XML and Databases

Lecture 13
Fast Substring Search

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Fast Substring Search

Recall the **`contains`**-predicate of XPath:

```
//book/abstract[contains(.,"fix")]
```

For instance the abstract node:

```
<book>..
  <abstract>This article dicusses the advantages of
  suffix arrays, for the purpose of substring search ..
  </abstract>..
</book>
```

will be returned, because it contains the substring "fix"
because it appears in the word "suffix" mentioned in the abstract text.

Fast Substring Search

Question

Given a ***very large text***, how do you search for

→ All occurrences of a given keyword?
→ All occurrences of a given substring?
→ Count them (can be done faster?)

Fast Substring Search

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What we know so far:

→ can use KMP-algorithm.
for a text of length n, it only takes $O(n)$ time to locate all occurrences of the substring.

→ in a database, that is *way* to slow!!
How do you think Google indexes text for fast search??

Fast Substring Search

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We want search time to be independent of the size n of the text, but should only depend on the length of the keyword.

We are allowed to preprocess the string in linear time ("indexing").

Fast Substring Search

Question

Given a ***very large text***, how do you search for

→ All occurrences of a given keyword?
→ All occurrences of a given substring?
→ Count them (can be done faster?)

Idea 1 --we search for exact WORDS, not substrings—

Make a "dictionary" of every WORD that occurs in the text:

```
1: this[0, 89, 2098]
2: article[8, 29300]
3: …
```

→ Sort it!

```
1: a[90, 183, 290, … ]
2: actual[450, 9812, … ]
3: article[8, 29300]
3: …
```

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Fast Substring Search

Given a keyword $a_1a_2\ldots a_m$ of length m,

How much time required to locate all occurrences of the keyword?

Easy: keep start rows of strings that “start with a_1” (for any letter), and within those rows, again those that “continue with letter a_2” (for all letters) Etc. (this is a tree of height=length of longest word, and branching=# different letters)

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1: a[90, 183, 290, … ]
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3: …
```

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Fast Substring Search

Given a keyword $a_1a_2\ldots a_m$ of length m,

How much time required to locate all occurrences of the keyword?

→ only time $O(m)$! ☺

Problems (1) indexing time?!
(2) how to do substring search??

Idea 1 --we search for exact WORDS, not substrings—

Make a “dictionary” of every WORD that occurs in the text:

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1: this[0, 89, 2098]
2: article[8, 29300]
3: …
```

Sort it!

```
1: a[90, 183, 290, … ]
2: actual[450, 9812, … ]
3: article[8, 29300]
3: …
```

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Fast Substring Search

Given the text of length n, how many **substrings** are there?

→ (begin position, end position)

Quadratically many! That is, $O(n^2)$.
Thus, it is **impossible** in linear time to list all these substrings and put them into a (sorted) dictionary!

Idea 1 --we search for exact WORDS, not substrings—

Make a “dictionary” of every WORD that occurs in the text:

```
1: this[0, 89, 2098]
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3: …
```

Sort it!

```
1: a[90, 183, 290, … ]
2: actual[450, 9812, … ]
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3: …
```

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The Burrows-Wheeler Transform

Idea comes from compression.
`bzip2` is based on the Burrows-Wheeler Transform!

1) Add an end-marker “$” to the end of the text
2) End-marker $ is smallest in ordering:
‘$’ < ‘a’ < ‘b’ < ‘c’ < ….. < ‘z’ < ‘A’ < ….
3) Compute all cyclic shifts of text
4) Sort them lexicographically

Burrows-Wheeler Transform of text T

banana$		$banana
$banana		a$banan
a$banan		ana$ban
na$bana	sort	anana$b
ana$ban		banana$
nana$ba		na$bana
anana$b		nana$ba

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anana$b		nana$ba

Question
Why do you think is the BWT good for compression?

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ana$ban		banana$
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anana$b		nana$ba

First row: only tells us how many substrings
→ start with “a” (3)
→ how many start with “b” (1)
etc.
Same for any text with these letters!
We canNOT reconstruct T from row 1!

The Burrows-Wheeler Transform
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Idea comes from compression.
`bzip2` is based on the Burrows-Wheeler Transform!

1) Add an end-marker "$" to the end of the text
2) End-marker $ is smallest in ordering:
   '$' < 'a' < 'b' < 'c' < ….. < 'z' < 'A' < ….
3) Compute all cyclic shifts of text
4) Sort them lexicographically

canNOT reconstruct T from second row!

Second row: tells us how many substrings
→start with "n", if letter before is "a" (2)
→start with "a" if letter before is "n" (2)

Burrows-Wheeler Transform of text T

banana$		$banana
$banana		a$banan
a$banan		ana$ban
na$bana	sort	anana$b
ana$ban		banana$
nana$ba		na$bana
anana$b		nana$ba

First row: only tells us how many substrings
→ start with "a" (3)
→ how many start with "b" (1)
etc.
Same for any text with these letters!
We canNOT reconstruct T from row 1!

The Burrows-Wheeler Transform
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But, we can reconstruct T from the last row!! ☺
How?

Naïve way:
1. given "annb$aa", sort the letters. This gives row 1!

$		a
a		n
a		n
a	sort	b
b		$
n		a
n		a

What's next?
Hint: this tells us all two-letter substrings!

banana$		$banana
$banana		a$banan
a$banan		ana$ban
na$bana	sort	anana$b
ana$ban		banana$
nana$ba		na$bana
anana$b		nana$ba

Burrows-Wheeler Transform of text T

The Burrows-Wheeler Transform
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But, we can reconstruct T from the last row!! ☺
How?

Naïve way:
1. given "annb$aa", sort the letters. This gives row 1!

$		a
a		n
a		n
a	sort	b
b		$
n		a
n		a

What's next?
Hint: this tells us all two-letter substrings!

Text contains		
a$		$b
na		a$ ← This is row 2!
na		an
ba	sort	an
$b		ba
an		na
an		na

The Burrows-Wheeler Transform
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But, we can reconstruct T from the last row!! ☺
How?

Naïve way:
1. given "annb$aa", sort the letters. This gives row 1!
2. Constuct 2-letter substrings, sort. Gives row 2!
3. Construct 3-letter substrings, sort. Gives row 3!
etc

$		a
a		n
a		n
a	sort	b
b		$
n		a
n		a

pre-pend

Text contains					
a$		$b		a$b	$ba
na		a$		na$	a$b
na		an		nan	ana
ba	sort	an		ban	ana
$b		ba		$ba	ban
an		na		ana	na$
an		na		ana	nan

(a$b … ana → sort → $ba … nan)

The Burrows-Wheeler Transform
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But, we can reconstruct T from the last row!! ☺
How?

Naïve way:
1. given "annb$aa", sort the letters. This gives row 1!
2. Constuct 2-letter substrings, sort. Gives row 2!
3. Construct 3-letter substrings, sort. Gives row 3!
etc

$		a
a		n
a		n
a	sort	b
b		$
n		a
n		a

pre-pend

$ba		$banana
a$b		a$banan
ana		ana$ban
ana	prepend → sort → …. → prepend → sort	anana$b
ban		banana$ ← Original!
na$		na$bana
nan		nana$ba

BWT: Better Decompression
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→ In a real implementation we may NOT construct all cyclic shifts and sort… (because that takes quadratic time!!)
→ Same for decompression. May not do it the naïve way!

banana$	$banana
$banana	a$banan
a$banan	ana$ban
na$bana	anana$b
ana$ban	banana$
nana$ba	na$bana
anana$b	nana$ba

C	$	a	b	n
	0	1	4	5

LF-mapping
$LF(i)=C[L[i]] + rank_{L[i]}(L,i)$

O(log S) time using *wavelet tree*

Retrieving T: start from end marker, read backwards (by applying LF)

e.g.: LF(5)=1, LF(1)=2, LF(2)=6, LF(6)=3, LF(3)=7, LF(7)=4
L[.]= $ a n a n a b

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Backward Search

Here comes the ***magic***: we are now able to count the number of occurrences of a substring of length m, only in time O(m log S)!

S = size of alphabet

This is what makes fast keyword Search a la Google possible!

Search time is INDEPENDENT of the size of the text!!

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Backward Search

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S = size of alphabet

```
banana$    $banana
$banana    a$banan
a$banan    ana$ban
na$bana    anana$b
ana$ban    banana$
nana$ba    na$bana
anana$b    nana$ba
```

```
C $ a b n
  0 1 4 5
```

LF-mapping
LF(i)=C[L[i]] + $rank_{L[i]}$(L,i)

O(log S) time using *wavelet tree*

Backward search for Pattern P[1]..P[m]

Initial range: [sp,ep] with sp=C[P[m]]+1 and ep=C[P[m]+1]
Then [s,e] with
s = C[P[i]] + $rank_{L[i]}$(L,sp-1) + 1
e = C[P[i]] + $rank_{L[i]}$(L,ep)

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Backward Search

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```
banana$    $banana
$banana    a$banan
a$banan    ana$ban
na$bana    anana$b
ana$ban    banana$
nana$ba    na$bana
anana$b    nana$ba
```

```
C $ a b n
  0 1 4 5
```

123
P = ana
[sp,ep] = [2,4]

Backward search for Pattern P[1]..P[m]

➔ Initial range: [sp,ep] with sp=C[P[m]]+1 and ep=C[P[m]+1]
Then [s,e] with
s = C[P[i]] + $rank_{L[i]}$(L,sp-1) + 1
e = C[P[i]] + $rank_{L[i]}$(L,ep)

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Backward Search

Here comes the ***magic***: we are now able to count the number of occurrences of a substring of length m, only in time O(m log S)!

S = size of alphabet

```
banana$    $banana
$banana    a$banan
a$banan    ana$ban
na$bana    anana$b
ana$ban    banana$
nana$ba    na$bana
anana$b    nana$ba
```

```
C $ a b n
  0 1 4 5
```

123
P = ana
[sp,ep] = [2,4]

s=C["n"] + $rank_n$(L,1) +1
= 5 + 0 + 1 = 6

e= 5 + $rank_n$(L,4)
= 5 + 2 = 7

Backward search for Pattern P[1]..P[m]

Then [s,e] with
s = C[P[i]] + $rank_{L[i]}$(L,sp-1) + 1
e = C[P[i]] + $rank_{L[i]}$(L,ep)

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Backward Search

Here comes the ***magic***: we are now able to count the number of occurrences of a substring of length m, only in time O(m log S)!

S = size of alphabet

```
banana$    $banana
$banana    a$banan
a$banan    ana$ban
na$bana    anana$b
ana$ban    banana$
nana$ba    na$bana
anana$b    nana$ba
```

```
C $ a b n
  0 1 4 5
```

123
P = ana
[sp,ep] = [2,4]
sp=6
ep=7

s=C["a"] + $rank_a$(L,5) + 1
= 1 + 1 + 1 = 3

e = 1 + rank_a(L,7) =
1 + 3 = 4

Backward search for Pattern P[1]..P[m]

s = C[P[i]] + $rank_{L[i]}$(L,sp-1) + 1
e = C[P[i]] + $rank_{L[i]}$(L,ep)

Done!
[3,4]=final range
➔ 2 Occs of "ana"

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Backward Search

Here comes the ***magic***: we are now able to count the number of occurrences of a substring of length m, only in time O(m log S)!

S = size of alphabet

Backward search for Pattern P[1]..P[m] **Counting**: O(m log S) time

Locating
If every l=$\log^{1+epsilon}$ n position is sampled
then O(l log S) per occurrence,
by backward traversal using LF.

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Real Performance

```
/* In order : IsContains, Timing of IsContains, GlobalCount, Timing of
GlobalCount, CountContains, time of CountContains,
time of Full Report Contains */
-----------------
Sampling rate 64
"Bakst": 1, 0.038, 1, 0.004, 1, 0.04, 0.012, max_mem = 61
"ruminants": 1, 0.04, 22, 0.009, 19, 2.281, 1.588, max_mem = 61
"morphine": 1, 0.026, 392, 0.009, 144, 29.924, 32.668, max_mem = 61
"AUSTRALIA": 1, 0.028, 438, 0.009, 438, 4.616, 4.457, max_mem = 61
"molecule": 1, 0.051, 1472, 0.008, 966, 128.28, 122.014, max_mem = 61
"brain": 1, 0.02, 2685, 0.005, 1493, 218.462, 215.196, max_mem = 61
"human": 1, 0.019, 6897, 0.005, 4690, 553.496, 548.009, max_mem = 62
"blood": 1, 0.018, 10402, 0.005, 8534, 401.214, 399.674, max_mem = 62
"from": 1, 0.016, 20859, 0.004, 12073, 1722.95, 1717.83, max_mem = 62
"with": 1, 0.016, 63332, 0.004, 22974, 5084.14, 5083.77, max_mem = 63
" in": 1, 0.014, 238638, 0.003, 42586, 19641.8, 19630.3, max_mem = 64
"a": 1, 0.001, 2932251, 0, 595716, 189299, 188377, max_mem = 93
"\n": 1, 0.001, 9730750, 0.001, 5870474, 132780, 132241, max_mem = 86
```

use naïve

`CountContains/FullContains on naïve text:  ca. ` **`2700ms`**

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Real Performance

```
/* In order : IsContains, Timing of IsContains, GlobalCount, Timing of
GlobalCount, CountContains, time of CountContains,
time of Full Report Contains */
-----------------
Sampling rate 5
"Bakst": 1, 0.038, 1, 0.005, 1, 0.049, 0.013, max_mem = 100
"ruminants": 1, 0.038, 22, 0.01, 19, 0.156, 0.086, max_mem = 100
"morphine": 1, 0.027, 392, 0.009, 144, 1.718, 1.357, max_mem = 100
"AUSTRALIA": 1, 0.098, 438, 0.009, 438, 4.145, 3.942, max_mem = 100
"molecule": 1, 0.029, 1472, 0.009, 966, 6.247, 5.853, max_mem = 101
"brain": 1, 0.019, 2685, 0.006, 1493, 12.24, 11.588, max_mem = 101
"human": 1, 0.018, 6897, 0.005, 4690, 25.403, 27.344, max_mem = 101
"blood": 1, 0.026, 10402, 0.005, 8534, 77.175, 73.613, max_mem = 101
"from": 1, 0.016, 20859, 0.003, 12073, 84.012, 78.663, max_mem = 101
"with": 1, 0.015, 63332, 0.004, 22974, 242.834, 235.043, max_mem = 102
" in": 1, 0.012, 238638, 0.002, 42586, 1105.6, 1091.43, max_mem = 103
"b": 1, 0, 411409, 0.001, 135307, 1779.27, 1762.62, max_mem = 108
"g": 1, 0.001, 748326, 0, 320440, 3411.65, 3378.85, max_mem = 119
""a": 1, 0, 2932251, 0, 595716, 13183.4, 13173.4, max_mem = 133
"\n": 1, 0.001, 9730750, 0.001, 5870474, 87770.9, 88230.4, max_mem = 126
```

use naïve

`CountContains/FullContains on naïve text:  ca. ` **`2700ms`**

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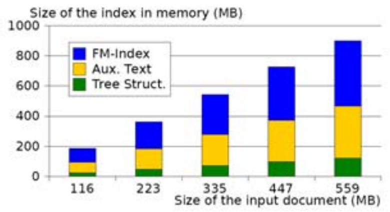
Construction Time

XMark data   174 different element labels
Max Depth: 14, Average Depth: 9.6

116MB XMark 6,074,297 nodes	Text: 7min 18s **TOTAL= 9min 20s**
559MB XMark 29,239,763 nodes	Text: 38min 45s **TOTAL= 53min 25s**
1GB XMark 58,472,941 nodes	Text: 1h 24min **TOTAL= 1h 55min**



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☺ Advertisement ☺

New course, will be first offered in Session 1 of 2011.

COMP9319 -- Web Data Compression and Search (PG, UOC: 6)

Contents

Data Compression : (a) Adaptive Coding, Information Theory
(b) Text Compression (ZIP, GZIP, BZIP, etc)
(c) Burrows-Wheeler Transform and Backward Search
(d) XML Compression

Search: (a) Indexing
(b) Pattern Matching and Regular Expression Search
(c) Distributed Querying
(d) Fast Index Construction
(e) Implementation
If time allows: Streaming Algorithms, On-Line Data Analytics

The lecture materials will be complemented by projects and assignments.

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END
Lecture 13 and of the course.

→ Thanks for your attention and hard work.
→ Hopefully you have enjoyed the lecture.
→ Good luck and all the best with
the exam on June 12th.