XML and Databases

Lecture 9
Properties of XPath

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CSE@UNSW -- Semester 1, 2010

Outline

1. XPath Equivalence
2. No Looking Back: How to Remove Backward Axes
3. Containment Test for XPath Expressions

A Note on Equality Test in XPath

Useful Functions (on Node Sets)

Careful with equality (“=“)

XPath 2.0 has clearer comparison operators!

```
<a>
 <b>
  <d>red</d>
  <d>green</d>
  <d>blue</d>
 </b>
 <c>
  <d>yellow</d>
  <d>orange</d>
  <d>green</d>
 </c>
</a>
```

XPath 1.0
Equality (“=“) is based on *string value* of a node!

`//a[b/d = c/d]` selects a-node!

there is a node in the node set for `b/d`
with same string value as a node in node set `c/d`

A Note on Equality Test

p1, p2 XPath (1.0) Expressions

(p1 == p2) is true if there exists *a node* selected by p1 that is *identical* to a node selected by p2

XPath 2.0
XQuery 1.0

```
<a>
 <b>
  <d>red</d>
  <d>green</d>
  <d>blue</d>
 </b>
 <c>
  <d>yellow</d>
  <d>orange</d>
  <d>green</d>
 </c>
</a>
```

`//a[b/d == c/d]` selects what?

A Note on Equality Test

p1, p2 XPath (1.0) Expressions

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XPath 2.0
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<a>
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 </b>
 <c>
  <d>yellow</d>
  <d>orange</d>
  <d>green</d>
 </c>
</a>
```

false (on *any* document)

`//a[b/d == c/d]` selects what?

```
//*[child::node()[1]
 == child::node()[position()=last()]]
```

A Note on Equality Test

Recall

child::* all child nodes that are elements
child::comment() all child nodes that are comments
child::processing-instruction() all child nodes that are proc. instr.'s
child::node() all child nodes that are element/comments/PI's

(only way to get to an *attribute*, is via the *attribute-axis*)

Question
Which axes can bring you from an attribute-node back to an element-node?

```
<a>
 <b>
  <d>red</d>
  <d>green</d>
  <d>blue</d>
 </b>
 <c>
  <d>yellow</d>
  <d>orange</d>
  <d>green</d>
 </c>
</a>
```

$\overbrace{\texttt{//a[b/d == c/d]}}^{\textbf{false}\text{ (on } \textit{any} \text{ document)}}$ selects what?

```
//*[child::node()[1]
 == child::node()[position()=last()]]
```

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A Note on Equality Test

p1, p2 XPath (1.0) Expressions

(p1 == p2) is true if there exists *a node* selected by p1
that is *identical* to a node selected by p2

XPath 2.0
XQuery 1.0

XPath 1.0 simulation of (node) equality test (==)

Instead of (p1 == p2) write:

(count(p1 | p2) < count(p1) + count(p2)) ☺

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A Note on Equality Test

Question

Can you give an XPath 1.0 filter expression
for checking whether the
node set of p1 is equal to the *node set* of p2?

XPath 1.0 simulation of (node) equality test (==)

Instead of (p1 == p2) write:

(count(p1 | p2) < count(p1) + count(p2)) ☺

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1. XPath Equivalence

p1, p2 XPath (1.0) Expressions

(p1 ≡ p2) p1 "*is equivalent to*" p2
is true if,
*for any document **D**, and any context node **N** of **D***,

p1 evaluated on ***D*** with context ***N*** gives the same result as
p2 evaluated on ***D*** with context ***N***.

Examples

```
/a//*/b          ≡  /a/*//b
//a/b/c/../..    ≡  //a[.b/c/]
//a[b | c]       ≡  //a[b] | //a[c]
//*[/a = /b]     ≡  /..
```

NOT equivalent: `child::*/parent::*` $\not\equiv$ `self::*`
→ show a counter example!

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1. XPath Equivalence

EBNF for XPaths that we want to consider now:

$$
\begin{array}{rl}
\textit{path} ::= & \textit{path}\ \texttt{|}\ \textit{path} \mid \texttt{/}\ \textit{path} \mid \textit{path}\ \texttt{/}\ \textit{path} \mid \textit{path}\ \texttt{[}\ \textit{qualif}\ \texttt{]} \mid \textit{axis}\ \texttt{::}\ \textit{nodetest} \mid \perp . \\
\textit{qualif} ::= & \textit{qualif}\ \texttt{and}\ \textit{qualif} \mid \textit{qualif}\ \texttt{or}\ \textit{qualif} \mid \texttt{(}\ \textit{qualif}\ \texttt{)} \mid \\
 & \textit{path}\ \texttt{=}\ \textit{path} \mid \textit{path}\ \texttt{==}\ \textit{path} \mid \textit{path} . \\
\textit{axis} ::= & \textit{reverse\_axis} \mid \textit{forward\_axis} . \\
\textit{reverse\_axis} ::= & \texttt{parent} \mid \texttt{ancestor} \mid \texttt{ancestor-or-self} \mid \\
 & \texttt{preceding} \mid \texttt{preceding-sibling} . \\
\textit{forward\_axis} ::= & \texttt{self} \mid \texttt{child} \mid \texttt{descendant} \mid \texttt{descendant-or-self} \mid \\
 & \texttt{following} \mid \texttt{following-sibling} . \\
\textit{nodetest} ::= & \textit{tagname} \mid \texttt{*} \mid \texttt{text()} \mid \texttt{node()} .
\end{array}
$$

An XPath starting with "/" (root node) is called *absolute*,
otherwise it is called *relative* (will be evaluated *relative* to a given context node).

(Note: This is Core XPath wo negation, but with = and == operators)

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1. XPath Equivalence

p1, p2 XPaths
p arbitrary XPath
q arbitrary qualifier

Rel→Abs If p1 ≡ p2, then /p1 ≡ /p2.

Adjunct If p1 ≡ p2 and p is a relative, then p1/p ≡ p2/p.
If p1 ≡ p2 and p1, p2 relative, then p/p1 ≡ p/p2.
If p1 ≡ p2, then p1[q] ≡ p2[q] and p[p1] ≡ p[p2].

Qualifier Flattening p[p1/p2] ≡ p[p1[p2]]

ancestor-or-self::n ≡ ancestor::n | self::n
descendant-or-self::n ≡ descendant::n | self::n

p[p1 = /p2] ≡ p[p1[self::node() = /p2]]
p[p1 == /p2] ≡ p[p1[self::node() == /p2]]

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1. XPath Equivalence

“no backward at root node”

Lemma 3.2. *Let m and n be node tests, i.e. m and n are tag names or one of the xPath constructs* `*`, `node()`, *or* `text()`.

- *Let a be one of the axes* `parent`, `ancestor`, `preceding`, `preceding-sibling`, `self`, `following`, *or* `following-sibling`. *Then the following holds:*

$$/a::n \equiv \begin{cases} / & \text{if } a = \texttt{self} \text{ and } n = \texttt{node()} \\ \bot & \text{otherwise} \end{cases}$$

- *Let a be the* `preceding` *or* `ancestor` *axis. Then the following equivalences hold:*

$$/\texttt{child}::m/a::n \equiv \begin{cases} /\texttt{self::node()[child::}m\texttt{]} & \text{if } a = \texttt{ancestor} \text{ and } n = \texttt{node()} \\ \bot & \text{otherwise} \end{cases}$$

$$/\texttt{child}::m[a::n] \equiv \begin{cases} /\texttt{child::}m & \text{if } a = \texttt{ancestor} \text{ and } n = \texttt{node()} \\ \bot & \text{otherwise} \end{cases}$$

(same holds for a = parent)

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2. No Looking Back

Dual	**backward**	**forward**

	`parent`
	`ancestor`
	`ancestor-or-self`
	`preceding`
	`preceding-sibling`
		`child`
		`descendant`
		`descendant-or-self`
		`following`
		`following-sibling`

Thus: dual(parent) = child
dual(following) = preceding
etc.

Rewrite rule #1 (p,s: relative paths, ax: reverse axis)

```
p[ax::m/s]   ➔
      p[/descendant::m[s]/dual(ax)::node() == self::node()]
```

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Rewrite rule #1 (p,s: relative paths, ax: reverse axis)

```
p[ax::m/s]   ➔
      p[/descendant::m[s]/dual(ax)::node() == self::node()]
```

any “m[s]-node” in the tree

but, via dual axis, must reach context node

E.g. ax = ancestor

```
p[ancestor::m]   ➔
    p[/descendant::m/descendant::node()==self::node()]
```

“any m-node from which the context node can be reached via descendant, must be an ancestor of the context node.”

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Rewrite rule #1 (p,s: relative paths, ax: reverse axis)

```
p[ax::m/s]   ➔
      p[/descendant::m[s]/dual(ax)::node() == self::node()]
```

any “m[s]-node” in the tree

but, via dual axis, must reach context node

E.g. ax = preceding-sibling

```
p[preceding-sibling::m]   ➔
    p[/descendant::m/following-sibling::node()==self::node()]
```

“any m-node from which the context node can be reached via following-sibling, must be a preceding-sibling of the context node.”

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Rewrite rule #1 (p,s: relative paths, ax: reverse axis)

```
p[ax::m/s]   ➔
      p[/descendant::m[s]/dual(ax)::node() == self::node()]
```

any “m[s]-node” in the tree

but, via dual axis, must reach context node

E.g. ax=preceding-sibling

```
p[preceding-sibling::m]   ➔
    p[/descendant::m/following-sibling::node()==self::node()]
```

“any m-node from which the context node can be reached via following-sibling, must be a preceding-sibling of the context node.”

Similar for parent and preceding. (ancestor-or-self not really needed. **Why?**)

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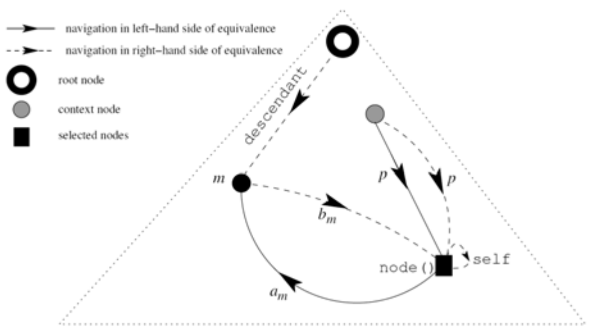
Rewrite rule #1 (p,s: relative paths, ax: reverse axis)

```
p[ax::m/s]   ➔
      p[/descendant::m[s]/dual(ax)::node() == self::node()]
```



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Rewrite rule #1 (p,s: relative paths, ax: reverse axis)

```
p[ax::m/s]    →
        p[/descendant::m[s]/dual(ax)::node() == self::node()]
```

Removes first reverse axis inside a filter (qualifier).

Use *qualifier flattening* to replace *any* reverse axis from inside a filter.

Qualifier Flattening   p[p1/p2] ≡ p[p1[p2]]

Similar rules for **absolute paths**:

```
/p/fAx::n/ax::m  → /descendant::m[dual(ax)::n == /p/fAx::n]
/fAx::n/ax::m    → /descendant::m[dual(ax)::n == /fAx::n]
```

Rewrite rules #2 and #2a

E.g.

```
/descendant::price/preceding::name
```

is rewritten via Rule #2a into:

```
/descendant::name[following::price==/descendant::price]
```

Similar rules for **absolute paths**:

```
/p/fAx::n/ax::m  → /descendant::m[dual(ax)::n == /p/fAx::n]
/fAx::n/ax::m    → /descendant::m[dual(ax)::n == /fAx::n]
```

Rewrite rules #2 and #2a

E.g.

```
/descendant::price/preceding::name
```

is rewritten via Rule #2a into:

a tautology

```
/descendant::name[following::price==/descendant::price]
```

Not needed, in this example.

Of course, the "join" can be removed in this example:

```
/descendant::name[following::price]
```

Similar rules for **absolute paths**:

```
/p/fAx::n/ax::m  → /descendant::m[dual(ax)::n == /p/fAx::n]
/fAx::n/ax::m    → /descendant::m[dual(ax)::n == /fAx::n]
```

Rewrite rules #2 and #2a

E.g.

```
/descendant::journal[child::title]/descendant::price/preceding::name
```

becomes

```
/descendant::name[following::price==
                  /descendant::journal[child::title]/descendant::price]
```

Can you avoide the join, also for this example??

Similar rules for **absolute paths**:

```
/p/fAx::n/ax::m  → /descendant::m[dual(ax)::n == /p/fAx::n]
/fAx::n/ax::m    → /descendant::m[dual(ax)::n == /fAx::n]
```

Rewrite rules #2 and #2a

path ::= *path* | *path* | / *path* | *path* / *path* | *path* [*qualif*] | *axis* :: *nodetest* | ⊥ .
qualif ::= *qualif* and *qualif* | *qualif* or *qualif* | (*qualif*) |
path = *path* | *path* == *path* | *path* .
axis ::= *reverse_axis* | *forward_axis* .
reverse_axis ::= parent | ancestor | ancestor-or-self |
preceding | preceding-sibling .
forward_axis ::= self | child | descendant | descendant-or-self |
following | following-sibling .
nodetest ::= *tagname* | * | text() | node() .

```
(1)   p[ax::m/s]    →
          p[/descendant::m[s]/dual(ax)::node() == self::node()]
(2)   /p/fAx::n/ax::m  → /descendant::m[dual(ax)::n == /p/fAx::n]
(2a)  /fAx::n/ax::m    → /descendant::m[dual(ax)::n == /fAx::n]
```

Rules (1),(2),(2a) suffice to remove ALL backward axes from above queries!
Why?
→ Size Increase?
→ How many joins?

2. No Looking Back

Dual	**backward**	**forward**	
	parent	child	
	ancestor	descendant	
	~~ancestor-or-self~~	~~descendant-or-self~~	not needed
	preceding	following	
	preceding-sibling	following-sibling	

Joins (==) are expensive! (typically quadratic wrt data)

To obtain queries with fewer joins
consider the **forward-axis** left of the reverse-axis to be removed!

New rules will be of the form

```
p/forw/back   →  p_new
p/forw[back]  →  p_new
```

2. No Looking Back

Interaction of `back=parent` with forward axes:

$$\texttt{descendant::}n\texttt{/parent::}m \equiv \texttt{descendant-or-self::}m\texttt{[child::}n\texttt{]} \tag{3}$$

2. No Looking Back

Interaction of `back=parent` with forward axes:

$$\texttt{descendant::}n\texttt{/parent::}m \equiv \texttt{descendant-or-self::}m\texttt{[child::}n\texttt{]} \tag{3}$$
$$\texttt{child::}n\texttt{/parent::}m \equiv \texttt{self::}m\texttt{[child::}n\texttt{]} \tag{4}$$

2. No Looking Back

Interaction of `back=parent` with forward axes:

$$\texttt{descendant::}n\texttt{/parent::}m \equiv \texttt{descendant-or-self::}m\texttt{[child::}n\texttt{]} \tag{3}$$
$$\texttt{child::}n\texttt{/parent::}m \equiv \texttt{self::}m\texttt{[child::}n\texttt{]} \tag{4}$$
$$p\texttt{/self::}n\texttt{/parent::}m \equiv p\texttt{[self::}n\texttt{]/parent::}m \tag{5}$$

2. No Looking Back

Interaction of `back=parent` with forward axes:

$$\texttt{descendant::}n\texttt{/parent::}m \equiv \texttt{descendant-or-self::}m\texttt{[child::}n\texttt{]} \tag{3}$$
$$\texttt{child::}n\texttt{/parent::}m \equiv \texttt{self::}m\texttt{[child::}n\texttt{]} \tag{4}$$
$$p\texttt{/self::}n\texttt{/parent::}m \equiv p\texttt{[self::}n\texttt{]/parent::}m \tag{5}$$
$$p\texttt{/following-sibling::}n\texttt{/parent::}m \equiv p\texttt{[following-sibling::}n\texttt{]/parent::}m \tag{6}$$

2. No Looking Back

Interaction of `back=parent` with forward axes:

$$\texttt{descendant::}n\texttt{/parent::}m \equiv \texttt{descendant-or-self::}m\texttt{[child::}n\texttt{]} \tag{3}$$
$$\texttt{child::}n\texttt{/parent::}m \equiv \texttt{self::}m\texttt{[child::}n\texttt{]} \tag{4}$$
$$p\texttt{/self::}n\texttt{/parent::}m \equiv p\texttt{[self::}n\texttt{]/parent::}m \tag{5}$$
$$p\texttt{/following-sibling::}n\texttt{/parent::}m \equiv p\texttt{[following-sibling::}n\texttt{]/parent::}m \tag{6}$$
$$\begin{aligned} p\texttt{/following::}n\texttt{/parent::}m &\equiv p\texttt{/following::}m\texttt{[child::}n\texttt{]} \\ &\mid p\texttt{/ancestor-or-self::*[following-sibling::}n\texttt{]} \\ &\quad \texttt{/parent::}m \end{aligned} \tag{7}$$

2. No Looking Back

Interaction of `back=parent` with forward axes:

$$\texttt{descendant::}n\texttt{/parent::}m \equiv \texttt{descendant-or-self::}m\texttt{[child::}n\texttt{]} \tag{3}$$
$$\texttt{child::}n\texttt{/parent::}m \equiv \texttt{self::}m\texttt{[child::}n\texttt{]} \tag{4}$$
$$p\texttt{/self::}n\texttt{/parent::}m \equiv p\texttt{[self::}n\texttt{]/parent::}m \tag{5}$$
$$p\texttt{/following-sibling::}n\texttt{/parent::}m \equiv p\texttt{[following-sibling::}n\texttt{]/parent::}m \tag{6}$$
$$\begin{aligned} p\texttt{/following::}n\texttt{/parent::}m &\equiv p\texttt{/following::}m\texttt{[child::}n\texttt{]} \\ &\mid p\texttt{/ancestor-or-self::*[following-sibling::}n\texttt{]} \\ &\quad \texttt{/parent::}m \end{aligned} \tag{7}$$

$$\texttt{descendant::}n\,\texttt{[parent::}m\texttt{]} \equiv \texttt{descendant-or-self::}m\texttt{/child::}n \tag{8}$$
$$\texttt{child::}n\texttt{[parent::}m\texttt{]} \equiv \texttt{self::}m\texttt{/child::}n \tag{9}$$
$$p\texttt{/self::}n\texttt{[parent::}m\texttt{]} \equiv p\texttt{[parent::}m\texttt{]/self::}n \tag{10}$$
$$p\texttt{/following-sibling::}n\texttt{[parent::}m\texttt{]} \equiv p\texttt{[parent::}m\texttt{]/following-sibling::}n \tag{11}$$
$$\begin{aligned} p\texttt{/following::}n\texttt{[parent::}m\texttt{]} &\equiv p\texttt{/following::}m\texttt{/child::}n \\ &\mid p\texttt{/ancestor-or-self::*[parent::}m\texttt{]} \\ &\quad \texttt{/following-sibling::}n \end{aligned} \tag{12}$$

2. No Looking Back

Interaction of `back=ancestor` with forward axes:

$$p/\texttt{descendant::}n/\texttt{ancestor::}m \equiv p[\texttt{descendant::}n]/\texttt{ancestor::}m \mid p/\texttt{descendant-or-self::}m[\texttt{descendant::}n] \tag{13}$$

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2. No Looking Back

Interaction of `back=ancestor` with forward axes:

$$p/\texttt{descendant::}n/\texttt{ancestor::}m \equiv p[\texttt{descendant::}n]/\texttt{ancestor::}m \mid p/\texttt{descendant-or-self::}m[\texttt{descendant::}n] \tag{13}$$

$$/\texttt{descendant::}n/\texttt{ancestor::}m \equiv /\texttt{descendant-or-self::}m[\texttt{descendant::}n] \tag{13a}$$

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2. No Looking Back

Interaction of `back=ancestor` with forward axes:

$$p/\texttt{descendant::}n/\texttt{ancestor::}m \equiv p[\texttt{descendant::}n]/\texttt{ancestor::}m \mid p/\texttt{descendant-or-self::}m[\texttt{descendant::}n] \tag{13}$$

$$/\texttt{descendant::}n/\texttt{ancestor::}m \equiv /\texttt{descendant-or-self::}m[\texttt{descendant::}n] \tag{13a}$$

$$p/\texttt{child::}n/\texttt{ancestor::}m \equiv p[\texttt{child::}n]/\underline{\texttt{ancestor-or-self}}\texttt{::}m \tag{14}$$

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2. No Looking Back

Interaction of `back=ancestor` with forward axes:

$$p/\texttt{descendant::}n/\texttt{ancestor::}m \equiv p[\texttt{descendant::}n]/\texttt{ancestor::}m \mid p/\texttt{descendant-or-self::}m[\texttt{descendant::}n] \tag{13}$$

$$/\texttt{descendant::}n/\texttt{ancestor::}m \equiv /\texttt{descendant-or-self::}m[\texttt{descendant::}n] \tag{13a}$$

$$p/\texttt{child::}n/\texttt{ancestor::}m \equiv p[\texttt{child::}n]/\underline{\texttt{ancestor-or-self}}\texttt{::}m \tag{14}$$

$$p/\texttt{self::}n/\texttt{ancestor::}m \equiv p[\texttt{self::}n]/\underline{\texttt{ancestor}}\texttt{::}m \tag{15}$$

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2. No Looking Back

Interaction of `back=ancestor` with forward axes:

$$p/\texttt{descendant::}n/\texttt{ancestor::}m \equiv p[\texttt{descendant::}n]/\texttt{ancestor::}m \mid p/\texttt{descendant-or-self::}m[\texttt{descendant::}n] \tag{13}$$

$$/\texttt{descendant::}n/\texttt{ancestor::}m \equiv /\texttt{descendant-or-self::}m[\texttt{descendant::}n] \tag{13a}$$

$$p/\texttt{child::}n/\texttt{ancestor::}m \equiv p[\texttt{child::}n]/\underline{\texttt{ancestor-or-self}}\texttt{::}m \tag{14}$$

$$p/\texttt{self::}n/\texttt{ancestor::}m \equiv p[\texttt{self::}n]/\underline{\texttt{ancestor}}\texttt{::}m \tag{15}$$

$$p/\texttt{following-sibling::}n/\texttt{ancestor::}m \equiv p[\texttt{following-sibling::}n]/\underline{\texttt{ancestor}}\texttt{::}m \tag{16}$$

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2. No Looking Back

Interaction of `back=ancestor` with forward axes:

$$p/\texttt{descendant::}n/\texttt{ancestor::}m \equiv p[\texttt{descendant::}n]/\texttt{ancestor::}m \mid p/\texttt{descendant-or-self::}m[\texttt{descendant::}n] \tag{13}$$

$$/\texttt{descendant::}n/\texttt{ancestor::}m \equiv /\texttt{descendant-or-self::}m[\texttt{descendant::}n] \tag{13a}$$

$$p/\texttt{child::}n/\texttt{ancestor::}m \equiv p[\texttt{child::}n]/\underline{\texttt{ancestor-or-self}}\texttt{::}m \tag{14}$$

$$p/\texttt{self::}n/\texttt{ancestor::}m \equiv p[\texttt{self::}n]/\underline{\texttt{ancestor}}\texttt{::}m \tag{15}$$

$$p/\texttt{following-sibling::}n/\texttt{ancestor::}m \equiv p[\texttt{following-sibling::}n]/\underline{\texttt{ancestor}}\texttt{::}m \tag{16}$$

$$p/\texttt{following::}n/\texttt{ancestor::}m \equiv p/\texttt{following::}m[\texttt{descendant::}n] \mid p/\underline{\texttt{ancestor-or-self}}\texttt{::*}[\texttt{following-sibling::*/descendant-or-self::}n]/\texttt{ancestor::}m \tag{17}$$

Similar rules for `ancestor` in a filters.

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2. No Looking Back

Interaction of `back=ancestor` with forward axes:

```
p/descendant::n/ancestor::m ≡ p[descendant::n]/ancestor::m                          (13)
                            | p/descendant-or-self::m[descendant::n]
/descendant::n/ancestor::m ≡ /descendant-or-self::m[descendant::n]                  (13a)
p/child::n/ancestor::m ≡ p[child::n]/ancestor-or-self::m                            (14)
p/self::n/ancestor::m ≡ p[self::n]/ancestor::m                                      (15)
p/following-sibling::n/ancestor::m ≡ p[following-sibling::n]/ancestor::m            (16)
p/following::n/ancestor::m ≡ p/following::m[descendant::n]                          (17)
                            | p/ancestor-or-self::*
                              [following-sibling::*/descendant-or-self::n]
                              /ancestor::m
```

Similar rules for `ancestor` in a filters.

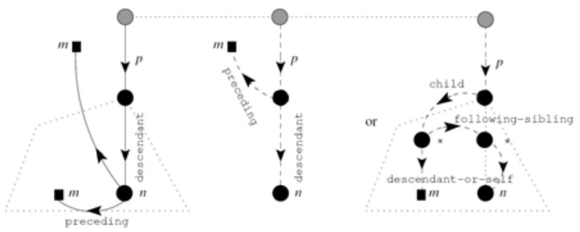
E.g., what is the forward query for: `//*[ancestor::a]`

2. No Looking Back

Interaction of `back=preceding` with forward axes:

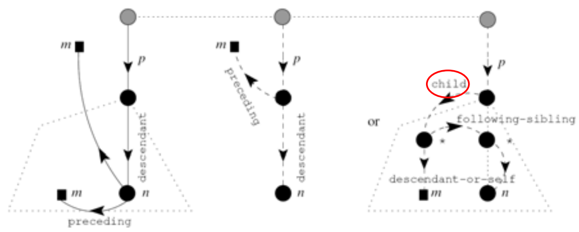
```
p/descendant::n/preceding::m ≡ p[descendant::n]/preceding::m                        (33)
                             | p/child::*
                               [following-sibling::*/descendant-or-self::n]
                               /descendant-or-self::m
/descendant::n/preceding::m ≡ /descendant::m[following::n]                          (33a)
p/child::n/preceding::m ≡ p[child::n]/preceding::m                                  (34)
                             | p/child::*[following-sibling::n]
                               /descendant-or-self::m
p/self::n/preceding::m ≡ p[self::n]/preceding::m                                    (35)
p/following-sibling::n/preceding::m ≡ p[following-sibling::n]/preceding::m          (36)
                             | p/following-sibling::*[following-sibling::n]
                               /descendant-or-self::m
                             | p[following-sibling::n]/descendant-or-self::m
p/following::n/preceding::m ≡ p[following::n]/preceding::m                          (37)
                             | p/following::m[following::n]
                             | p[following::n]/descendant-or-self::m
```


Rule 33



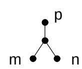

```
p/descendant::n/preceding::m ≡ p[descendant::n]/preceding::m
      | p/child::*[following-sibling::*/descendant-or-self::n]/descendant-or-self::m
```


Rule 33




```
p/descendant::n/preceding::m ≡ p[descendant::n]/preceding::m
      | p/child::*[following-sibling::*/descendant-or-self::n]/descendant-or-self::m
```

Wrong.
Should be `descendant` instead!



2. No Looking Back

`/descendant::price/preceding::name`

is rewritten via Rule #2a into:

`/descendant::name[following::price==/descendant::price]`

Now, let us use Rule (33a)

`/descendant::n/preceding::m` ➔ `/descendant::m[following::n]`

We obtain

`/descendant::name[following::price]`


```
p
/descendant::journal[child::title]/descendant::price/preceding::name
```

becomes

```
/descendant::name[following::price==
                  /descendant::journal[child::title]/descendant::price]
```

Rule (33a)

`/descendant::n/preceding::m` ➔ `/descendant::m[following::n]`

doesn't work because descendant is absolute here.

Rule (33):

```
p/descendant::n/preceding::m  ➔  p[descendant::n]/preceding::m
                 | p/child::*[following-sibling::*/descendant-or-self::n]
                   /descendant-or-self::m
```

We obtain

```
p[descendant::price]/preceding::name
 | p/child::*[following-sibling::*/descendant-or-self::price]
             /descendant-or-self::name
```

```
                 p
/descendant::journal[child::title]/descendant::price/preceding::name
```

becomes

```
/descendant::name[following::price==
                   /descendant::journal[child::title]/descendant::price]
```

Rule (33a)
`/descendant::n/preceding::m` ➔ `/descendant::m[following::n]`
doesn't work because descendant is absolute here.
Rule (33):
```
p/descendant::n/preceding::m  ➔  p[descendant::n]/preceding::m
               | p/child::*[following-sibling::*/descendant-or-self::n]
                 /descendant-or-self::m
```

→ Rule (33a) with n = `journal[child::title][descendant::price]`

```
p[descendant::price]/preceding::name
 | p/child::*[following-sibling::*/descendant-or-self::price]
             /descendant-or-self::name
```

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```
                 p
/descendant::journal[child::title]/descendant::price/preceding::name
```

becomes

```
/descendant::name[following::price==
                   /descendant::journal[child::title]/descendant::price]
```

Rule (33a)
`/descendant::n/preceding::m` ➔ `/descendant::m[following::n]`
doesn't work because descendant is absolute here.

```
/descendant::name[following::journal[child::title][descendant::price]]
          | p/child::*[following-sibling::*/descendant-or-self::price]
                      /descendant-or-self::name
```

→ Rule (33a) with n = `journal[child::title][descendant::price]`

```
p[descendant::price]/preceding::name
 | p/child::*[following-sibling::*/descendant-or-self::price]
             /descendant-or-self::name
```

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```
                 p
/descendant::journal[child::title]/descendant::price/preceding::name
```

becomes =n

```
/descendant::name[following::price==
                   /descendant::journal[child::title]/descendant::price]
```

Rule (33a)
`/descendant::n/preceding::m` ➔ `/descendant::m[following::n]`
~~doesn't work because descendant is absolute here.~~ seems it does work! ☺

```
/descendant::name[following::journal[child::title][descendant::price]]
          | p/child::*[following-sibling::*/descendant-or-self::price]
                      /descendant-or-self::name
```

p[p1/p2]
≡ p[p1[p2]]

What about this one:

`/descendant::name[following::journal[child::title]/descendant::price]`

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Theorem
(from D. Olteanu, H. Meuss, T. Furche, F. Bry
XPath: Looking Forward. EDBT Workshops 2002: 109-127)

Given an XPath expression p that has no joins of the form (p1 == p2) with both p1,p2 relative, an equivalent expression u without reverse axes can be computed.

Time needed: at most **exponential** in length of p
Length of u: at most **exponential** in length of p

(moreover: *no joins* are introduced when computing u)

Questions

→ Why rewriting takes exponential time?
→ Can you find a subclass for which *Time* to compute u is linear or polynomial?
→ What is the problem with joins (p1 == p2) for removal of reverse axes?

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Theorem
Given an XPath expression p that has no joins of the form (p1 == p2) with both p1,p2 relative, an equivalent expression u without reverse axes can be computed.

Time needed: at most **exponential** in length of p
(moreover: *no joins* are introduced when computing u)

More Questions

→ Give an example of Core backward XPath with **negation**, for which there is no forward XPath query.

→ Give an example of Core backward XPath with **data values**, for which there is no forward XPath query.

→ Give an example of a Core backward XPath with **counting**, for which there is no forward XPath query.

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3. XPath Containment Test

Given two XPath expressions p, q:
Are all nodes selected by p, also selected by q? (on *any* document)
(p "contained in" q)

Has many applications! Boolean query

Want to select documents that "match p".
→ If a document matches p, and p contained in q,
then we know the document also matches q!

→ If a document does not match q, and p contained in q,
then we know the document does not match p!

Applications

➔ Decrease online-time of publish/subscribe systems based on XPath
➔ Decrease query-time by making use of materialized intermediate results
➔ Optimization by ruling out queries with empty result set
etc, etc

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3. XPath Containment Test

Given two XPath expressions p, q

"0-containment" $p \subseteq_0 q$ — For every tree, if p selects a node then so does q.

"1-containment" $p \subseteq_1 q$ — For every tree, all nodes selected by p are also selected by q.

"2-containment" $p \subseteq_2 q$ — For every tree, and every context node N, all nodes selected by p starting from N, are also selected by q starting from N.

1. Inclusion on *Booleans*
2. Inclusion on *Node Sets*

$\}$ start from root

3. Inclusion on *Node Relations*

(If only child and descendant axes are allowed then $\subseteq_1$ and $\subseteq_2$ are the same! -- **Why?**)

3. XPath Containment Test

Given two XPath expressions p, q

"0-containment" $p \subseteq_0 q$ — For every tree, if p selects a node then so does q.

"1-containment" $p \subseteq_1 q$ — For every tree, all nodes selected by p are also selected by q.

Question

Given p, q and the fact $p \subseteq_1 q$,
how can you determine from a *result set of nodes* for q,
the correct *result set of nodes* for p?

3. XPath Containment Test

Given two XPath expressions p, q

Sometimes we want to test containment wrt a given DTD:

```
p = /a/b//d
q = /a//c
```

Boolean!

Want to check if $p \subseteq_0 q$.

NO!

```
a
|
b
|
d
```

But, what if documents are valid wrt to this DTD?

```
root → a*
a    → b* | c*
b    → d+c+
c    → b?c?
```


<table>
<tr><td>PTIME</td><td>XP(/,//,∗) [21]
XP(/,[],∗) (see [19])
XP(/,//,[]) [2], with fixed bounded SXICs [9]
XP(/,//) + DTDs [22]
XP[/,[]] + DTDs [22]</td></tr>
<tr><td>coNP</td><td>XP(/,//,[],∗) [19]
XP(/,//,[],∗,|), XP(/,|), XP(//,|) [22]
XP(/,[]) + DTDs [22]
XP(//,[]) + DTDs [22]</td></tr>
<tr><td>Π_2^p</td><td>XP(/,//,[],|) + existential variables + path equality + ancestor-or-self axis + fixed bounded SXICs [9]
XP(/,//,[],∗,|) + existential variables + all backward axes + fixed bounded SXICs [9]
XP(/,//,[],|) + existential variables with inequality [22]</td></tr>
<tr><td>PSPACE</td><td>XP(/,//,[],∗,|) and XP(/,//,|) if the alphabet is finite [22]
XP(/,//,[],∗,|) + variables with XPath semantics [22]</td></tr>
<tr><td>EXPTIME</td><td>XP(/,//,[],|) + existential variables + bounded SXICs [9]
XP(/,//,[],∗,|) + DTDs [22]
XP(/,//,|) + DTDs [22]
XP(/,//,[],∗) + DTDs [22]</td></tr>
<tr><td>Undecidable</td><td>XP(/,//,[],|) + existential variables + unbounded SXICs [9]
XP(/,//,[],|) + existential variables + bounded SXICs + DTDs [9]
XP(/,//,[],∗,|) + nodeset equality + simple DTDs [22]
XP(/,//,[],∗,|) + existential variables with inequality[22]</td></tr>
</table>

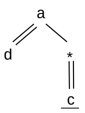
2. XPath Containment Test

from:

T. Schwentick
XPath query containment.
SIGMOD Record 33(1): 101-109 (2004)

Pattern trees

E.g. p = `a[.//d]/*//c`



Note: child order has no meaning in pattern trees!

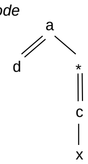
selection node (unique)

Test $\subseteq_1$ (node set inclusion) using $\subseteq_0$ (Boolean inclusion)

→ Simply add a new node below the *selection node*

New tree is Boolean (no selection node)

In a given XML tree:
pattern matches / does not match.



3. XPath Containment Test

4 techniques of testing XPath (Boolean) containment:

(1) The Canonical Model Technique

(2) The Homomorphism Technique

(3) The Automaton Technique

(4) The Chase Technique

3. XPath Containment Test

Canonical Model - XPath(/,//,[],*)

Idea: if there exists a tree that matches p but not q, then such a tree exists of **size polynomial in the size** of p an q.

Simple: remember, if you know that the XML document is only of height 5, then **//**a/b/*/c could be enumerated by /a/b/*/c **|** /*/a/b/*/c **|** /*/*/a/b/*/c **|** /*/*/*/a…

Similarly, we try to construct a counter example tree, by replacing in p

→ every * by some new symbol "z"
→ every // by z/, z/z/, z/z/z/, … z/z/../z/

N+1 many z's

N = length of longest */../* chain in q

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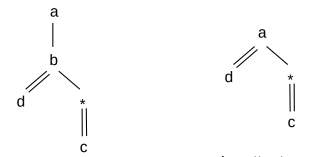
3. XPath Containment Test

Canonical Model - XPath(/,//,[],*)

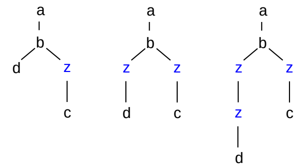
Example



p's patter tree

q's patter tree

Test for q-match:



Formally, must test 1 and 2 more z's at *right branch* of each of the trees.

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3. XPath Containment Test

Homomorphism h maps each node of q's query tree Q to a node of p's query tree P such that

(1) root of Q is mapped to root of P
(2) if (u,v) is child-edge of Q then (h(u),h(v)) is child-edge of P
(3) if (u,v) is descendant-edge of Q, then h(v) is a "below" h(u) in P
(4) if u is labeled by "e" (not *), then h(u) is also labeled by "e".

p,q expressions in XPath(/,//,[])

Theorem
p $\subseteq_0$ q *if and only if* there is a homomorphism from Q to P.

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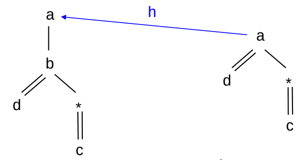
Homomorphism h maps each node of q's query tree Q to a node of p's query tree P such that



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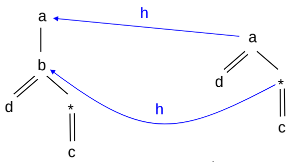

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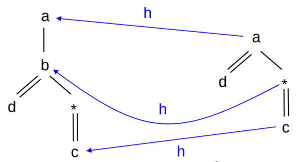

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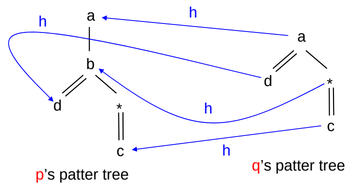

p's patter tree

q's patter tree

→ hom. h exists from Q to P, thus $p \subseteq_0 q$ must hold!

(1) root of Q is mapped to root of P
(2) if (u,v) is child-edge of Q then (h(u),h(v)) is child-edge of P
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"1-containment" For every tree, all nodes selected by p are also selected by q.
$p \subseteq_1 q$

Question

Given p, q and the fact $p \subseteq_1 q$,
how can you determine from a *result set of nodes* for q,
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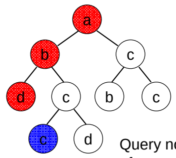
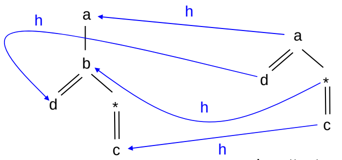


Query nodes
of a p result

→With homomorphism technique:

Use a result node of q together with
run-time info on pattern nodes.
Enables to search "inside", only on paths
between pattern nodes.

3. XPath Containment Test

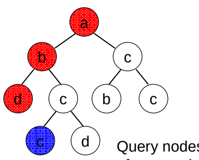


p's patter tree

q's patter tree

1. x := parent of q's c-node
2. Check if q's *-node is
   (a) ancestor of x
   (b) labeled b
   (c) has a-parent
   (d) is ancestor of q's d-node

Cave: we will have to try all homomorphisms …



Query nodes
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h(v) is a "below" h(u) in P
(4) if u is labeled by "e" (not *), then h(u) is also labeled by "e".

p,q expressions in XPath(`/`,`//`,`[]`)

Theorem
$p \subseteq_0 q$ *if and only if* there is a homomorphism from Q to P.

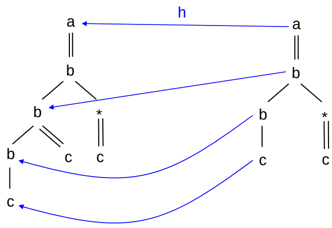
Cave If we add the star (*) then homomorphism need not exist!

→ there are p,q ∈ XPath(`/`,`//`,`[]`,`*`) such that $p \subseteq_0 q$ and
there is **no** homomorphism from Q to P ☹

3. XPath Containment Test

[/a//b[./b/c]//c]/*/c]

[/a//b[./b/c]/*//c]

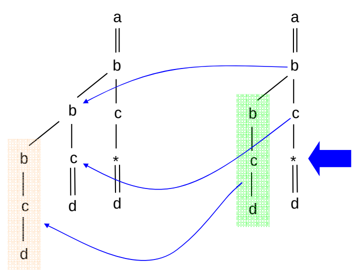


IS there a homomorphism??

Cave If we add the star (*) then homomorphism need not exist!

→ there are p,q ∈ XPath(`/`,`//`,`[]`,`*`) such that $p \subseteq_0 q$ and
there is **no** homomorphism from Q to P ☹


```
p = /a[.//b[c/*//d]/b[c//d]/b[c/d]]
q = /a[.//b[c/*//d]/b[c/d]]
```



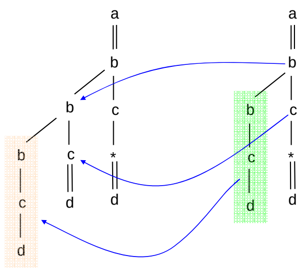
Where to map??

Cave If we add the star (*) then homomorphism need not exist!

→ there are p,q ∈ XPath(`/`,`//`,`[]`,`*`) such that $p \subseteq_0 q$ and
there is **no** homomorphism from Q to P ☹

p = `/a[.//b[c/*//d]/b[c//d]/b[c/d]]`
q = `/a[.//b[c/*//d]/b[c/d]]`

Is p contained in q??
→ Test this, using the canonical model!!



Where to map??

Cave If we add the star (*) then homomorphism need not exist!

→ there are p,q ∈ XPath(`/`,`//`,`[]`,`*`) such that p $\subseteq_0$ q and there is **no** homomorphism from Q to P ☹

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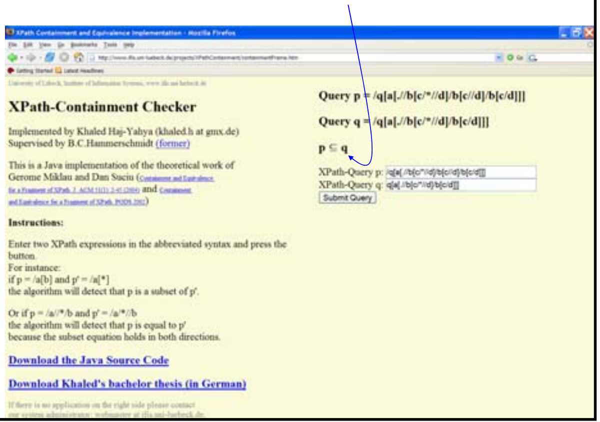
Let's check the web… → **YES** p contained in q!



p = `a/*//a`
q = `a//*/a`

Clearly, p is equivalent to q. (containment holds in both directions)

But, no homomorphisms exist.

Cave If we add the star (*) then homomorphism need not exist!

→ there are p,q ∈ XPath(`/`,`//`,`[]`,`*`) such that p $\subseteq_0$ q and there is **no** homomorphism from Q to P ☹

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3. XPath Containment Test

Automaton Technique

Recall: for any DTD there is a tree automaton which recognized the corresponding trees.

Similarly, for any XPath(`/`,`//`,`[]`,`*`,`|`) expression ex we can construct a (*non-deterministic* bottom-up) tree automaton A which accepts a tree if and only if ex matches the tree.

Theorem
Containment test of XPath(`/`,`//`,`[]`,`*`,`|`) in the presence of DTDs can be solved in EXPTIME.

Exponential (deterministic) time
Blow-up due to non-determinism of tree automaton.

BUT: no hope for improvement:
The problem is actually ***complete*** for EXPTIME.

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3. XPath Containment Test

Automaton Technique

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Similarly, for any XPath(`/`,`//`,`[]`,`*`,`|`) expression ex we can construct a (*non-deterministic* bottom-up) tree automaton A which accepts a tree if and only if ex matches the tree.

Theorem
Containment test of XPath(`/`,`//`,`[]`,`*`,`|`) in the presence of DTDs can be solved in EXPTIME.

Union of automata *Intersection* of automata ("product construction")

Proof Idea construct automaton for all possible counter example trees. Test if this automaton accepts any tree.

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3. XPath Containment Test

Automaton Technique

Recall: for any DTD there is a tree automaton which recognized the corresponding trees.

Similarly, for any XPath(`/`,`//`,`[]`,`*`,`|`) expression ex we can construct a (*non-deterministic* bottom-up) tree automaton A which accepts a tree if and only if ex matches the tree.

Theorem
Containment test of XPath(`/`,`//`,`[]`,`*`,`|`) in the presence of DTDs can be solved in EXPTIME.

→Automata can also be Tested for Finiteness!

Is p $\subseteq_0$ q, for all trees but finitely many exceptions?

Emptiness test for automata

solvable!

Proof Idea construct automaton for all possible counter example trees. Test if this automaton accepts any tree.

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3. XPath Containment Test

Chase Technique -- 1979 relational DB's to check query containment in the presence of *integrity constraints*.

("the chase" extends the relational homomorphsim technique)

Example

DTD E =

```
root → a*
a    → b* | c*
b    → d+c+
c    → b?c?
```

p = /a/b//d
q = /a//c

Is p contained in q for E-conform documents?

First Possibility: use tree automata

→ Construct automata Ap, Aq, AE
→ Construct Bq for the complement of Aq (=not q)
→ Intersect Bq with Ap with AE (gives automaton A)
→ Check if A accepts any tree.

3. XPath Containment Test

Chase Technique -- 1979 relational DB's to check query containment in the presence of *integrity constraints*.

("the chase" extends the relational homomorphsim technique)

Example

DTD E =

```
root → a*
a    → b* | c*
b    → d+c+
c    → b?c?
```

p = /a/b//d
q = /a//c

Is p contained in q for E-conform documents?

Each b-element has a d-child and a c-child
→ *constraints*

c1: b→d
c2: b→c

p's pattern tree

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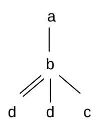

p's pattern tree
after *chasing* with c1,c2

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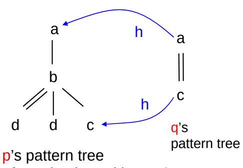

p is contained in q in the presence of the DTD E ←

p's pattern tree
after *chasing* with c1,c2

END
Lecture 8