# XML and Databases
# Efficient XPath evaluation

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Week 8

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\begin{array}{lcl}
p & ::= & [(a_1, l_1, p_1); \ldots ; (a_n, l_n, p_n)] \\
a & ::= & \texttt{child} | \texttt{descendant} | \ldots \\
l & ::= & * | \mathit{tagname} | \texttt{text()}
\end{array}
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$$p_i \quad ::= \quad [(a_{i1}, l_{i1}, [\ldots]); \ldots; (a_{in}, l_{in}, [\ldots])]$$

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$$

# Node Set Algorithm (1/6)

```
NodeSet eval(Path p, NodeSet nodes, bool all)
```

Applies the path `p` to the set of nodes `nodes` and returns:

- All the nodes matching the query if `all` is `true`
- The first node matching the query if `all` is `false`

```
NodeSet eval_axis(Axis a, Label l, NodeSet nodes,bool all)
```

Given a set of nodes `nodes` of a document returns the nodes in the axis `a` with label `l`

- if `all` is `true`, returns all the matching nodes.
- if `all` is `false`, returns the first matching node

# Node Set Algorithm (2/6)

```
NodeSet eval(Path p, NodeSet nodes,bool all){
  NodeSet r = nodes;
  //we apply the steps one after another
  for each (a,l,f) in p {
    //we select all the node matching the axis and label
    r = eval_axis(a,l,r,all);
    if (filter != []) {
      r' = Empty;
      for each n in r
        if (eval(f,{ n },false) != Empty)
           r' = add(r',n);
      r = r';
    };
  return r;
}
```

# Node Set Algorithm (3/6)

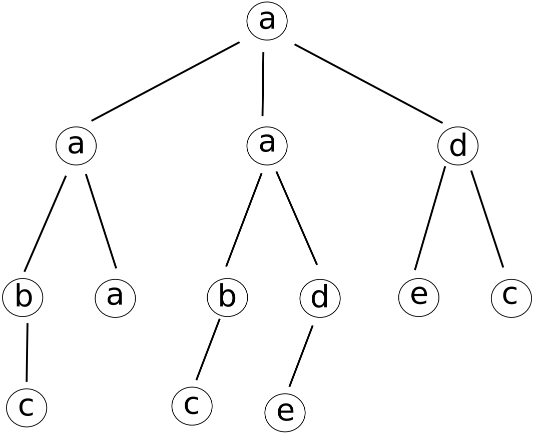
```
NodeSet eval_axis(Axis a,Label l,NodeSet n,bool all)
{
  switch (a){
    child:
         return eval_child(l,n,all);
    descendant:
         return eval_descendant(l,n,all);
    //continue for all the axes
         ...
  }
}
```

# Node Set Algorithm (4/6)

```
NodeSet eval_descendant(Label l,NodeSet n,bool all)
{
  NodeSet r = Empty;
  for each t in n {
    for each tc in children(t) {
      if (label(tc) == l){
        r = add(r,tc);
        if (!(all))   //we only want the first result
           return r;
      }
  }; //r contains all the children of t tagged l
  r = r ∪ eval_descendant(l,children(t));
}
  return r;
}
```

# Node Set Algorithm (5/6)

Example: XPath expresison `//a[d//e]/b//c`
Called initially with the `NodeSet` containing the **root**



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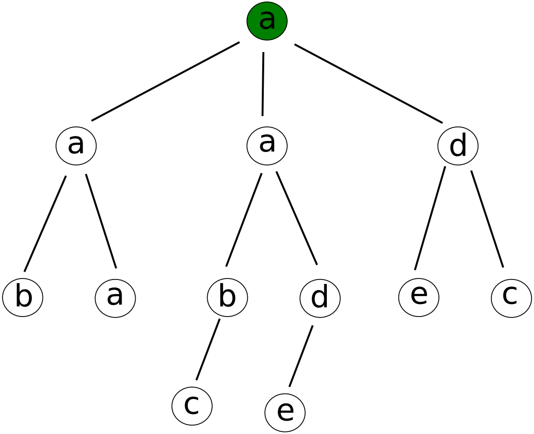

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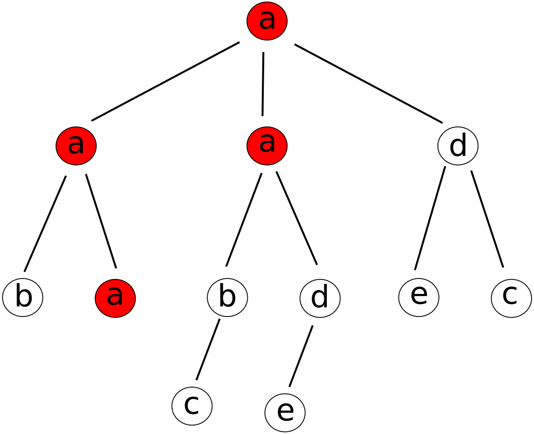


eval_axis(`desc`,a,. . . ,`true`)

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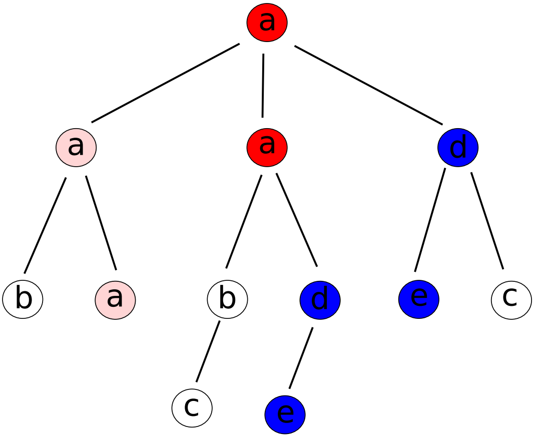


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eval(d//e,...,`false`)

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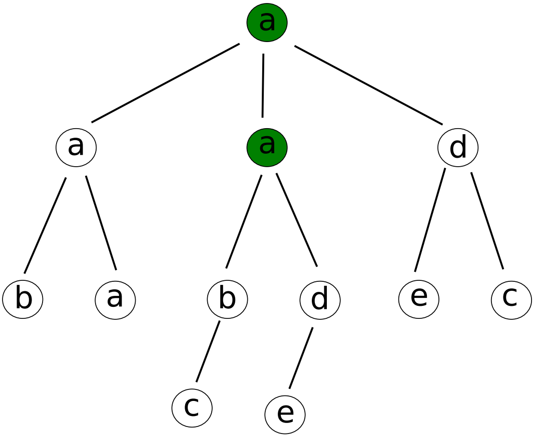


eval_axis(desc,a,. . . ,`true`)
eval(d//e,. . . ,`false`)
Result of the first step

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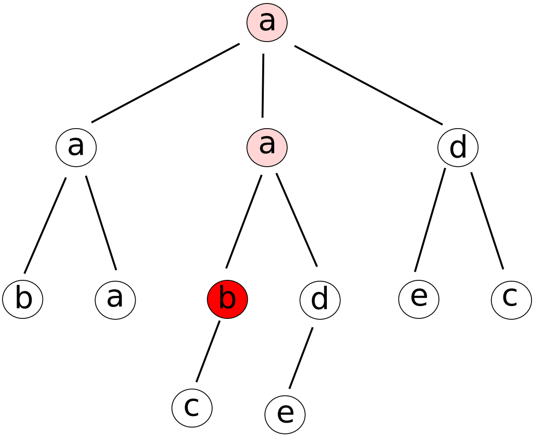


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eval(d//e,. . . ,`false`)
Result of the first step
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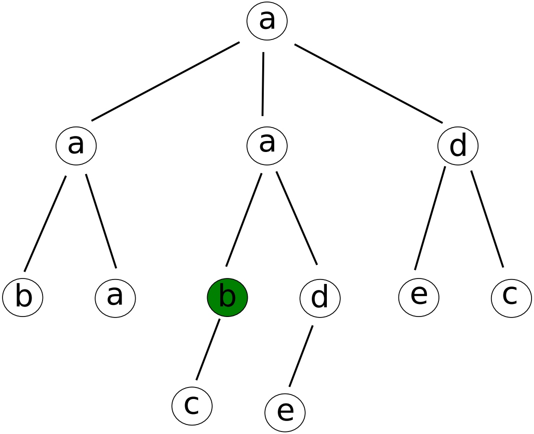


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eval_axis(`child`,b,. . . ,`true`)
Result of the $2^{nd}$ step

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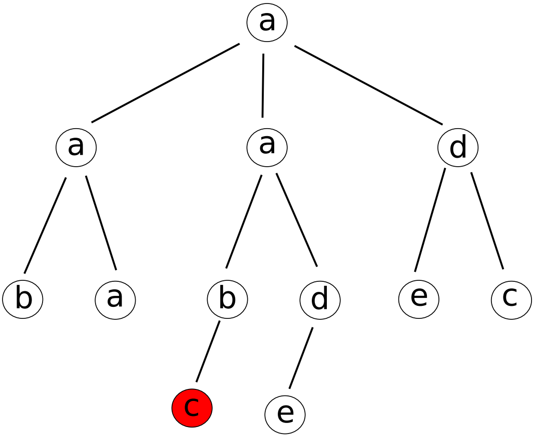


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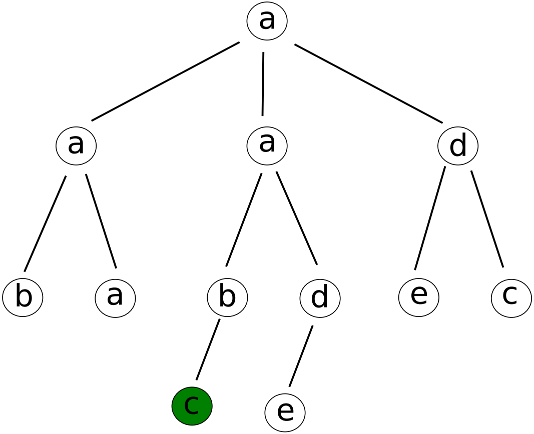


eval_axis(`desc`,a,...,`true`)
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Result of the $2^{nd}$ step
eval_axis(`desc`,c,...,`true`)
Final result

# Node Set Algorithm (6/6)

Pros and cons of the algorithm:

+ Easy to implement

Remains very inefficient: $O(|D|^2)$ for forward XPath, $O(2^{|Q|} + |D|^2)$ for full XPath (cf. Lecture)

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- May return several copies of the same node, thus either use a Set datastructure for the result, or sort and sieve the result at the end.

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+ Easy to implement
+ Can can be extended to all XPath axes easily
- May return several copies of the same node, thus either use a Set datastructure for the result, or sort and sieve the result at the end.
- Need to traverse many times the tree, cannot be done in streaming

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# Automata based algorithm

We proceed in two steps:

- first we see how this works for XPath expressions without filters

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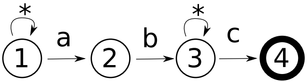

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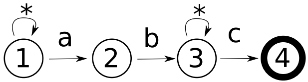
`//a/b//c` becomes: 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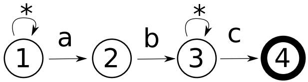


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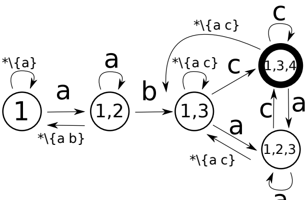

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Good news: we don't need to determinize!
Reference:

*Processing XML streams with deterministic automata and stream indexes*
By T.J. Green, A. Gupta, G. Miklau, M. Onizuka, D. Suciu, TODS 2004

# Topdown XPath evaluation

```
// Takes a NFA, a set of states and a document node
// Returns the set of nodes matched by the automaton
NodeSet eval(Automaton a, States S, Node t){
  //The empty tree yields no result
  if (t == null) return Empty
  else { //Everything is done here, see next slide
    S'={q' | ∀q ∈ S, s.t. q,l → q' ∈ a, l = label(t) or *}
    r = Empty;
    for each t' in children(t) {
      r = r ∪ eval(a,S',t');
    };
    if (finalstate(a) ∈ S')
      r = r ∪ {t}
  };
  return r;
}
```

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What does this do?

$$S' = \{q' \mid \forall q \in S,\ \text{s.t. } q, l \rightarrow q' \ \in \ a,\ l = \texttt{label}(t) \ or \ *\}$$

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For each state $q$ of the NFA in $S$ it computes the set of states in which we can go with the label of the current node $t$

- Then we recursively evaluate $S'$ on all the children of $t$
- If we took a transition which lead us to an accept state, then we also need to add $t$ to the final result

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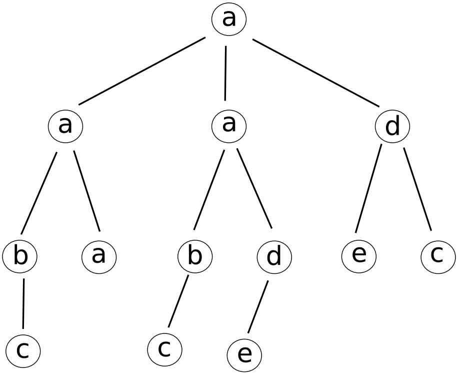
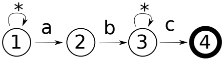
To represent the NFA, we need:

- The set of all states, $Q$, the initial state $I$, the final state $F$
- a hash table mapping pairs of states$\times$labels to states

# Topdown XPath evaluation




$Q = \{1, 2, 3, 4\}$

$I = \{1\}$

$F = \{4\}$

| | | | |
|---|---|---|---|
| 1, | a | $\mapsto$ | 2 |
| 1, | * | $\mapsto$ | 1 |
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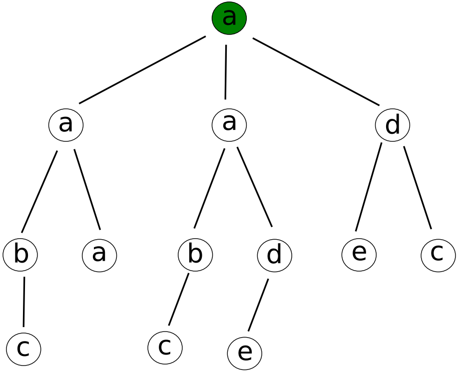
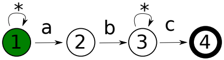



We start on the root, with the initial state

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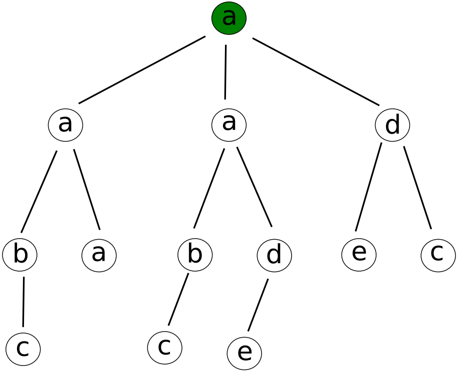
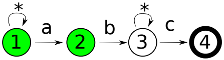



For label "a" in state 1, the NFA can end up in two states, 1 and 2...

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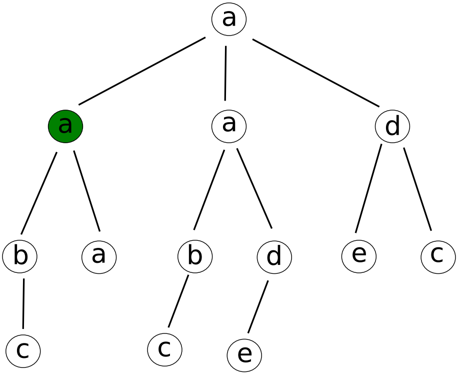
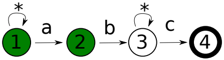



So we call recursively, with $S = \{1, 2\}$ on the first child of the root...

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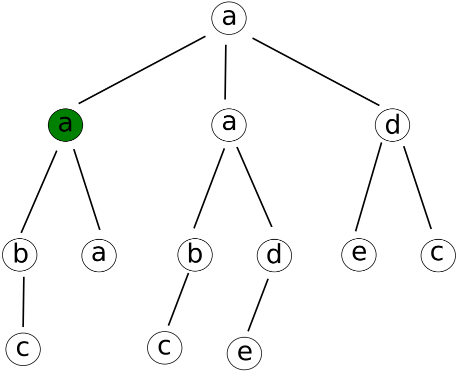


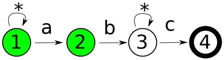


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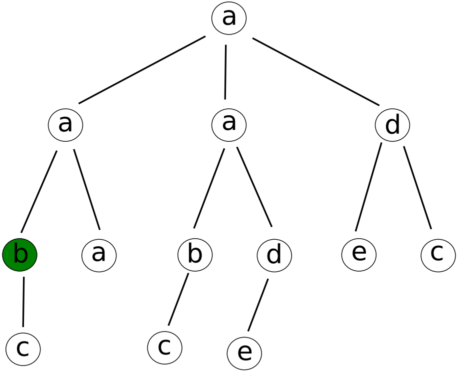


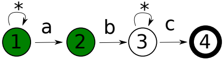


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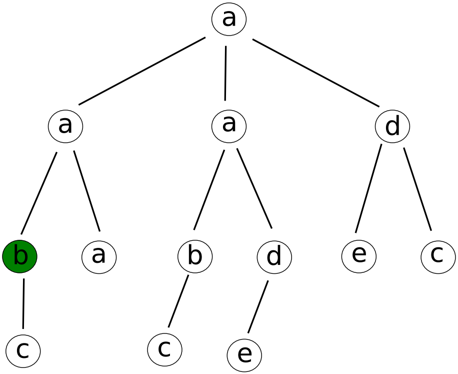
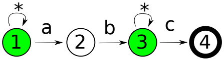



Here label “b” allows us to go in state 3 and also stays in state 1

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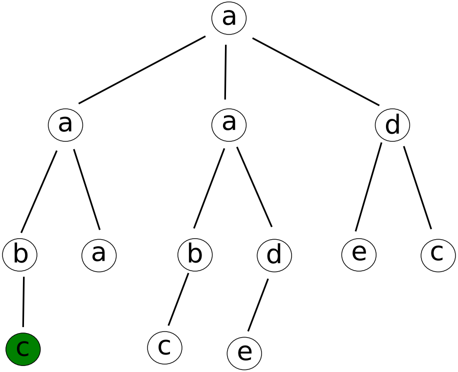


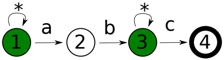


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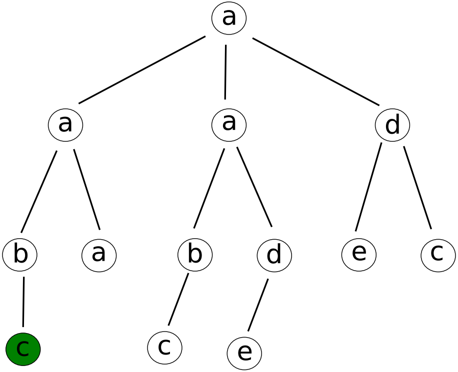
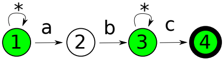



We arrive in "c". The call on the children returns `Empty`. One of our state is final, so there is a run of the automaton which accepts this path, we mark the node as **selected**.

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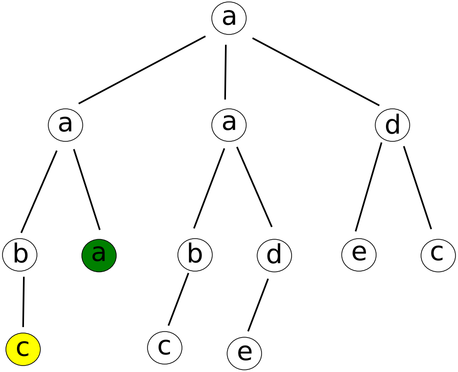


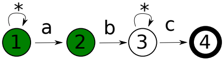


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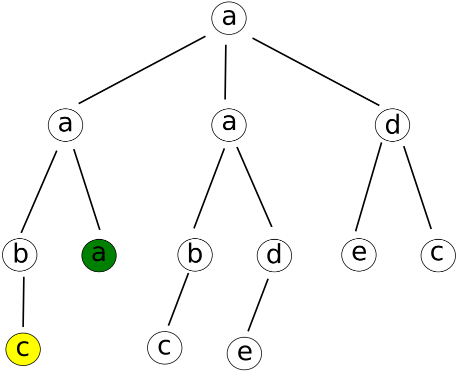


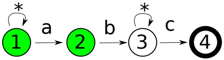


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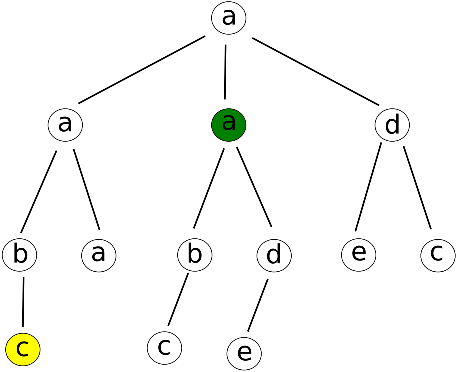


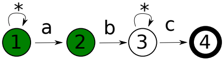


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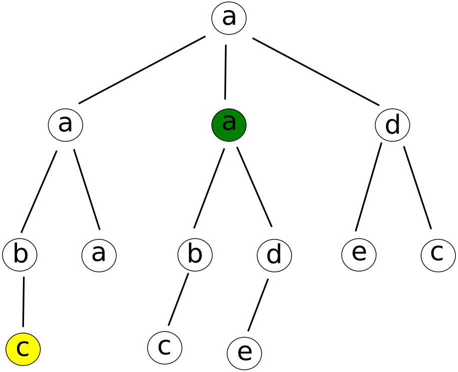


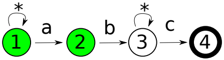


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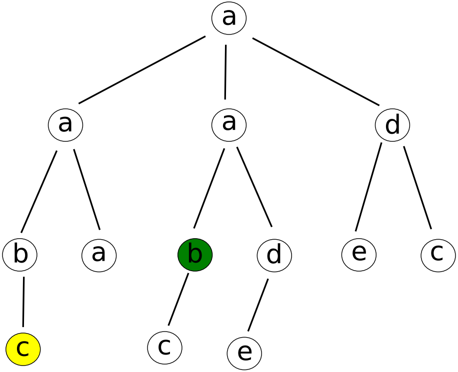


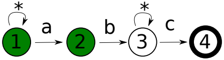


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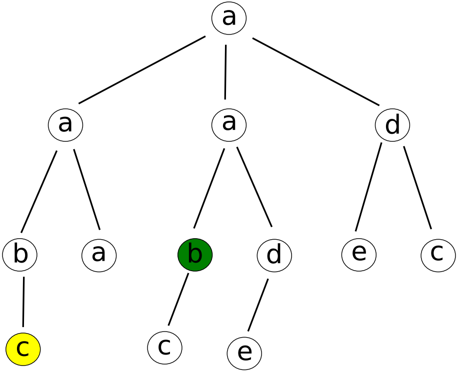


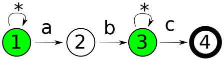


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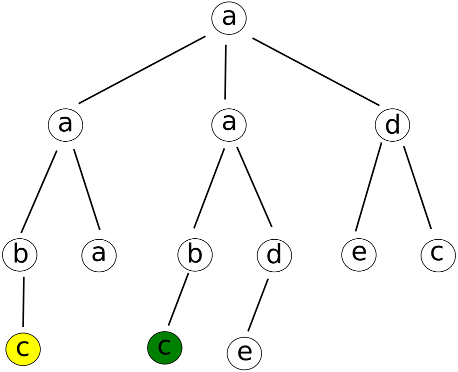


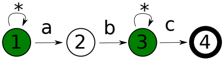


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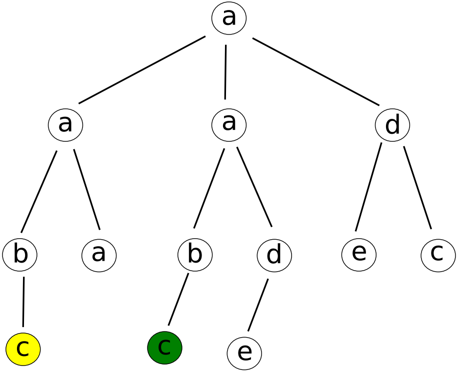


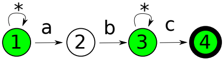


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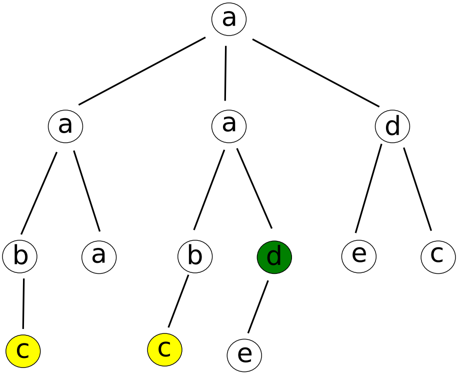


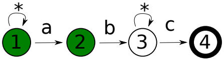


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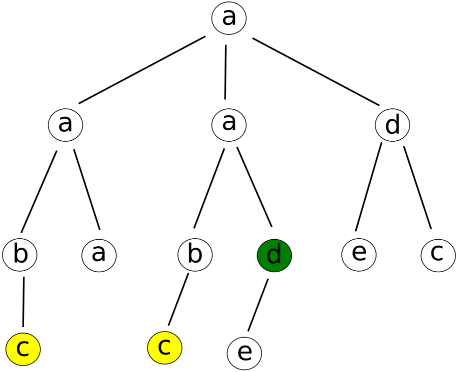


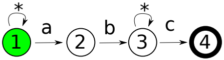


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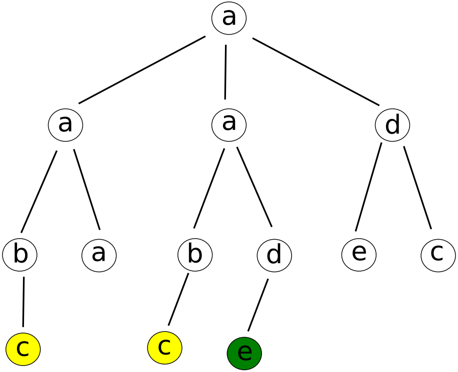


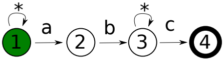


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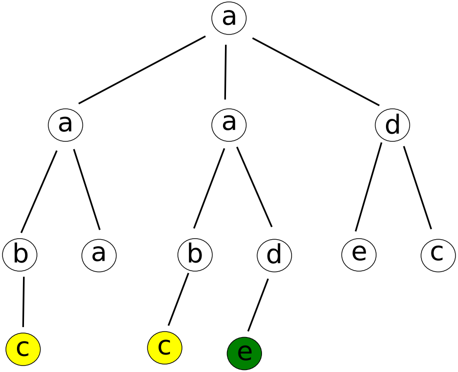

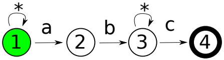


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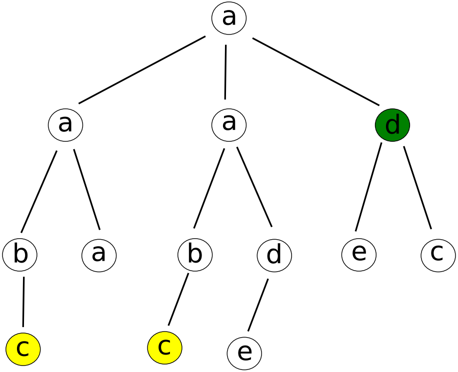


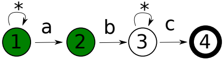


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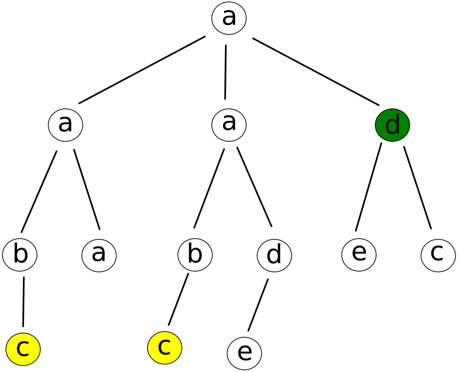


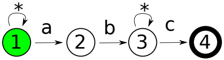


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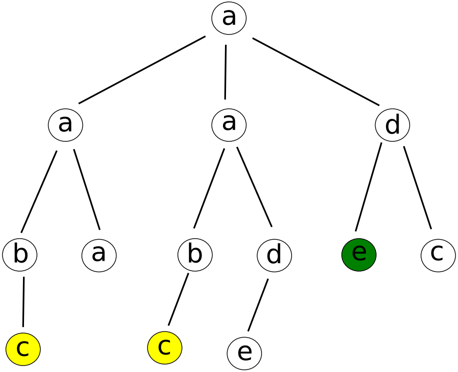


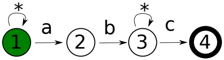


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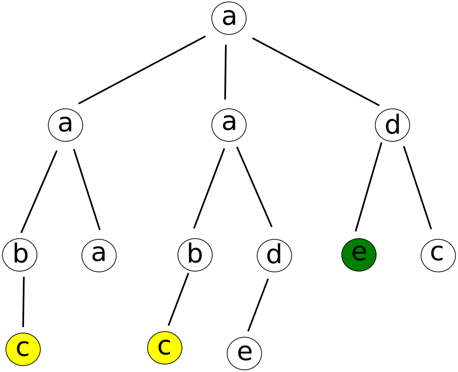


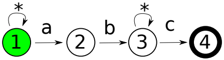


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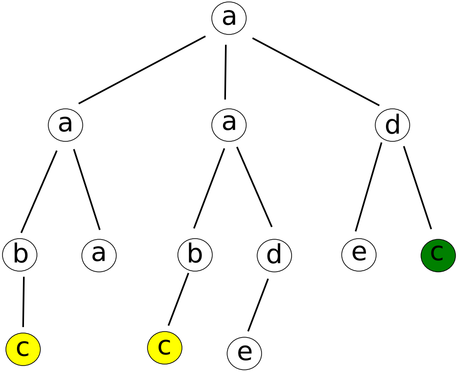


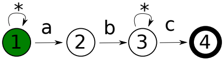


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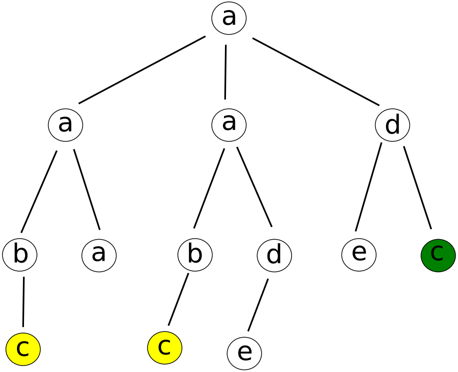


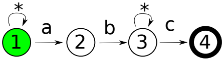


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Complexity is $O(|Q| \times |D|)$, which is the best complexity for this problem (cf lecture).

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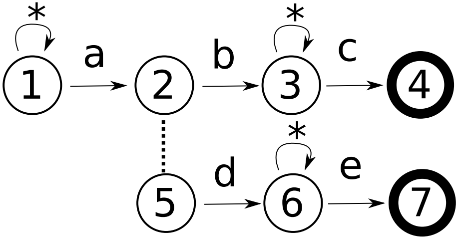

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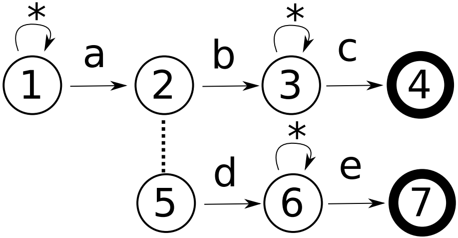

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```
NodeSet eval(Automata a, States S, Node t,
             FilterStack FS){
  if (t == null) return Empty,FS
  else {
    S'={q' | q,l → q' ∈ a, l = label(t) or *}
    FilterSet f = {{InitState(FilterAuto(q))} |q ∈ S}
    FS'=push(f,FS);
    FS''=EmptyStack;
    for each fs in FS' {
      fs'=Empty;
      for each (_,s) in fs
      fs' = fs' ∪ {s×{q' | q,l → q' ∈ a_i, l = label(t) or *}}
      push(FS'',fs');
    }
```

...

# Topdown XPath evaluation

```
r = Empty;
fs = Empty;
for each t' in children(t) {
  r',FS''' = eval(a,S',t',FS'');
  r = r∪r';
  fs''' = pop(FS''');
  fs'' = pop(FS'');
  for each (s,s') in fs'''
     if (finalstate(a')␣∈ s')
        remove (_,s) from fs'';
  FS'' = push(FS'',fs'');
};
```

# Topdown XPath evaluation

```
   fs = peek(FS");
   if (isempty(fs))
     if (finalstate(a) ∈ S)
     r = r ∪ {t};
   else
     r = Empty
return (r,FS");
```