi \in \Phi(X)$ must always hold during this phase. Note that the semantics does not immediately assure that a state where the target formula holds is eventually reached. Formulations of the adaptability requirement are given in Section 4.
- Rule AdaptEnd describes the end of the adaptation phase, i.e. a transition $\xrightarrow{r,\varphi,r'}$ from an adapting state $(q,r,\{(\varphi,r')\})$ where q satisfies the set of target constraints ($q' \models L(r')$), to the steady non-adapting state $(q,r',\emptyset)$.

Note that rules Steady+AdaptStart ensure that there cannot exist a non-adapting state with both an outgoing non-adapting transition $\xrightarrow{r}$ and an outgoing adapting transition $\xrightarrow{r,\varphi,r'}$. Conversely, rules Adapt+AdaptEnd ensure that there cannot exist an adapting state with both an outgoing non-adapting transition and an adapting transition.

The flattened transitional semantics of the two systems $S_0[B]$ and $S_1[B]$ in the adaptive 1-predator 2-prey food web example presented in Section 2.1 is depicted in Figure 4. First, we observe that the flat $S_0[B]$ system has a larger state space than the flat $S_1[B]$, due to the higher number of S-transitions in S_0. In both cases two different adaptation phases can be noticed, the first starting from the flat state $((0,0,1,\mathsf{true},\mathsf{false}),r_0,\emptyset)$ and the second starting from $((1,0,0,\mathsf{true},\mathsf{false}),r_1,\emptyset)$. While in $S_0[B]$ it is possible to adapt to the migration region also in the first phase, in $S_1[B]$ this is possible only in the second phase, i.e. when both prey become unavailable. Moreover in $S_0[B]$, we notice that in each adaptation phase there always exists an adaptation path leading to a target stable region, but some adaptation paths cannot proceed because they violate the invariant. Conversely, in $S_1[B]$ every adaptation path leads to a target S-state. Therefore the same behavioural level B possesses different adaptation capabilities, depending on the structure S it is embedded in. These two different kinds of adaptability are formalized in Section 4.

Although, depending on the structure S, the flat semantics could possibly lead to a model larger than the behavioural model B, the flat $S[B]$-system lends itself quite naturally to on-the-fly representation techniques. Indeed, during non-adapting phases it would be necessary to keep in memory just the subsystem restricted to the set $[[L(r)]] \subseteq B$ of B-states that satisfy the current constraints $L(r)$. On the other hand, as soon as an adaptation of the form $(q,r,\emptyset) \xrightarrow{r,\varphi,r'} (q',r,\{(\varphi,r')\})$ takes place, it would be sufficient to store those B-states q'' such that $q'' \models \varphi \wedge q'' \not\models L(r')$, i.e. those state where the invariant is met, but the target constraints are not.

4 Adaptability relations

The above described transitional semantics for $S[B]$-systems does not guarantee that an adaptation process always leads to a state satisfying the target constraints, or that the system can always start adapting when the current constraints are not met. We characterize this requirements on the adaptability of an $S[B]$-system by means of two binary relations over the set of B-states and the set of S-states, namely the *weak adaptability relation* $\mathscr{R}_w$ and the *strong adaptability relation* $\mathscr{R}_s$.

Informally, B is *weak adaptable* to S if any active B-state q satisfies the constraints imposed by the active S-state r, or it can start adapting and there exists a finite path reaching a B-state q' satisfying the constraints dictated by a target S-state r'. On the other hand, B is *strong adaptable* to S if any active B-state q satisfies the constraints imposed by the active S-state r, or it can start adapting towards a target S-state r' and all paths reach a B-state q' satisfying the constraints $L(r')$ in a finite number of transitions.

In the following definitions the notation $\to^i$ with $i \in \mathbb{N}$ indicates the exponentiation of the transition relation $\to$, i.e. $\to^i = (\to)^i = \to (\to)^{i-1}$. We use this notation to remark that adaptation paths must be of finite length.

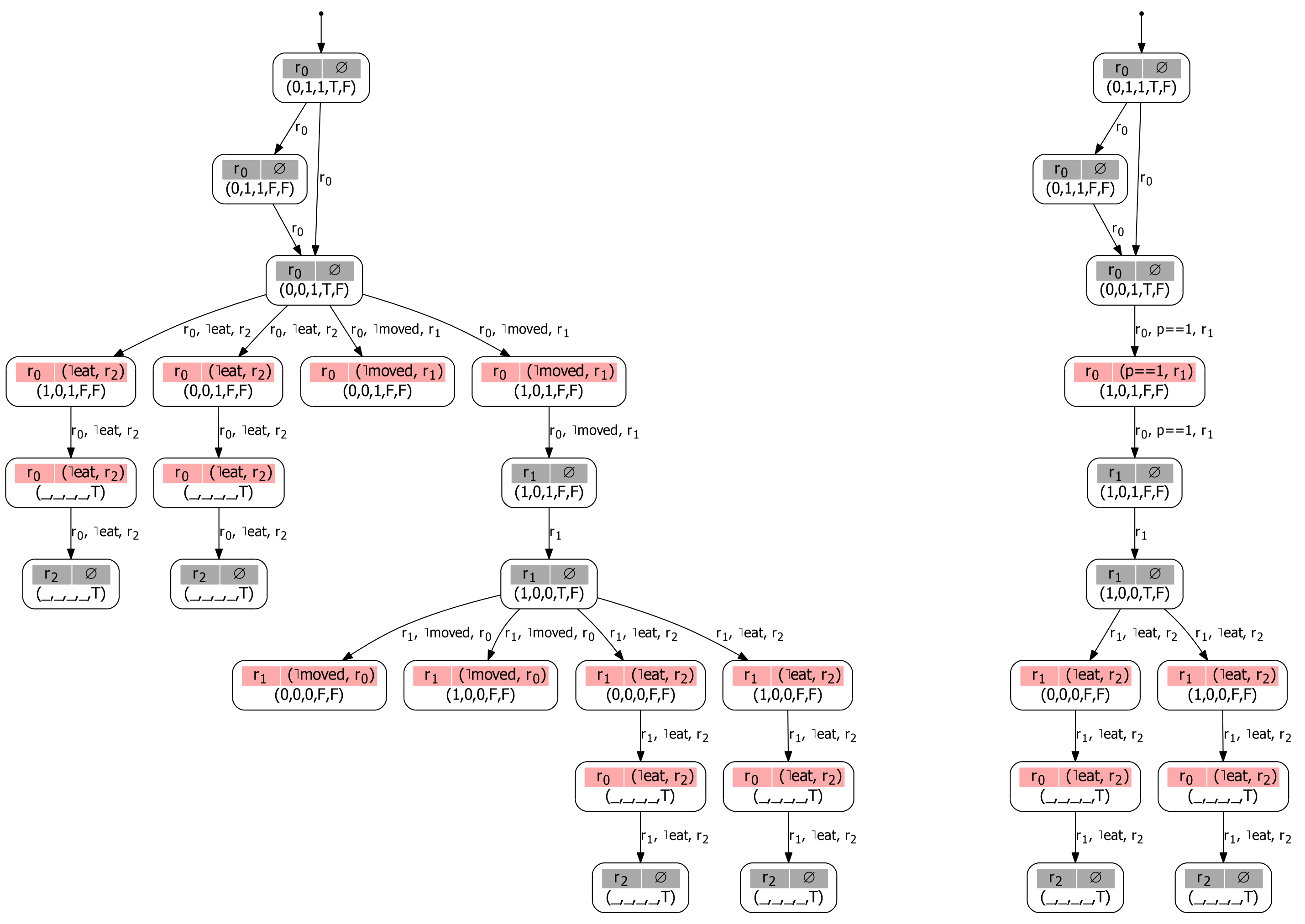
Definition 7 (Weak adaptability) *Weak-adaptability is a binary relation $\mathscr{R}_w \subseteq Q \times R$ defined as follows. Let $q \in Q$ be a B-state and $r \in R$ be an S-state. Then, $q \, \mathscr{R}_w \, r$ iff*

- *$q \models L(r)$ and*
- *for all $q' \in Q$, whenever $q \to_B q'$, it holds that either*
  - *$q' \mathscr{R}_w \, r$, or*
  - *there exists $q'' \in Q, \varphi \in \Phi(X), r' \in R, i \in \mathbb{N}$,*
    $(q,r,\emptyset) \xrightarrow{r,\varphi,r'} (q',r,\{(\varphi,r')\}) \xrightarrow{r,\varphi,r'}{}^{i} (q'',r',\emptyset)$ and $q'' \mathscr{R}_w \, r'$.

Let $S[B]$ be an $S[B]$-system. Then B is weak adaptable to S if their initial states are weak adaptable, i.e. $q_0 \, \mathscr{R}_w \, r_0$.

Definition 8 (Strong adaptability) *Strong-adaptability is a binary relation $\mathscr{R}_s \subseteq Q \times R$ defined as follows. Let $q \in Q$ be a B-state and $r \in R$ be an S-state. Then, $q \, \mathscr{R}_s \, r$ iff*

- *$q \models L(r)$ and*
- *for all $q' \in Q$, whenever $q \to_B q'$, it holds that either*
  - *$q' \mathscr{R}_s \, r$, or*
  - *$(q,r,\emptyset) \xrightarrow{r,\varphi,r'} (q',r,\{(\varphi,r')\})$ for some $\varphi \in \Phi(X), r' \in R$ **and** every path starting from $(q',r,\{(\varphi,r')\})$ leads, in a finite number of consecutive $\xrightarrow{r,\varphi,r'}$ transitions, to a state $(q'',r',\emptyset)$ such that $q'' \mathscr{R}_s \, r'$.*


r0 ∅ (0,1,1,T,F)
r0 ∅ (0,1,1,F,F)
r0 ∅ (0,0,1,T,F)
r0, ⌉eat, r2
r0, ⌉moved, r1
r0, p==1, r1
r0 (⌉eat, r2) (1,0,1,F,F)
r0 (⌉eat, r2) (0,0,1,F,F)
r0 (⌉moved, r1) (0,0,1,F,F)
r0 (⌉moved, r1) (1,0,1,F,F)
r0 (p==1, r1) (1,0,1,F,F)
r0 (⌉eat, r2) (_,_,_,_,T)
r2 ∅ (_,_,_,_,T)
r1 ∅ (1,0,1,F,F)
r1 ∅ (1,0,0,T,F)
r1, ⌉moved, r0
r1, ⌉eat, r2
r1 (⌉moved, r0) (0,0,0,F,F)
r1 (⌉moved, r0) (1,0,0,F,F)
r1 (⌉eat, r2) (0,0,0,F,F)
r1 (⌉eat, r2) (1,0,0,F,F)

(a) (b)

Figure 4: The flat semantics of the two systems $S_0[B]$ (fig. 4 (a)) and $S_1[B]$ (fig. 4 (b)) in the adaptive 1-predator 2-prey example. Different structural levels lead to different adaptation capabilities. Two adaptation phases (light red marked ones) can be recognized: the first occurs when the predator stops consuming the prey p_0, the second when it stops consuming p_1. In both systems there always exists an adaptation path leading to a target stable region, but in $S_0[B]$ some paths violate the invariant and cannot proceed. In $S_1[B]$ every adaptation path leads to a target S-state.

Let $S[B]$ be an $S[B]$-system. Then B is strong adaptable to S if their initial states are strong adaptable, $q_0 \mathscr{R}_s r_0$.

In the remainder of the paper we will alternatively say that a system $S[B]$ is weak (strong) adaptable, in the sense that B is weak (strong) adaptable to S. It is straightforward to see that strong adaptability implies weak adaptability, since the strong version of the relation requires that every adaptation path reaches a target S-state, while the weak version just requires that at least one adaptation path reaches a target S-state. Now that a relational characterization of adaptability has been given, a concept of equivalence between B-states that are adaptable to the same S-states naturally arises. Therefore we define the weak adaptation equivalence and the strong adaptation equivalence over the set of B-states as follows.

Definition 9 (Weak adaptation equivalence) *Two B-states $q_1, q_2 \in Q$ are said to be equivalent under weak adaptation, written $q_1 \approx_w q_2$, iff for each S-state $r \in R$, $q_1 \mathscr{R}_w r \iff q_2 \mathscr{R}_w r$.*

Definition 10 (Strong adaptation equivalence) *Two B-states $q_1, q_2 \in Q$ are said to be equivalent under strong adaptation, written $q_1 \approx_s q_2$, iff for each S-state $r \in R$, $q_1 \mathscr{R}_s r \iff q_2 \mathscr{R}_w r$.*

As discussed in Section 3, the adaptive 1-predator 2-prey system possesses different adaptation capabilities depending on the structural level S. In particular we notice that the system $S_0[B]$ is *weak adaptable*, since in each adaptation phase there always exists an adaptation path leading to a target S-state. Nevertheless, it is not strong adaptable because there are adaptation paths that violate the invariant and consequently cannot end adapting. On the other hand, $S_1[B]$ is *strong adaptable*, because every adaptation path leads to a target S-state.

4.1 A logical characterization for adaptability

In this part we formulate the above introduced adaptability requirements in terms of temporal formulae that can be statically checked on the flat $S[B]$-system. To this purpose we describe such properties in the well known *CTL (Computational Tree Logic)* [8], a branching-time logic whose semantics is defined in term of states. The set of well-formed CTL formulas are given by the following grammar:

$$\phi ::= \mathsf{false} \mid \mathsf{true} \mid p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \mathbf{AX}\phi \mid \mathbf{EX}\phi \mid \mathbf{AF}\phi \mid \mathbf{EF}\phi \mid \mathbf{AG}\phi \mid \mathbf{EG}\phi \mid \mathbf{A}[\phi\mathbf{U}\phi] \mid \mathbf{E}[\phi\mathbf{U}\phi],$$

where p is an atomic proposition, logical operators are the usual ones $(\neg, \wedge, \vee)$ and temporal operators ($\mathbf{X}$ next, $\mathbf{G}$ globally, $\mathbf{F}$ finally, $\mathbf{U}$ until) are preceded by the universal path quantifier $\mathbf{A}$ or the existential path quantifier $\mathbf{E}$. Starting from a state s, CTL operators are interpreted as follows. $\mathbf{AX}\phi$: for all paths, ϕ holds in the next state; $\mathbf{EX}\phi$: there exists a path s.t. ϕ holds in the next state; $\mathbf{AF}\phi$: for all paths, ϕ eventually holds; $\mathbf{EF}\phi$: there exists a path s.t. ϕ eventually holds; $\mathbf{AG}\phi$: for all paths, ϕ always holds; $\mathbf{EG}\phi$: there exists a path s.t. ϕ always holds; $\mathbf{A}[\phi_1\mathbf{U}\phi_2]$: for all paths, ϕ_1 holds until ϕ_2 holds; and $\mathbf{E}[\phi_1\mathbf{U}\phi_2]$: there exists a path s.t. ϕ_1 holds until ϕ_2 holds).

In the following we provide the CTL formulas characterizing a weak adaptable and a strong adaptable $S[B]$-system. Formulas are evaluated over the flat semantics and we employ the proposition *adapt* to denote an adapting state. More formally, given a flat $S[B]$-system F and a state $s = (q_s, r_s, \rho_s)$, $\langle F, s\rangle \models$ *adapt* if and only if $\rho_s \neq \emptyset$. Additionally, the connective $\phi_1 \implies \phi_2$ has the usual meaning: $\neg\phi_1 \vee \phi_2$.

- **Weak adaptability:** for all paths, it always holds that as soon as adaptation starts, there exists at least one path for which the system eventually ends the adaptation phase leading to a target S-state.

$$\mathbf{AG}((\neg adapt \wedge \mathbf{EX}\, adapt) \implies \mathbf{EF}\, \neg adapt) \tag{4.1}$$

- **Strong adaptability:** for all paths, it always holds that whenever the system is in an adapting state, for all paths it eventually ends the adaptation phase leading to a target S-state.

$$\mathbf{AG}(adapt \implies \mathbf{AF}\,\neg adapt) \tag{4.2}$$

Proposition 1 (Equivalent formulations of weak adaptability) *Let $S[B]$ be an $S[B]$-system. Then, $S[B]$ is weak adaptable if and only if $S[B]$ satisfies the weak adaptability CTL formula (equation 4.1). Formally, $q_0 \mathscr{R}_s r_0 \iff \langle F, f_0\rangle \models \mathbf{AG}((\neg adapt \wedge \mathbf{EX}\, adapt) \implies \mathbf{EF}\,\neg adapt)$, where F is the flat semantics of $S[B]$, q_0, r_0 and f_0 are the initial states of the behavioural level B, of the structural level S and of the flattened system F, respectively.*

Proposition 2 (Equivalent formulations of strong adaptability) *Let $S[B]$ be an $S[B]$-system. Then, $S[B]$ is strong adaptable if and only if $S[B]$ satisfies the strong adaptability CTL formula (equation 4.2). Formally, $q_0 \mathscr{R}_w r_0 \iff \langle F, f_0\rangle \models \mathbf{AG}(adapt \implies \mathbf{AF}\,\neg adapt)$, where F is the flat semantics of $S[B]$, q_0, r_0 and f_0 are the initial states of the behavioural level B, of the structural level S and of the flattened system F, respectively.*

Note that since we assume that the behavioural and the structural state machines are finite state, then the CTL adaptability properties can be model checked. This means that the defined notions of weak and strong adaptability are decidable.

5 Discussion and conclusion

In this work we presented $S[B]$-systems, a general multi-level model for self-adaptive systems, where the lower B-level is a state machine describing the behaviour of the system and the upper S-level is a second-order state machine accounting for the dynamical constraints with which the system has to comply. Higher-order S-states identify stable regions that the B-level may reach by performing adaptation paths. An intriguing (but here simplified) case study from ecology has been provided to demonstrate the capabilities of $S[B]$-systems: the adaptive 1-predator 2-prey system. The semantics of the multi-level system is given by a flattened transition system and two different concepts of adaptability (namely, weak and strong adaptability) have been formalized, both in a relational flavour and with CTL formulas that can be model checked. We report that this work gives a formal computational characterization of self-adaptive systems, based on concepts like multiple levels and higher-order structures that are well-established in the science of complex systems.

Note also that in this work we defined in details just two levels, namely the S-level and the B-level. However, our approach can be easily extended in order to consider multiple levels arising from the composition of multiple $S[B]$-systems. Let $\{S^n[B^n]_i \mid i \in I\}$ be a set of $S[B]$-systems at a certain level n. Their parallel composition would be defined as $\|_{i\in I}\, S^n[B^n]_i$. Then, if we let $B^{n+1} = \|_{i\in I}\, S^n[B^n]_i$ be the behavioural state machine at level $n+1$, an higher-level $S[B]$-system $S^{n+1}[B^{n+1}]$ can be built by defining a structure S^{n+1} at level $n+1$, together with a set of observable variables X^{n+1} and with an observation function $\mathscr{O}^{n+1}$.

The present work is just an initial attempt and several extensions can be integrated into the model in the next future. First, the definition of a higher-level algebraic language for specifying $S[B]$-systems would be useful in order to handle more complex and larger models of adaptive systems. Additionally, we are currently investigating further adaptability relations and different models for the structural level, where adaptation can occur not only when no possible future behaviours satisfy the current constraints, but also when stability conditions are met. Then, another possible research direction would be embedding quantitative aspects into the two levels of an $S[B]$-system. In this way, an S-transition would have associated a measure of its cost/propensity, for distinguishing the adaptation paths more likely to occur (e.g. in the 1-predator 2-prey example, the predator adapting its diet), to those less probable (e.g. the predator migrating even under prey availability conditions).

Finally we assume that the reciprocal knowledge between the two levels is limited: they see each other as black-box systems. However, this approach could be extended in order that the structure S has a more comprehensive knowledge of the behaviour B. Under the *white-box assumption*, the structure could act as a sort of monitor that is able to statically check the behavioural model for properties of safe adaptation. In this way, the system will know in advance if an adaptation path eventually leads to a target S-state and if not, it will avoid that path. In other words, runtime model checking techniques allows the system to behave in an *anticipatory way*. Anticipation is a crucial property in complex self-adaptive systems, since it makes possible to adjust present behaviour in order to address future faults. A well-know definition is given by Rosen [20]: *"An anticipatory system is a system containing a predictive model of itself and/or its environment, which allows it to change state at an instant in accord with the model's predictions pertaining to a later instant"*. In the settings of $S[B]$-systems, the predictive model of the system could be the behavioural level itself, or a part of it if we assume that S does not have a complete knowledge of B and is able to "look ahead" only at a limited number of future steps. The verdict of runtime model checking would be what Rosen refers to as model's predictions.

Acknowledgements

The authors thank TOPDRIM: *Topology Driven Methods for Complex Systems* funded by the European Commission (FP7 ICT FET Proactive - N. 318121). The authors thank Marianna Taffi for helping in the definition of the case study.

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