COMP9311: Database Systems

Relational Algebra

(textbook: chapter 8)

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Week 3 Relational Algebra and SQL

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Disclaimer: the course materials are sourced from

- previous offerings of COMP9311 and COMP3311
- Prof. Werner Nutt on Introduction to Database Systems (http://www.inf.unibz.it/~nutt/Teaching/IDBs1011/)

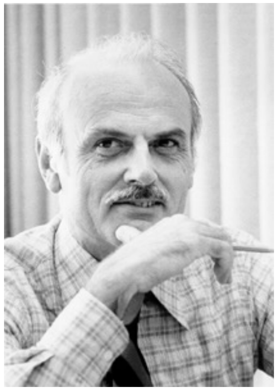
Motivation

We know how to store data … i.e., we modelled our data in relational data model -> then created tables to store them into a relational database.

How do we manipulate or retrieve (interesting) the data?

A data model must include a set of operations to manipulate the database, in addition to the concepts for defining database's structure and constraints.

The basic set of operations for the relational model is ***Relational Algebra***



- Edgar F. Codd (1970): Relational Algebra, mathematical foundation for relational data management
- supports basic retrieval requests (queries) -> the result of a query is also a ***relation***
- A sequence of relational algebra operations form a relational algebra expression -> results in a ***relation***

Motivation

The relational algebra is very important for several reasons.

- First, it provides a formal foundation for relational model operations.
- Second, and perhaps more important, it is used as a basis for implementing and optimizing queries in the query processing and optimization modules that are integral parts of relational database management systems (RDBMSs),
- Third, some of its concepts are incorporated into the SQL, the standard query language for relational database management systems

Characteristics of an Algebra

An algebra expression:

- is constructed with operators from atomic operands (constants, variables, ….)
- can be evaluated
- can be equivalent to another expression
  - …if they return the same result for all values of the variables

Atomic expressions:

numbers and variables

Operators: $+, -, \cdot, :$

This equivalence concept gives rise to an algebraic identity between expressions

An **algebraic identity** is an equality that holds for any values of its variables.

The value of an expression is independent of its context

- e.g., 5 + 3 has the same value, no matter whether it occurs as

  $10 - (5 + 3)$ or $4 \cdot (5 + 3)$

Identitities:

$x + y = y + x$

$x \cdot (y + z) = x \cdot y + x \cdot z$

… and so on

Consequence: subexpressions can be replaced by equivalent expressions without changing the meaning of the entire expression

Relational Algebra: Principles

Atoms are relations

Operators are defined for arbitrary instances of a relation

The following two results have to be defined for each operator:

- result schema
- result instance

Set theoretic operators

- union "$\cup$", intersection "$\cap$", difference "\"

Renaming operator ρ

Removal operators

- projection π, selection σ

Combination operators

- Cartesian product "$\times$", joins "$\bowtie$"

Extended operators

- duplicate elimination, grouping, aggregation, sorting, outer joins, etc.

- *"Equivalent" to SQL query language … Relational Algebra concepts reappear in SQL*
- *Used inside a DBMS, to express query plans*

Set Operators

Observations:

Instances of relations are sets
➔ we can form **unions**, **intersections**, and **differences**

Set algebra operators can only be applied to relations with identical attributes,

- same number of attributes
- same attribute names
- same domains

- (i.e., set operation compatibility)

Union (∪)

CS-Student

Studno	Name	Year
s1	Egger	5
s3	Rossi	4
s4	Maurer	2

Master-Student

Studno	Name	Year
s1	Egger	5
s2	Neri	5
s3	Rossi	4

CS-Student ∪ Master-Student

Studno	Name	Year
s1	Egger	5
s2	Neri	5
s3	Rossi	4
s4	Maurer	2

Intersection (∩)

CS-Student

Studno	Name	Year
s1	Egger	5
s3	Rossi	4
s4	Maurer	2

Master-Student

Studno	Name	Year
s1	Egger	5
s2	Neri	5
s3	Rossi	4

CS-Student ∩ Master-Student

Studno	Name	Year
s1	Egger	5
s3	Rossi	4

Difference (\)

CS-Student

Studno	Name	Year
s1	Egger	5
s3	Rossi	4
s4	Maurer	2

Master-Student

Studno	Name	Year
s1	Egger	5
s2	Neri	5
s3	Rossi	4

CS-Student \ Master-Student

Studno	Name	Year
s4	Maurer	2

Set difference, formally:

$$B \setminus A = \{x \in B \mid x \notin A\}.$$

Renaming ρ

- The renaming operator ρ (reads 'rho') changes the name of relation schema (both for relation name and relation attributes)
- It changes the schema, but only within a query
- $\rho_{\mathtt{x}}(\mathtt{E})$ where `E` is the relation name and `x` is the new name for `E`, usually a shorter name
  - $\rho_{\mathtt{FC}}(\mathtt{Father\text{-}Child})$
- $\rho_{\mathtt{a/b}}(\mathtt{E})$ where `E` is the relation name, `a`, `b` are attribute names, `b` is an attribute of `E`
  - $\rho_{\mathtt{parent/father}}(\mathtt{Father\text{-}Child})$

Father-Child

Father	Child
Adam	Abel
Adam	Cain
Abraham	Isaac

$\rho_{\mathtt{FC}}(\mathtt{Father\text{-}Child})$

FC

Father	Child
Adam	Abel
Adam	Cain
Abraham	Isaac

$\rho_{\mathtt{parent/father}}(\mathtt{Father\text{-}Child})$

Parent	Child
Adam	Abel
Adam	Cain
Abraham	Isaac

Union example after renaming

$\rho_{\text{Parent / Father}}$ (Father-Child)

Parent	Child
Adam	Abel
Adam	Cain
Abraham	Isaac

$\rho_{\text{Parent / Mother}}$ (Mother-Child)

Parent	Child
Eve	Abel
Eve	Seth
Sara	Isaac

$\rho_{\text{Parent / Father}}$ (Father-Child)

$\cup$

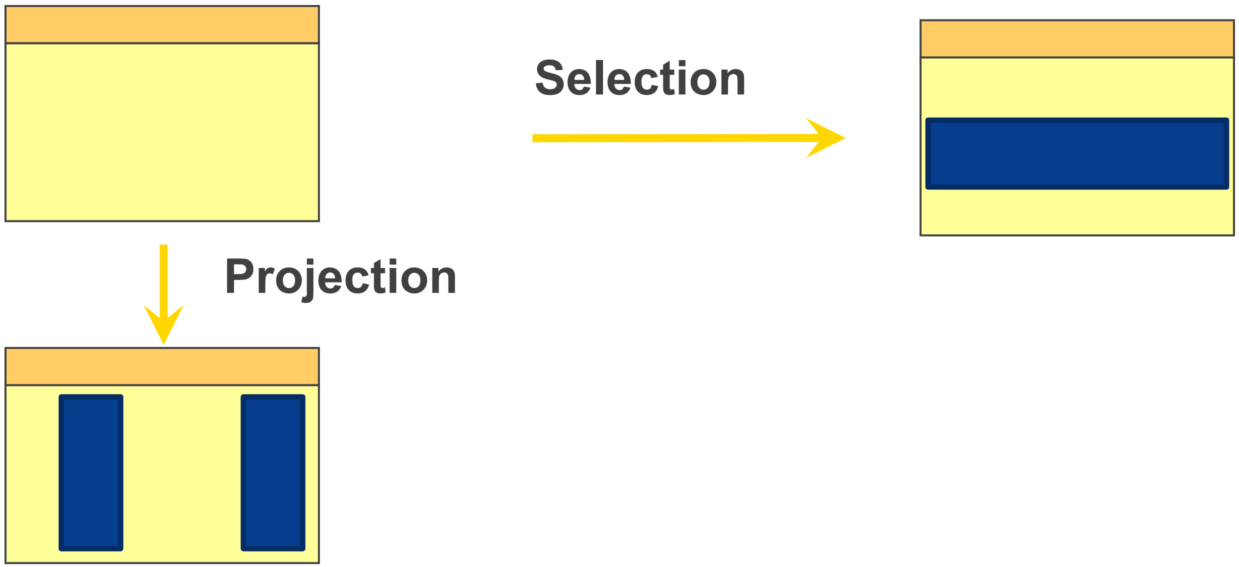
$\rho_{\text{Parent / Mother}}$ (Mother-Child)

Parent	Child
Adam	Abel
Adam	Cain
Abraham	Isaac
Eve	Abel
Eve	Seth
Sara	Isaac

Projection and Selection

Two “orthogonal” operators

- Selection:
  - horizontal decomposition
- Projection:
  - vertical decomposition



Projection (π)

General form: $\pi_{A1,\ldots,Ak}(R)$

where R is a relation and $A_1,\ldots,A_k$ are attributes of R.

Result:

- Schema: $(A_1,\ldots,A_k)$
- Instance: the set of all subtuples $t[A_1,\ldots,A_k]$ where $t \in R$

Intuition: "removes" all attributes that are not in projection list

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Projection: Example

STUDENT

studno	name	hons	tutor	year
s1	jones	ca	bush	2
s2	brown	cis	kahn	2
s3	smith	cs	goble	2
s4	bloggs	ca	goble	1
s5	jones	cs	zobel	1
s6	peters	ca	kahn	3

$\pi_{tutor}(\text{STUDENT})$ =

tutor
bush
kahn
goble
zobel

Note:

- *result relations don't have a name*
- *If duplicates?*

Selection (σ)

General form: $\sigma_C(R)$

with a relation R and a condition C on the attributes of R.

Result:

- Schema: the schema of R
- Instance: the set of all $t \in R$ that satisfy C

Intuition: Filters out all tuples that do not satisfy C

Selection: Example

STUDENT

studno	name	hons	tutor	year
s1	jones	ca	bush	2
s2	brown	cis	kahn	2
s3	smith	cs	goble	2
s4	bloggs	ca	goble	1
s5	jones	cs	zobel	1
s6	peters	ca	kahn	3

$\sigma_{name='bloggs'}(STUDENT)$ =

STUDENT

studno	name	hons	tut or	year
s4	bloggs	ca	goble	1

Note:

- *result relation has a name*

Selection Conditions

Elementary conditions:

<attr> op <val> or <attr> op <attr> or <expr> op <expr>

where op is “=”, “<”, “≤”, (on numbers and strings)
“LIKE” (for string comparisons),…

Example:

- age ≤ 24
- phone LIKE ‘0039%’
- salary + commission ≥ 24000

Combined conditions (using Boolean connectives):

C1 and C2 or C1 or C2 or not C

Selection conditions

STUDENT

studno	name	hons	tutor	year
s1	jones	ca	bush	2
s2	brown	cis	kahn	2
s3	smith	cs	goble	2
s4	bloggs	ca	goble	1
s5	jones	cs	zobel	1
s6	peters	ca	kahn	3

$$\sigma_{((hons=\text{‘cs’})\ or\ (hons\ =\ \text{‘ca’}))\ and\ (tutor=\text{‘goble’})}(STUDENT) \quad =$$

STUDENT

studno	name	hons	tut or	year
s3	smiths	cs	goble	2
s4	bloggs	ca	goble	1

Operators Can Be Nested

Who is the tutor of the student named “Bloggs”?

STUDENT

studno	name	hons	tutor	year
s1	jones	ca	bush	2
s2	brown	cis	kahn	2
s3	smith	cs	goble	2
s4	bloggs	ca	goble	1
s5	jones	cs	zobel	1
s6	peters	ca	kahn	3

$$\pi_{tutor}\left(\sigma_{name='bloggs'}(STUDENT) \right)$$

=

tutor
goble

Identities for Selection and Projection

For all conditions C1, C2, more generally predicates p,q and relation R we have:

Selection splitting:

$\sigma_{C1 \text{ and } C2}(R) = \sigma_{C1}(\ \sigma_{C2}(R)\ )$

Also, selection is commutative:

$\sigma_{C1}(\ \sigma_{C2}(R)\ ) = \sigma_{C2}(\ \sigma_{C1}(R)\ )$

What about these – commutativity of selection and projection

$\pi_{A1,\dots,Am}(\sigma_{C}(R)) = \sigma_{C}(\pi_{A1,\dots,Am}(R))$

Selection Conditions and "NULL"

Does the following identity hold?

$$\text{Student} = \sigma_{\text{year} \leq 3}(\text{Student}) \cup \sigma_{\text{year} > 3}(\text{Student}) \text{ ?}$$

What if Student contains a tuple t with t[year] = null ?

Convention: Only comparisons with non-null values are TRUE or FALSE. Comparisons involving null yield a value UNKNOWN. To test, whether a value is null or not null, there are two conditions:

<attr> IS NULL or <attr> IS NOT NULL

Thus, the following identities hold:

$$\text{Student} = \sigma_{\text{year} \leq 3}(\text{Student}) \cup \sigma_{\text{year} > 3}(\text{Student}) \cup \sigma_{\text{year IS NULL}}(\text{Student})$$

$$= \sigma_{\text{year} \leq 3 \text{ OR year} > 3 \text{ OR year IS NULL}}(\text{Student})$$

Cartesian Product (X)

General form:

where R and S are arbitrary relations $R \times S$

Result:

- Schema: (A1,…,Am,B1,…,Bn), where (A1,…,Am) is the schema of R and (B1,…,Bn) is the schema of S.

  (If A is an attribute of both, R and S, then $R \times S$ contains the disambiguated attributes R.A and S.A.)

- Instance: the set of all concatenated tuples (t,s) where $t \in R$ and $s \in S$

Cartesian Product: Student × Staff

STUDENT

studno	name	hons	tutor	year
s1	jones	ca	bush	2
s2	brown	cis	kahn	2
s3	smith	cs	goble	2
s4	bloggs	ca	goble	1
s5	jones	cs	zobel	1
s6	peters	ca	kahn	3

STAFF

lecturer	roomno
kahn	IT206
bush	2.26
goble	2.82
zobel	2.34
watson	IT212
woods	IT204
capon	A14
lindsey	2.10
barringer	2.125

Brings all information from relations into one ***without applying any conditions***

What's the point of this?

studno	name	hons	tutor	year	lecturer	roomno
s1	jones	ca	bush	2	kahn	IT206
s1	jones	ca	bush	2	bush	2.26
s1	jones	ca	bush	2	goble	2.82
s1	jones	ca	bush	2	zobel	2.34
s1	jones	ca	bush	2	watson	IT212
s1	jones	ca	bush	2	woods	IT204
s1	jones	ca	bush	2	capon	A14
s1	jones	ca	bush	2	lindsey	2.10
s1	jones	ca	bush	2	barringer	2.125
s2	brown	cis	kahn	2	kahn	IT206
s2	brown	cis	kahn	2	bush	2.26
s2	brown	cis	kahn	2	goble	2.82
s2	brown	cis	kahn	2	zobel	2.34
s2	brown	cis	kahn	2	watson	IT212
s2	brown	cis	kahn	2	woods	IT204
s2	brown	cis	kahn	2	capon	A14
s2	brown	cis	kahn	2	lindsey	2.10
s2	brown	cis	kahn	2	barringer	2.125
s3	smith	cs	goble	2	kahn	IT206
s3	smith	cs	goble	2	bush	2.26
s3	smith	cs	goble	2	goble	2.82
s3	smith	cs	goble	2	zobel	2.34
s3	smith	cs	goble	2	watson	IT212
s3	smith	cs	goble	2	woods	IT204
s3	smith	cs	goble	2	capon	A14
s3	smith	cs	goble	2	lindsey	2.10
s3	smith	cs	goble	2	barringer	2.125
s4	bloggs	ca	goble	1	kahn	IT206

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"Where are the Tutors of Students?"

STAFF

lecturer	roomno
kahn	IT206
bush	2.26
goble	2.82
zobel	2.34
watson	IT212
woods	IT204
capon	A14
lindsey	2.10
barringer	2.125

To answer the query

"For each student, identified by name and student number, return the name of the tutor and their office number"

we have to

- combine tuples from Student and Staff
- that satisfy "`Student.tutor=Staff.lecturer`"
- and keep the attributes `studno`, `name`, (`tutor` or `lecturer`), `and roomno`.

STUDENT

studno	name	hons	tutor	year
s1	jones	ca	bush	2
s2	brown	cis	kahn	2
s3	smith	cs	goble	2
s4	bloggs	ca	goble	1
s5	jones	cs	zobel	1
s6	peters	ca	kahn	3

In relational algebra:

$$\pi_{studno,name,lecturer,roomno}(\sigma_{tutor=lecturer}(Student \times Staff))$$

The part $\sigma_{tutor=lecturer}(Student \times Staff)$ is a "<u>join</u>".

Example: Student Marks in Courses

STUDENT

studno	name	hons	tutor	year
s1	jones	ca	bush	2
s2	brown	cis	kahn	2
s3	smith	cs	goble	2
s4	bloggs	ca	goble	1
s5	jones	cs	zobel	1
s6	peters	ca	kahn	3

ENROL

stud no	course no	lab mark	exam mark
s1	cs250	65	52
s1	cs260	80	75
s1	cs270	47	34
s2	cs250	67	55
s2	cs270	65	71
s3	cs270	49	50
s4	cs280	50	51
s5	cs250	0	3
s6	cs250	2	7

"For each student,
show the courses in which they are enrolled and their marks"

First,

$$R \leftarrow \sigma_{\text{Student.studno= Enrol.studno}}(\text{Student} \times \text{Enrol}),$$

then

$$\text{Result} \leftarrow \pi_{\text{studno,name, …,exam\_mark}}(R)$$

$$\pi_{\text{studno,name, …,exam\_mark}}(\sigma_{\text{Student.studno= Enrol.studno}}(\text{Student} \times \text{Enrol}))$$

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Join ($\bowtie$)

- The most used operator in the relational algebra.

  Allows us to establish connections among data in different relations, taking advantage of the ”data-based" nature of the relational model.

- Three main versions of the join:
  - "**natural**" join: takes attribute names into account;
  - "**theta**" join.
  - “**equi**” join (a special form of theta join)
  - all denoted by the symbol $\bowtie$

Natural Join

STUDENT

studno	name	hons	tutor	year
s1	jones	ca	bush	2
s2	brown	cis	kahn	2
s3	smith	cs	goble	2
s4	bloggs	ca	goble	1
s5	jones	cs	zobel	1
s6	peters	ca	kahn	3

ENROL

stud no	course no	lab mark	exam mark
s1	cs250	65	52
s1	cs260	80	75
s1	cs270	47	34
s2	cs250	67	55
s2	cs270	65	71
s3	cs270	49	50
s4	cs280	50	51
s5	cs250	0	3
s6	cs250	2	7

$$\text{Student} \bowtie \text{Enrol}$$

- **Implicit** join based on **common** attributes
- The tuples in the resulting relation are obtained by combining tuples in the operands with equal values on the common attributes
- Common attributes appear once in the results

Student ⋈ Enrol

STUDENT

studno	name	hons	tutor	year
s1	jones	ca	bush	2
s2	brown	cis	kahn	2
s3	smith	cs	goble	2
s4	bloggs	ca	goble	1
s5	jones	cs	zobel	1
s6	peters	ca	kahn	3

ENROL

stud no	course no	lab mark	exam mark
s1	cs250	65	52
s1	cs260	80	75
s1	cs270	47	34
s2	cs250	67	55
s2	cs270	65	71
s3	cs270	49	50
s4	cs280	50	51
s5	cs250	0	3
s6	cs250	2	7

```
 stuno |  name  | hons | tutor | year | courseno | labmark | exammark
-------+--------+------+-------+------+----------+---------+----------
 s1    | jones  | ac   | bush  |    2 | cs250    |      65 |       52
 s1    | jones  | ac   | bush  |    2 | cs260    |      80 |       75
 s1    | jones  | ac   | bush  |    2 | cs270    |      47 |       34
 s2    | brown  | is   | kahn  |    2 | cs250    |      67 |       55
 s2    | brown  | is   | kahn  |    2 | cs270    |      65 |       71
 s3    | smith  | cs   | goble |    2 | cs270    |      49 |       50
 s4    | bloggs | ac   | goble |    1 | cs250    |      50 |       51
 s5    | jones  | cs   | zobel |    1 | cs250    |       0 |        3
 s6    | peters | ac   | kahn  |    3 | cs250    |       2 |        7
(9 rows)
```

Natural Join (another example)

Offences

Code	Date	Officer	Dept	Registartion
143256	25/10/1992	567	75	5694 FR
987554	26/10/1992	456	75	5694 FR
987557	26/10/1992	456	75	6544 XY
630876	15/10/1992	456	47	6544 XY
539856	12/10/1992	567	47	6544 XY

Cars

Registration	Dept	Owner	…
6544 XY	75	Cordon Edouard	…
7122 HT	75	Cordon Edouard	…
5694 FR	75	Latour Hortense	…
6544 XY	47	Mimault Bernard	…

Offences $\bowtie$ Cars

Code	Date	Officer	Dept	Registration	Owner	…
143256	25/10/1992	567	75	5694 FR	Latour Hortense	…
987554	26/10/1992	456	75	5694 FR	Latour Hortense	…
987557	26/10/1992	456	75	6544 XY	Cordon Edouard	…
630876	15/10/1992	456	47	6544 XY	Mimault Bernard	…
539856	12/10/1992	567	47	6544 XY	Mimault Bernard	…

Example from CSC343 Intro to Database, University of Toronto

θ-Joins (read "Theta"-Joins), Equi-Joins

Theta-Join:

- *The most general form of JOIN …*
- Theta join combines tuples from different relations provided they satisfy the theta condition. The join condition is denoted by the symbol **θ**.
- Theta join can use comparison operators and common attributes are not required.

$$\text{Student} \bowtie_{\text{student.year} < \text{enrol.labmark}} \text{Enrol}$$

- The results include the 'joined' attributes from both relations
- The attribute names do not have to match (but their domains have to be compatible)

Equi-Join:

A special form of Theta-join, and *the most common form of JOIN …*

with a **join** condition containing an equality operator (i.e., explicitly stating the joining attributes)

$$\text{Student} \bowtie_{\text{stuno=stuno}} \text{Enrol}$$

STUDENT

<u>studno</u>	name	hons	tutor	year
s1	jones	ca	bush	2
s2	brown	cis	kahn	2
s3	smith	cs	goble	2
s4	bloggs	ca	goble	1
s5	jones	cs	zobel	1
s6	peters	ca	kahn	3

STAFF

<u>lecturer</u>	roomno
kahn	IT206
bush	2.26
goble	2.82
zobel	2.34
watson	IT212
woods	IT204
capon	A14
lindsey	2.10
barringer	2.125

Student $\bowtie_{\text{tutor=lecturer}}$ Staff

(equivalent to: $\sigma_{\text{tutor=lecturer}}(\text{Student} \times \text{Staff})$)

stud no	name	hons	tutor	year	lecturer	roomno
s1	jones	ca	bush	2	bush	2.26
s2	brown	cis	kahn	2	kahn	IT206
s3	smith	cs	goble	2	goble	2.82
s4	bloggs	ca	goble	1	goble	2.82
s5	jones	cs	zobel	1	zobel	2.34
s6	peters	ca	kahn	3	kahn	IT206

Join: An Observation

Some tuples don’t contribute to the result, they get lost.

Employee	Department
Brown	A
Jones	B
Smith	B

$\bowtie$

Department	Head
B	Black
C	White

Employee	Department	Head
Jones	B	Black
Smith	B	Black

Outer Join

An outer join extends those tuples with null values that would get lost by a join like natural join or equi join (a.k.a. inner joins)

The outer join comes in three versions

- left: keeps the tuples of the left argument, extending them with nulls if necessary
- right: ... of the right argument ...
- full: ... of both arguments ...

(Natural) Left Outer Join

Employee

Employee	Department
Brown	A
Jones	B
Smith	B

Department

Department	Head
B	Black
C	White

Employee $\bowtie_{\text{Left}}$ Department

Employee	Department	Head
Brown	A	null
Jones	B	Black
Smith	B	Black

(Natural) Right Outer Join

Employee

Employee	Department
Brown	A
Jones	B
Smith	B

Department

Department	Head
B	Black
C	White

Employee $\bowtie_{\text{Right}}$ Department

Employee	Department	Head
Jones	B	Black
Smith	B	Black
null	C	White

(Natural) Full Outer Join

Employee

Employee	Department
Brown	A
Jones	B
Smith	B

Department

Department	Head
B	Black
C	White

Employee $\bowtie_{Full}$ Department

Employee	Department	Head
Brown	A	null
Jones	B	Black
Smith	B	Black
null	C	White

Duplicate Elimination

Real DBMSs implement a version of relational algebra that operates on multisets (“bags”) instead of sets.

(Which of these operators may return bags,
even if the input consists of sets?)

For the bag version of relational algebra, there exists a duplicate elimination operator δ.

If R =

A	B
1	2
3	4
3	4
1	2

, then $\delta(R)$ =

A	B
1	2
3	4

Aggregation

Often, we want to retrieve aggregate values, like the “sum of salaries” of employees, or the “average age” of students.

This is achieved using aggregation functions, such as SUM, AVG, MIN, MAX, or COUNT.

Such functions are applied by the grouping and aggregation operator γ.

If $R =$

A	B
1	2
3	4
3	5
1	1

, then $\gamma_{SUM(A)}(R) =$

SUM(A)
8

and $\gamma_{AVG(B)}(R) =$

AVG(B)
3

Grouping and Aggregation

More often, we want to retrieve aggregate values for groups, like the “sum of employee salaries” per department, or the “average student age” per faculty.

As additional parameters, we give γ attributes that specify the criteria according to which the tuples of the argument are grouped.

E.g., the operator γA,SUM(B) (R)

- partitions the tuples of R in groups that agree on A,
- returns the sum of all B values for each group.

If R =

A	B
1	2
3	4
3	5
1	3

, then $\gamma_{A,SUM(B)}(R)$ =

A	SUM(B)
1	5
3	9