

# On the Existence of Universal Finite or Pushdown Automata

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We investigate the (non)-existence of universal automata for some classes of automata, such as finite automata and pushdown automata, and in particular the influence of the representation and encoding function. An alternative approach, using transition systems, is presented too.

## 1 Introduction

It is well known that there exist universal Turing machines (UTM). Such a UTM simulates any special Turing machine (TM)  $M$  in a certain way. There are several ways of simulation. One is that a UTM  $U$  simulating a TM  $M$  with input  $w$  halts if and only if  $M$  halts on input  $w$ . Another possibility is that any computation step of  $M$  is simulated by  $U$  using some number of steps which are restricted by some complexity function. In very small UTM's this can be exponential.

Almost all UTM's constructed so far are deterministic, simulating deterministic TM's. In [4] it has been shown that there exist UTM's simulating all special TM's with complexity constraints. These complexity constraints, for space or time, are from a subclass of all primitive recursive functions over one variable. The UTM's have the same complexity constraints.

In both cases, general TM's and those with complexity constraint, the specific TM  $M$  and its input  $w \in \Sigma(M)^*$ , where  $\Sigma(M)$  is the alphabet of  $M$ , have to be encoded. Such an encoding, and also the decoding, can be achieved by deterministic finite state transducers (DFST), which means that encoding and decoding is bijective. The input for a UTM  $U$ , to simulate  $M$  with input  $w$ , can then have the form  $c_m(M)\#c_i(w)$  where  $c_m, c_i$  are the encoding functions for  $M, w$ , respectively.

If one intends to construct universal machines for weaker automata classes it should be kept in mind that encoding and decoding for such automata should not exceed the power of deterministic versions of those machines. Otherwise too much power and information could be hidden in the encoding.

In [3] it has been shown, under this condition, that there don't exist universal 1-way finite automata (FA), neither deterministic (DFA) nor nondeterministic (NFA) ones. The proof uses arguments on the number of states of such automata.

So the question arises whether there exist universal pushdown automata (UPDA), and if so if encoding and decoding can be achieved by DFST's, or if deterministic pushdown transducers (DPDT) are necessary. For general TM's DFST's suffice for encoding and decoding (e.g. [5]).

## 2 Transducers

Pushdown transducers (see e.g. [1, 2]) are just the analogon to finite state transducers, i.e. pushdown automata with output.

Formally:

A (*non-deterministic*) pushdown transducer (PDT) is a construct  $(Q, \Sigma_i, \Sigma_o, \Delta, \$, q_0, Q_f, \rho)$  where

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$Q$	set of states
$\Sigma_i$	input alphabet
$\Sigma_o$	output alphabet
$\Delta$	pushdown alphabet
$\$ \in \Delta$	stack bottom symbol
$q_o \in Q$	initial state
$Q_f \subseteq Q$	set of final states
$\rho \subseteq Q \times \Sigma_i^* \times \Sigma_o^* \times \Delta^* \times \Delta^* \times Q$	the transition relation .

A *deterministic pushdown transducer (DPDT)* is the deterministic version of PDT. That is,  $\rho$  is a function  $\rho : Q \times \Sigma_i^* \times \Delta^* \rightarrow \Sigma_o^* \times Q$ .

In the sequel a normal form of PDT's will be considered, being quasi lettering in input and pushdown, i.e.  $\rho \subseteq Q \times (\Sigma_i \cup \{\lambda\}) \times (\Sigma_o \cup \{\lambda\}) \times (\Delta \cup \{\lambda\}) \times (\Delta \cup \{\lambda\}) \times Q$ .

Contrary to regular languages (**REG**) which are closed under finite state transductions the context-free languages (**CF**) are not closed under pushdown transductions, neither non-deterministic nor deterministic. Even linear context-free languages give sets outside **CF** if a deterministic pushdown transduction is applied. This is shown by the following examples of DPDT's where  $\tau$  denotes the function defined by a deterministic transducer.

Let  $L_1 = \{0^n 1 \mid n \geq 0\} \in \mathbf{REG}$  and

$$T_1 = (\{q_0, q_1\}, \{0, 1\}, \{0, 1\}, \{q_0\}, \{q_1\}, \rho_1) \text{ with}$$

$$\rho_1 = \{(q_0, 0, 0, \$, 0$, $, q_0), (q_0, 0, 0, 0, 00, q_0), (q_0, 1, \lambda, \$, \$, q_2),$$

$$(q_0, 1, \lambda, 0, 0, q_1), (q_1, \lambda, 1, 0, \lambda, q_1), (q_1, \lambda, \lambda, \$, \$, q_2)\}.$$

Then  $\tau(L_1) = \{0^n 1^n \mid n \geq 0\} \notin \mathbf{REG}$ .

Let  $L_2 = \{0^n 1^n 0 \mid n > 0\} \in \mathbf{LIN}$  and

$$T_2 = (\{q_0, q_1, q_2, q_3\}, \{0, 1\}, \{0, 1\}, \{q_0\}, \{q_3\}, \rho_2) \text{ with}$$

$$\rho_2 = \{(q_0, 0, 0, \$, 0$, $, q_0), (q_0, 0, 0, 0, 00, q_0), (q_0, 1, 1, 0, 10, q_1),$$

$$(q_1, 1, 1, 1, 11, q_1), (q_1, 0, \lambda, 1, 1, q_2), (q_2, \lambda, 0, 1, \lambda, q_2),$$

$$(q_2, \lambda, 1, 0, \lambda, q_2), (q_2, \lambda, \lambda, \$, \$, q_3)\}.$$

Then  $\tau(L_2) = \{0^n 1^n 0^n 1^n \mid n \geq 0\} \notin \mathbf{CF}$ .

Let  $L_3 = \{w c w^R c \mid w \in \{0, 1\}^+\} \in \mathbf{LIN}$  and

$$T_3 = (\{q_0, q_1, q_2, q_3\}, \{0, 1\}, \{0, 1\}, \{q_0\}, \{q_3\}, \rho_3) \text{ with}$$

$$\rho_3 = \{(q_0, 0, 0, \$, 0$, $, q_0), (q_0, 1, 1, \$, 1$, $, q_0), (q_0, 0, 0, 0, 00, q_0),$$

$$(q_0, 0, 0, 1, 01, q_0), (q_0, 1, 1, 0, 10, q_0), (q_0, 1, 1, 1, 11, q_0),$$

$$(q_0, c, c, 0, 0, q_1), (q_0, c, c, 1, 1, q_1), (q_1, 0, 0, \$, 0$, $, q_1),$$

$$(q_1, 1, 1, \$, 1$, $, q_1), (q_1, 0, 0, 0, 00, q_1), (q_1, 0, 0, 1, 01, q_1),$$

$$(q_1, 1, 1, 0, 10, q_1), (q_1, 1, 1, 1, 11, q_1), (q_1, c, c, 0, 0, q_2),$$

$$(q_1, c, c, 1, 1, q_2), (q_2, \lambda, \lambda, 0, 0, q_2), (q_2, \lambda, \lambda, 1, 1, q_2),$$

$$(q_2, \lambda, \lambda, c, c, q_2), (q_2, \lambda, \lambda, \$, \$, q_3)\}.$$

Then  $\tau(L_3) = \{w c w^R c w c w^R c \mid w \in \{0, 1\}^+\} \notin \mathbf{CF}$ .

One also might consider 2-way FST's (2FST). However, **REG** is not closed under 2-way finite state transductions, as can be seen from the following example of a 2DFST ( $L, R, M$  denote move to left, right, or not, respectively).

Let  $L_4 = \{a w b \mid w \in \{0, 1\}^*\} \in \mathbf{REG}$  and

$T_4 = (\{q_0, q_1, q_2, q_3\}, \{0, 1, a, b\}, \{0, 1, a\}, \{q_0\}, \{q_4\}, \rho_2)$  with  
 $\rho_4 = \{(q_0, a, a, R, q_0), (q_0, 0, 0, R, q_0), (q_0, 1, 1, R, q_0), (q_0, b, \lambda, L, q_1),$   
 $(q_1, 0, \lambda, L, q_1), (q_1, 1, \lambda, L, q_1), (q_1, a, a, R, q_2),$   
 $(q_2, 0, 0, R, q_2), (q_2, 1, 1, R, q_2), (q_2, b, \lambda, M, q_3)\}.$   
 Then  $\tau(L_4) = \{awaw \mid w \in \{0, 1\}^*\} \notin \mathbf{CF}.$

### 3 Representations

A representation of FA, PDA, TM's etc. has to contain information on the set of states, initial and final states, alphabets, and the set of transitions:

$(Q, \Sigma, \Delta, Q_0, Q_f, R)$  with e.g.  $R \subseteq Q \times \Sigma^* \times \Delta^* \times \Delta^* \times Q$  for a PDA.

Usually  $R$  is represented by an ordered list of elements from  $R$ , together with lists for  $Q_0$  and  $Q_f$ . For non-deterministic machines one might also allow repetitions of list elements. This can give a regular set of representations if  $Q$ ,  $\Sigma$ , and  $\Delta$  are fixed.

FA and PDA usually are represented by the list  $R$  of their transitions, putting together the tuples  $(q, x, q')$  or  $(q, x, y, y', q')$  for FA or PDA respectively, where  $x \in (\Sigma \cup \{\lambda\})$ , and  $y, y' \in (\Delta \cup \{\lambda\})$ .

For FA one has a representation  $R \in (Q \cdot (\Sigma \cup \{*\}) \cdot Q)^+$ , and for PDA  $R \in (Q \cdot (\Sigma \cup \{*\}) \cdot (\Delta \cup \{*\}) \cdot (\Delta \cup \{*\}) \cdot Q)^+$  where  $*$  stands for  $\lambda$ . Together with the input  $w$  this gives a representation  $R(M)\#w$ . But one might think also of a representation  $w\#R(M)$  or even  $R(M)\sqcup\#\sqcup w$  where  $\sqcup$  is the shuffle operation, and  $\#$  a separation symbol.

If the first version of representation is encoded by a DFST  $T$  the result is a word  $\tau(R(M))\tau(\#)\tau(w)$  where  $\tau$  is the function associated to  $T$ . This follows from the fact that  $T$  is working 1-way. The same conditions hold for the 'inverse', namely for a DPDT  $T'$  with associated function  $\tau'$ . Furthermore,  $\tau'(\tau(R(M))\tau(\#)\tau(w)) = R(M)\#w$  has to hold.

Considering representations of FA with arbitrary sets of states  $Q$  and arbitrary alphabets  $\Sigma$ , states  $q_k$  can be represented by  $qa^k$  ( $0 < k$ ) and symbols  $x_m$  by  $xa^m$  ( $0 < m$ ) over the finite alphabet  $\{q, x, a\}$ . Then the set of all representations of finite automata is given by

$$F = (\{q\} \cdot \{a\}^+ \cdot (\{x\} \cdot \{a\}^+ \cup \{*\}) \cdot \{q\} \cdot \{a\}^+)^+.$$

Clearly,  $F \in \mathbf{REG}$ . But note that this holds only for non-deterministic FA. In the case of DFA there is the condition that a pair  $(q_k, x_m)$  of state and symbol, represented by  $qa^k$  and  $xa^m$ , can appear only once as first components in the list of transitions.

An analogous property also holds for PDA.

Since  $\mathbf{REG}$  and  $\mathbf{CF}$  are closed under FST mappings, encoding (and decoding) will not lead out from these classes. However, the examples above show that this does not hold for PDT mappings. Therefore, not to gain too much power it might be reasonable to have a condition that those PDT mappings used for encoding and decoding are not leading out from the class  $\mathbf{CF}$ .

### 4 Universality?

In the sequel we shall consider PDA  $M$  accepting languages  $L(M) \subseteq \{0, 1\}^*$ . Assume that there exists a universal PDA  $U$  simulating all specific PDA  $M$  over  $\{0, 1\}$ . Denote this class by  $\mathbf{CF}_2$ . Let  $\Sigma_U$  be the alphabet of  $U$ . Then, with the first version of representation,

$$L(U) = \{\tau(R(M))\tau(\#)\tau(w) \mid L(M) \in \mathbf{CF}_2, w \in L(M)\} \in \mathbf{CF}.$$

Consider now the special regular (context-free) languages  $\{w\} \subset \{0,1\}^*$ . A representation of a DFA  $M$ , being also a PDA or 2DFA, accepting exactly the language  $\{w\}$ , e.g. looks like  $R(M) = q_0 w q_1 = \varphi(w)$ . Together with an input  $v \in \{0,1\}^*$  the representation has the form  $R(M)\#v = q_0 w q_1 \#v$ . A DFST maps this into  $\tau(\varphi(w))\tau(\#)\tau(v)$ . Clearly,

$$\begin{aligned} S &= \{\tau(\varphi(w))\tau(\#)\tau(v) \mid v, w \in \{0,1\}^*\} \\ &= \{\tau(\varphi(w)) \mid w \in \{0,1\}^*\} \cdot \{\tau(\#)\} \cdot \tau(\{0,1\}^*) \in \mathbf{REG}. \end{aligned}$$

Now

$$\begin{aligned} L(U) \cap S &= \{\tau(\varphi(w))\tau(\#)\tau(v) \mid w, v \in \{0,1\}^*, v = w\} \\ &= \{\tau(\varphi(w))\tau(\#)\tau(w) \mid w \in \{0,1\}^*\} \in \mathbf{CF} \end{aligned}$$

since **CF** is closed under intersection with regular sets.

Applying the ‘inverse’ DFST mapping  $\tau'$  yields

$$\tau'(L(U) \cap S) = \{\varphi(w)\#w \mid w \in \{0,1\}^*\} \in \mathbf{CF}.$$

Another DFST mapping  $\psi$  with  $\psi(q_0) = \psi(q_1) = \lambda$ ,  $\psi(0) = 0$ ,  $\psi(1) = 1$  gives

$$\psi(\tau'(L(U) \cap S)) = \{w\#w \mid w \in \{0,1\}^*\} \in \mathbf{CF},$$

a contradiction.

These considerations can be summarized as

**Theorem 1:** If encoding and decoding of specific finite or pushdown automata have to be achieved by DFST then there doesn't exist a universal finite automaton, or 2-way finite automaton or pushdown automaton, simulating all specific finite automata.  $\square$

It should be remarked, however, that the proof of this theorem cannot be used to show that the statement also holds for all quasi lettering finite automata. The reason is that the DFST for encoding has to know the length of  $w$  for the representation  $q_0 x_1 q_1 \cdots x_k q_k$  where  $x_i \in \{0,1\}$  and all  $q_i$  are different.

Therefore we give another proof that this theorem also holds for quasi-lettering automata. For non-deterministic (quasi-lettering) FA it can be assumed that there is exactly one initial and exactly one final state, and that the first element in the list has the form  $(q_1, x, q)$  where  $q_1$  is the initial state and  $x \in \Sigma$ , and that the last element has the form  $(q_2, \lambda, q_2)$  where  $q_2$  is the final state.

Let  $M_\mu = (Q, \{0,1\}, q_1, \{q_2\}, R_\mu)$  a FA, illustrated in Figure 1, where

$\mu = (i, i', i'', j, j', j'', k, k', k'', \ell, \ell', \ell'')$  and

$2 < i, i', i'', j, j', j'', k, k', k'', \ell, \ell', \ell'' \leq |Q|$  are fixed and

$$\begin{aligned} R(M_\mu) &= \{(q_1, 0, q_i), (q_1, 1, q_k), (q_{i'}, 0, q_{j'}), (q_{i''}, 1, q_{\ell''}), \\ &\quad (q_{k''}, 0, q_{j''}), (q_{k'}, 1, q_{\ell'}), (q_j, 0, q_2), (q_\ell, 1, q_2), (q_2, \lambda, q_2)\}. \end{aligned}$$

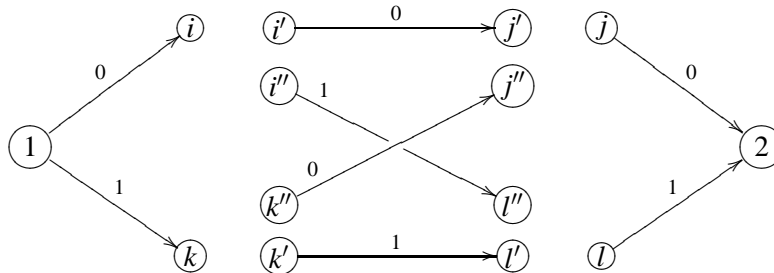


Figure 1: FA

$L(M_\mu) = \{000, 011, 100, 111\}$  implies  $i = i' = i'', j = j' = j'', k = k' = k'', \ell = \ell' = \ell''$ . Note that  $M_\mu$  is also a PDA.

An encoding of  $R(M_\mu)$  is given e.g. by

$$\tau(R(M_\mu)) = qaxbqa^i \cdot qaxcqa^k \cdot qa^jxbqa^j \cdot qa^{\ell'}xcqa^{\ell''} \cdot qa^{\ell''}xbqa^{\ell''} \cdot qa^{\ell'}xcqa^{\ell''} \cdot qa^{\ell'}xbqa^2 \cdot qa^{\ell'}xcqa^2 \cdot qa^2 * qa^2$$

$\tau$  is a DFST mapping. Note that  $\tau$  actually depends on  $Q$ .

Now assume that

$$U = \{ \tau(R(M))\tau(\#) \mid M \in \text{QLFA}, \{000, 011, 100, 111\} \subseteq L(M) \} \\ \cdot \tau(\{000, 011, 100, 111\})$$

is context-free, where QLFA denotes the class of quasi-lettering FA with set of states  $Q$ , initial state  $q_1$ , final state  $q_2$ , and alphabet  $\{0, 1\}$ .

$$\text{Define } S = \{ \tau(R(M_\mu))\tau(\#) \mid 2 < i, i', i'', j, j', j'', k, k', k'', \ell, \ell', \ell'' \leq |Q| \} \\ \cdot \tau(\{000, 011, 100, 111\}).$$

Obviously,  $S$  is regular, implying that  $U \cap S$  is context-free. But

$$U \cap S = \{ qaxbqa^i \cdot qaxcqa^k \cdot qa^jxba^\ell \cdot q^jxcqa^\ell \cdot qa^kxbqa^j \cdot qa^kxcqa^\ell \cdot qa^jxbqa^2 \\ \cdot qa^\ell xcqa^2 \cdot qa^2 * qa^2 \mid 2 < i, j, k, \ell \leq |Q| \}$$

is not context-free, a contradiction.

If 2-way pushdown transducers are allowed however, and the encoding is not required to have the form  $\tau(R(M))\tau(\#)\tau(w)$ , e.g. in a more general form  $\tau(R(M)\#w)$ , it is possible to construct a universal PDA, following an idea by Gh. Păun et. al.. To be more precise, there exists a quasi-lettering universal PDA simulating all lettering PDA in this way.

Let  $M = (Q, \Sigma, \Delta, \delta, q_0, \{q_f\})$  with  $\Sigma = \{0, 1\}$  be a lettering PDA, and  $R(M) \subseteq \{Q \times \Sigma \times \Delta \times \Delta \times Q\}^*$  be its representation. Suppose we allow  $\tau(R(M)\#w)$  to be transduced by a 2-way PDT (2PDT). Then one can choose the coding  $\tau(R(M)\#x_1x_2 \dots x_n) = x_1\sigma(R(M))x_2\sigma(R(M)) \dots x_n\% \sigma(R(M))$  where  $x_i \in \Sigma$ , and  $\sigma(R(M))$  is a coding of  $R(M)$  in a fixed alphabet. Note that  $\sigma$  depends on  $Q$ . For simplicity let  $\sigma = \iota$  at first where  $\iota$  is the identity function. Then  $\tau$  can be calculated by a 2PDT that works as follows:

1. First go right until the place after  $\#$  and print the first symbol of  $w$ .
2. Go left and push  $a \in \Delta$  to the stack until reading  $\#$ .
3. Go left to the beginning of the input.
4. Print  $R(M)$  until reading  $\#$ .
5. Go further right and simultaneously pop  $a \in \Delta$  from the stack. On empty stack check if the end of the input has been reached.
  - (a) If so, print ' $\%$ ', go to the beginning of the input and copy  $R(M)$  one last time and halt.
  - (b) Otherwise print the current input symbol and repeat from step 2.

Consider the following PDA  $U$  (Figure 2) with  $\Sigma_U = \{0, 1\} \cup (Q \times \Sigma \times \Delta \times \Delta \times \Sigma)$  and  $\Delta_U = \{a\} \cup Q$  for all possible  $Q, \Sigma, \Delta$ . The  $\Sigma_U, \Delta_U$  are not finite, but when chosen an appropriate encoding  $\sigma$  for the representation of simulated automata, the following idea for an UPDA works with finite alphabets  $\Sigma_U, \Delta_U$ . For now, assume any transition  $(q, x, a, b, q')$  in the simulated automaton to be an atomic symbol of our UPDA.

At any time  $U$ 's stack consists of a word  $q\delta_m\delta_{m-1} \dots \delta_0$  where  $\delta = \delta_m \dots \delta_0$  is the stack content and  $q$  the current state of  $M$  during a simulation. In state  $R$ ,  $U$  first reads a symbol  $x_i$  from the input (word  $w$ ) and afterwards checks whether the simulated PDA could have read  $x_i$  by travelling through the input

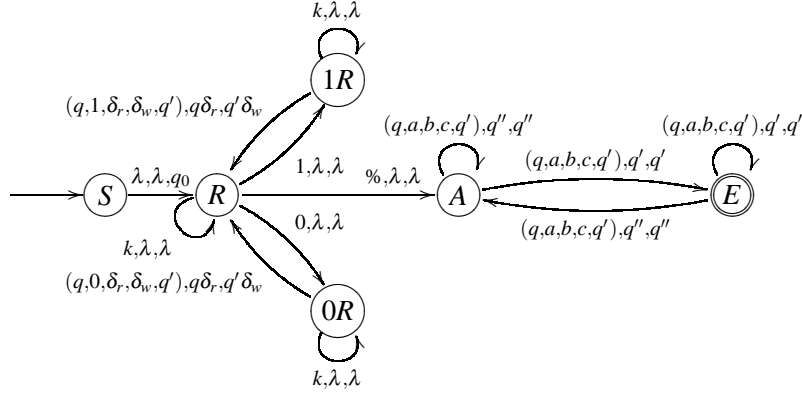


Figure 2: Universal PDA

$R(M)$  and looking for a transition  $(q, x_i, \delta_r, \delta_w, q')$  with  $q$  being the state  $M$  is currently in, and storing the new state and changed stack content of  $M$  in its own stack.

In the construction we have used  $(q, x, a, b, q')$  etc. as one symbol, but it works also if an encoding  $\sigma$  over a finite alphabet  $\Sigma_U$  is used. E.g.,  $\sigma((q_i, x, \delta_j, \delta_k, q_\ell)) = DSA^i x PB^j PB^k SA^\ell$  with  $x \in \Sigma = \{0, 1\}$  gives such an encoding. Then, point 4 in the transduction  $\tau$  would contain additional steps to encode a transition. E.g. to read symbol  $q_i \in Q$  and print  $SA^i$  and so on. Also, the UPDA  $U$  must have additional components for decoding in all states that have outgoing transitions reading a tuple  $(q, x, a, b, q')$  from the input.

This can be achieved as follows. Let  $\Sigma_U = \{A, B, D, P, S, T, 0, 1\}$  where  $D, P, S, T$  are markers.  $D$  is the delimitator of  $DSA^i y PB^j PB^k SA^\ell$  encoding  $(q_i, y, \delta_j, \delta_k, q_\ell)$ , and  $T$  the delimitator of a block  $\sigma(R(M))$ .

In a step  $U$ , after reading  $x \in \{0, 1\}$ , goes into different states according to  $x$ . The stack of  $U$  contains  $SA^m PB^n PB^r \dots \$$  where  $\$$  is the bottom symbol.  $U$  non-deterministically goes to some  $DSA^i y PB^j PB^k SA^\ell$  within  $\sigma(R(M))$ . If  $i = m$ ,  $x = y$ , and  $j = n$  then  $SA^m PB^n \dots \$$  in the stack is replaced by  $SA^\ell PB^k \dots \$$ . Otherwise,  $U$  goes into a sink. Note that  $U$  is a quasi-lettering PDA.

With a slight modification in the encoding it can be shown that the result also holds for PDA  $M$  not being lettering for the pushdown alphabet  $\Delta$ .

## 5 Transition Systems

In this section we present an alternative approach to investigate universal automata.

In order to specify what we mean by ‘universal PDA’ or ‘universal grammar’, we introduce the notion of *Transition Systems*, a generalisation of finite automata that models any kind of computational device having an internal *state* that can be altered by the occurrence of an *action* during the course of a computation.

### Definition 1: (Transition System)

A *Transition System* is a quintuple  $(S, \Sigma, \delta, S_0, S_F)$  with

$S$	a set of states,
$\Sigma$	an alphabet of transitions
$\delta \subseteq S \times (\Sigma \cup \{\lambda\}) \times Q$	a transition relation
$S_0 \subseteq S$	a set of initial states
$S_F \subseteq S$	a set of final states

We write  $s \xrightarrow{t} s'$  for  $(s, t, s') \in \delta$ . A transition system is called *finite* if  $S \cup T$  is finite. The transition relation in a transition system can be extended to finite sequences of transitions:

- $s \xrightarrow{\lambda} s$  for all states  $s$  and the empty sequence  $\lambda$ .
- $s \xrightarrow{wr} s'$  iff a state  $s''$  exists, such that  $s \xrightarrow{w} s'' \wedge s'' \xrightarrow{t} s'$ .

Let  $\xrightarrow{*}$  denote the transitive and reflexive closure of  $\rightarrow$ .

The *language* of the transition system  $A$  is

$$L(A) = \{w \in \Sigma^* \mid s_0 \xrightarrow{w} s_f, s_0 \in S_0, s_f \in S_F\}.$$

Any finite automaton  $A = (Q, \Sigma, \delta, q_0, Q_F)$  is also a transition system by definition. However, in a transition system  $Q$  and  $Q_F$  are generally not finite. Any PDA  $A = (Q, \Sigma, \Delta, \delta, q_0, Q_F)$  defines a transition system whose states are all possible configurations  $c \in Q \times \Delta^*$  of the PDA, and transitions are defined by  $(q, wx) \xrightarrow{a} (q', wy) \iff (q, a, x, y, q') \in \delta$ .

### Definition 2: (Universal Transition System)

Let  $\mathbf{X} \subseteq \mathbf{RE}$  be a language class below  $\mathbf{RE}$ . The TS  $A = (S, \Sigma, \delta, S_0, S_F)$  is called  $\mathbf{X}$ -*universal* iff for any language  $L \in \Sigma^*$  in  $\mathbf{X}$  there is a state  $s_L \in S$  such that  $L((S, \Sigma, \delta, s_L, S_F)) = L$ .

Any computing device that unambiguously defines a transition system will be considered  $\mathbf{X}$ -universal if its transition system is.

Obviously there *is* a **REG**-universal system, namely the disjoint union of all the possible NFA. The interesting question is however, whether or not such a **REG**-universal system can be defined by an NFA. The following lemma recovers a theorem from [3] from a new perspective.

**Lemma 1:** There is no **REG**-universal NFA.

*Proof:* For arbitrary  $n \in \mathbb{N}, a \in \Sigma, L_n = \{a^n\} \in \mathbf{REG}$ . Any **REG**-universal system  $U$  must have a state  $s_{L_n}$  from that on exactly  $n$  steps can be made. If the set of states in  $U$  was finite, this state could not exist for  $n > |S|$ , so  $U$  must have an infinite set of states. Since any NFA defines a finite transition system, no **REG**-universal system can be defined by an NFA, and therefore no **REG**-universal NFA exists.  $\square$

## 6 Outlook

We have shown that there doesn't exist universal 1-way or 2-way finite automata nor pushdown automata if encoding and decoding have to be done by deterministic finite state transducers. However, if 2-way deterministic pushdown transducers are allowed and for encoding of a repetition of the specific pushdown automaton to be simulated, depending on the length of input, a universal pushdown automaton can be constructed. Further research has to be done on encoding and decoding device. Our conjecture is that 1-way deterministic are not sufficient for the existence of a universal pushdown automaton.

## 7 Acknowledgement

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