

XML and Databases

Lecture 9

Properties of XPath

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CSE@UNSW -- Semester 1, 2010

Outline

1. XPath Equivalence
2. No Looking Back: How to Remove Backward Axes
3. Containment Test for XPath Expressions

A Note on Equality Test in XPath

Useful Functions (on Node Sets)

Careful with **equality** (“=“)

XPath 2.0 has clearer comparison operators!

```
<a>
  <b>
    <d>red</d>
    <d>green</d>
    <d>blue</d>
  </b>
  <c>
    <d>yellow</d>
    <d>orange</d>
    <d>green</d>
  </c>
</a>
```

XPath 1.0

Equality (“=“) is based on *string value* of a node!

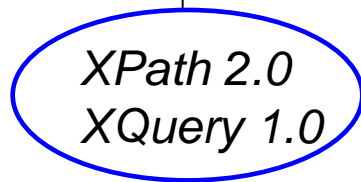
//a[b/d = c/d] selects a-node!

there is a node in the node set for **b/d**
with same string value as a node in node set **c/d**

A Note on Equality Test

p1, p2 XPath (1.0) Expressions

(p1 == p2) is true if there exists *a node* selected by **p1** that is *identical* to a node selected by **p2**



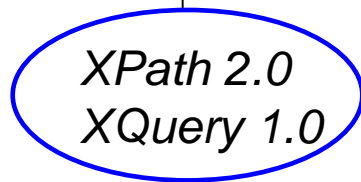
```
<a>
  <b>
    <d>red</d>
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  </b>
  <c>
    <d>yellow</d>
    <d>orange</d>
    <d>green</d>
  </c>
</a>
```

//a[b/d == c/d] selects what?

A Note on Equality Test

p1, p2 XPath (1.0) Expressions

(p1 == p2) is true if there exists *a node* selected by **p1** that is *identical* to a node selected by **p2**



```
<a>
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  </b>
  <c>
    <d>yellow</d>
    <d>orange</d>
    <d>green</d>
  </c>
</a>
```

false (on *any* document)

//a[b/d == c/d] selects what?

**//*[child::node()[1]
== child::node()[position()=last()]]**

A Note on Equality Test

Recall

child::***** all child nodes that are **elements**
child::**comment()** all child nodes that are **comments**
child::**processing-instruction()** all child nodes that are **proc. instr.'s**
child::**node()** all child nodes that are **element/comments/PI's**

(only way to get to an *attribute*, is via the *attribute-axis*)

Question

Which axes can bring you from an attribute-node back to an element-node?

```
<a>
  <b>
    <d>red</d>
    <d>green</d>
    <d>blue</d>
  </b>
  <c>
    <d>yellow</d>
    <d>orange</d>
    <d>green</d>
  </c>
</a>
```

false (on *any* document)

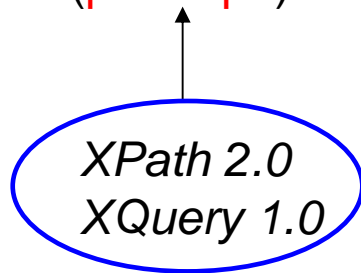
//a[b/d == c/d] selects what?

//*[child::node()][1]
== child::node()[position()=last()]

A Note on Equality Test

$p1, p2$ XPath (1.0) Expressions

$(p1 == p2)$ is true if there exists *a node* selected by $p1$ that is *identical* to a node selected by $p2$



XPath 1.0 simulation of (node) equality test ($==$)

Instead of $(p1 == p2)$ write:

$(\text{count}(p1 \mid p2) < \text{count}(p1) + \text{count}(p2))$ ☺

A Note on Equality Test

Question

Can you give an XPath 1.0 filter expression for checking whether the *node set* of **p1** is equal to the *node set* of **p2**?

XPath 1.0 simulation of (node) equality test (**==**)

Instead of (**p1 == p2**) write:

(count(**p1** | **p2**) < count(**p1**) + count(**p2**)) ☺

1. XPath Equivalence

p1, p2 XPath (1.0) Expressions

(p1 \equiv p2) **p1** “*is equivalent to*” **p2**
is true if,
for any document **D**, and any context node **N** of **D**,

p1 evaluated on **D** with context **N** gives the same result as
p2 evaluated on **D** with context **N**.

Examples

/a//*/b	\equiv	/a/*//b
//a/b/c/.. /..	\equiv	//a[. b/c/]
//a[b c]	\equiv	//a[b] //a[c]
//*[/a = /b]	\equiv	/..

NOT equivalent: $\text{child::}^*/\text{parent::}^* \not\equiv \text{self::}^*$
 \rightarrow show a counter example!

1. XPath Equivalence

EBNF for XPath paths that we want to consider now:

```
path ::= path | path | / path | path / path | path [ qualif ] | axis :: nodetest | ⊥ .
qualif ::= qualif and qualif | qualif or qualif | ( qualif ) |
          path = path | path == path | path .
axis ::= reverse_axis | forward_axis .
reverse_axis ::= parent | ancestor | ancestor-or-self |
               preceding | preceding-sibling .
forward_axis ::= self | child | descendant | descendant-or-self |
                following | following-sibling .
nodetest ::= tagname | * | text() | node() .
```

An XPath starting with “/” (root node) is called *absolute*, otherwise it is called *relative* (will be evaluated *relative* to a given context node).

(Note: This is **Core XPath** wo negation, but with = and == operators)

1. XPath Equivalence

$p1, p2$ XPaths

p arbitrary XPath

q arbitrary qualifier

Rel \rightarrow Abs If $p1 \equiv p2$, then $/p1 \equiv /p2$.

Adjunct If $p1 \equiv p2$ and p is a relative, then $p1/p \equiv p2/p$.

If $p1 \equiv p2$ and $p1, p2$ relative, then $p/p1 \equiv p/p2$.

If $p1 \equiv p2$, then $p1[q] \equiv p2[q]$ and $p[p1] \equiv p[p2]$.

Qualifier Flattening $p[p1/p2] \equiv p[p1[p2]]$

$\text{ancestor-or-self}::n \equiv \text{ancestor}::n \mid \text{self}::n$

$\text{descendant-or-self}::n \equiv \text{descendant}::n \mid \text{self}::n$

$p[p1 = /p2] \equiv p[p1[\text{self}::\text{node}() = /p2]]$

$p[p1 == /p2] \equiv p[p1[\text{self}::\text{node}() == /p2]]$

1. XPath Equivalence

“no backward at root node”

Lemma 3.2. *Let m and n be node tests, i.e. m and n are tag names or one of the *xPath* constructs `*`, `node()`, or `text()`.*

- *Let a be one of the axes `parent`, `ancestor`, `preceding`, `preceding-sibling`, `self`, `following`, or `following-sibling`. Then the following holds:*

$$/a::n \equiv \begin{cases} / & \text{if } a = \text{self} \text{ and } n = \text{node}() \\ \perp & \text{otherwise} \end{cases}$$

- *Let a be the `preceding` or `ancestor` axis. Then the following equivalences hold:*

$$\begin{aligned} /child::m/a::n &\equiv \begin{cases} /self::node()[child::m] & \text{if } a = \text{ancestor} \text{ and } n = \text{node}() \\ \perp & \text{otherwise} \end{cases} \\ /child::m[a::n] &\equiv \begin{cases} /child::m & \text{if } a = \text{ancestor} \text{ and } n = \text{node}() \\ \perp & \text{otherwise} \end{cases} \end{aligned}$$

(same holds for $a = \text{parent}$)

2. No Looking Back

Dual

backward

forward

parent
ancestor
ancestor-or-self
preceding
preceding-sibling

child
descendant
descendant-or-self
following
following-sibling

Thus: `dual(parent) = child`
`dual(following) = preceding`
etc.

Rewrite rule #1 (p,s: relative paths, **ax: reverse axis**)

`p[ax::m/s] → p[/descendant::m[s]/dual(ax)::node()] == self::node()]`

Rewrite rule #1 (p,s: relative paths, ax: reverse axis)

$p[\text{ax} :: m/s] \rightarrow p[/math>/ $\text{descendant} :: m[s]$ / $\text{dual(ax)} :: \text{node}()$ == sel f :: node()]$

↑

any “m[s]-node”
in the tree

↑

but, via **dual axis**, must
reach context node

E.g. **ax** = ancestor

$p[\text{ancestor} :: m] \rightarrow p[/math>/ $\text{descendant} :: m$ / $\text{descendant} :: \text{node}() == \text{self} :: \text{node}()$]$

“any m-node from which the context node can be reached via descendant, must be an ancestor of the context node.”

Rewrite rule #1 (p,s: relative paths, ax: reverse axis)

$p[\text{ax} :: m/s] \rightarrow p[/math>/ $\text{descendant} :: m[s]$ / $\text{dual(ax)} :: \text{node}()$ == sel f :: node())$

↑
any “m[s]-node”
in the tree

↑
but, via **dual axis**, must
reach context node

E.g. **ax** = preceding-sibling

p[preceding-sibling::m] → p[/descendant::m/following-sibling::node()=self::node()]

“any m-node from which the context node can be reached via following-sibling, must be a preceding-sibling of the context node.”

Rewrite rule #1 (p,s: relative paths, ax: reverse axis)

$p[ax :: m/s] \rightarrow p[/math>/ $\text{descendant} :: m[s]$ / $\text{dual}(ax) :: \text{node}()$ == sel f :: node()]$

↑
any “m[s]-node”
in the tree

↑
but, via **dual axis**, must
reach context node

E.g. ax=preceding-sibling

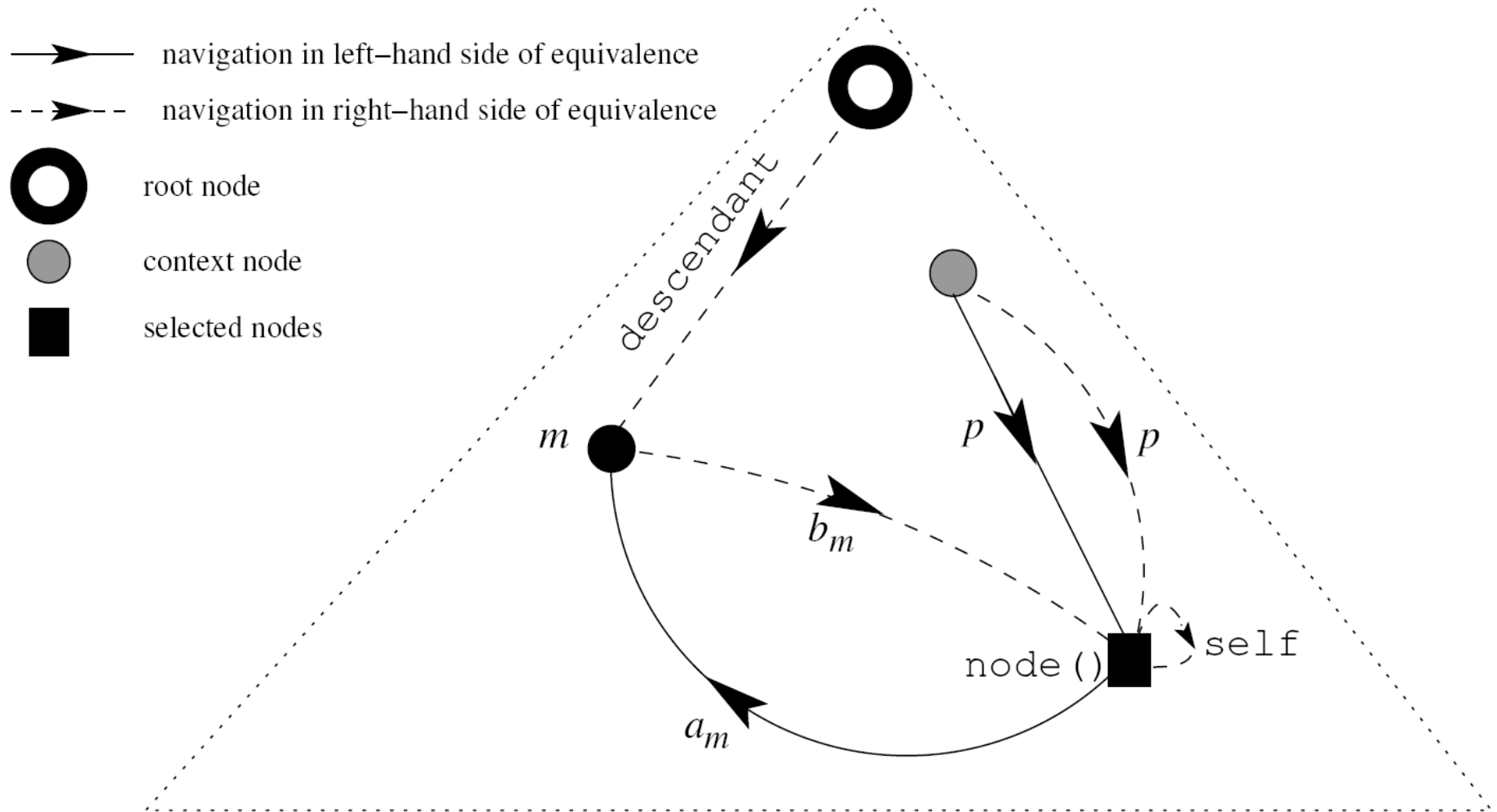
p[preceding-sibling::m] → p[/descendant::m/following-sibling::node()=self::node()]

“any m-node from which the context node can be reached via following-sibling, must be a preceding-sibling of the context node.”

Similar for **parent and preceding**. (ancestor-or-self not really needed. **Why?**)

Rewrite rule #1 (p,s: relative paths, **ax: reverse axis**)

$p[\text{ax} :: m/s] \rightarrow p[/math>/descendant :: $m[s]/\text{dual}(\text{ax}) :: \text{node}()$ == sel f :: $\text{node}()$]$



Rewrite rule #1 (p,s: relative paths, **ax: reverse axis**)

$$p[\text{ax} :: m/s] \rightarrow p[/\text{descendant} :: m[s] / \text{dual}(\text{ax}) :: \text{node}() == \text{sel f} :: \text{node}()]$$

Removes first reverse axis inside a filter (qualifier).

Use *qualifier flattening* to replace *any* reverse axis from inside a filter.

Qualifier Flattening $p[p1/p2] \equiv p[p1[p2]]$

Similar rules for **absolute paths**:

$$/p/f\text{Ax} :: n / \text{ax} :: m \rightarrow /\text{descendant} :: m[\text{dual}(\text{ax}) :: n == /p/f\text{Ax} :: n]$$
$$/f\text{Ax} :: n / \text{ax} :: m \rightarrow /\text{descendant} :: m[\text{dual}(\text{ax}) :: n == /f\text{Ax} :: n]$$

Rewrite rules #2 and #2a

E.g.

/descendant: : pri ce/**precedi ng**: : name

is rewritten via Rule #2a into:

/descendant: : name[**fol l owi ng**: : pri ce==/descendant: : pri ce]

Similar rules for **absolute paths**:

/p/fAx: : n/**ax**: : m → /descendant: : m[**dual(ax)**: : n == /p/fAx: : n]

/fAx: : n/**ax**: : m → /descendant: : m[**dual(ax)**: : n == /fAx: : n]

Rewrite rules #2 and #2a

E.g.

/descendant: : pri ce/**precedi ng**: : name

is rewritten via Rule #2a into:

/descendant: : name[**fol l owi ng**: : pri ce == /descendant: : pri ce]

a tautology

Of course, the “join” can be removed in this example:

/descendant: : name[**fol l owi ng**: : pri ce]

↑
Not needed, in this example.

Similar rules for **absolute paths**:

/p/fAx: : n/**ax**: : m → /descendant: : m[dual(**ax**): : n == /p/fAx: : n]

/fAx: : n/**ax**: : m → /descendant: : m[dual(**ax**): : n == /fAx: : n]

Rewrite rules #2 and #2a

E.g.

/descendant::journal[child::title]/descendant::price/preceding::name

becomes

/descendant::name[following::price==
/descendant::journal[child::title]/descendant::price]

Can you avoid the **join**, also for this example??

Similar rules for **absolute paths**:

/p/fAx::n/ax::m → /descendant::m[dual(ax)::n == /p/fAx::n]

/fAx::n/ax::m → /descendant::m[dual(ax)::n == /fAx::n]

Rewrite rules #2 and #2a

$path ::= path \mid path \mid / path \mid path / path \mid path [qualif] \mid axis :: nodetest \mid \perp .$
 $qualif ::= qualif \textbf{and} qualif \mid qualif \textbf{or} qualif \mid (qualif) \mid$
 $path = path \mid path == path \mid path .$
 $axis ::= reverse_axis \mid forward_axis .$
 $reverse_axis ::= parent \mid ancestor \mid ancestor\text{-or-self} \mid$
 $preceding \mid preceding\text{-sibling} .$
 $forward_axis ::= self \mid child \mid descendant \mid descendant\text{-or-self} \mid$
 $following \mid following\text{-sibling} .$
 $nodetest ::= tagname \mid * \mid \text{text}() \mid \text{node}() .$

- (1) $p[\text{ax} :: m/s] \rightarrow p[/math> /descendant :: m[s] /dual(ax) :: node() == self :: node()]
 (2) $/p/fAx :: n/\text{ax} :: m \rightarrow /descendant :: m[\text{dual}(\text{ax}) :: n == /p/fAx :: n]$
 (2a) $/fAx :: n/\text{ax} :: m \rightarrow /descendant :: m[\text{dual}(\text{ax}) :: n == /fAx :: n]$$

Rules (1),(2),(2a) suffice to remove ALL backward axes from above queries!

Why?

- Size Increase?
- How many joins?

2. No Looking Back

Dual

backward

forward

parent	child	not needed
ancestor	descendant	
ancestor or self	descendant or self	
preceding	following	
preceding-sibling	following-sibling	

Joins (==) are expensive! (typically quadratic wrt data)

To obtain queries with fewer joins

consider the **forward-axis** left of the **reverse-axis** to be removed!

New rules will be of the form

p/forw/back → p_new

p/forw[back] → p_new

2. No Looking Back

Interaction of **back=parent** with forward axes:

$$\text{descendant}::n/\text{parent}::m \equiv \text{descendant-or-self}::m[\text{child}::n] \quad (3)$$

2. No Looking Back

Interaction of **back=parent** with forward axes:

$$\text{descendant}::n/\text{parent}::m \equiv \text{descendant-or-self}::m[\text{child}::n] \quad (3)$$

$$\text{child}::n/\text{parent}::m \equiv \text{self}::m[\text{child}::n] \quad (4)$$

2. No Looking Back

Interaction of **back=parent** with forward axes:

$$\text{descendant}::n/\text{parent}::m \equiv \text{descendant-or-self}::m[\text{child}::n] \quad (3)$$

$$\text{child}::n/\text{parent}::m \equiv \text{self}::m[\text{child}::n] \quad (4)$$

$$p/\text{self}::n/\text{parent}::m \equiv p[\text{self}::n]/\text{parent}::m \quad (5)$$

2. No Looking Back

Interaction of **back=parent** with forward axes:

$$\text{descendant}::n/\text{parent}::m \equiv \text{descendant-or-self}::m[\text{child}::n] \quad (3)$$

$$\text{child}::n/\text{parent}::m \equiv \text{self}::m[\text{child}::n] \quad (4)$$

$$p/\text{self}::n/\text{parent}::m \equiv p[\text{self}::n]/\text{parent}::m \quad (5)$$

$$p/\text{following-sibling}::n/\text{parent}::m \equiv p[\text{following-sibling}::n]/\text{parent}::m \quad (6)$$

2. No Looking Back

Interaction of **back=parent** with forward axes:

$$\text{descendant}::n/\text{parent}::m \equiv \text{descendant-or-self}::m[\text{child}::n] \quad (3)$$

$$\text{child}::n/\text{parent}::m \equiv \text{self}::m[\text{child}::n] \quad (4)$$

$$p/\text{self}::n/\text{parent}::m \equiv p[\text{self}::n]/\text{parent}::m \quad (5)$$

$$p/\text{following-sibling}::n/\text{parent}::m \equiv p[\text{following-sibling}::n]/\text{parent}::m \quad (6)$$

$$p/\text{following}::n/\text{parent}::m \equiv p/\text{following}::m[\text{child}::n] \quad (7)$$

$$\begin{aligned} &| \text{p/ancestor-or-self}::*[\text{following-sibling}::n] \\ &\quad / \text{parent}::m \end{aligned}$$

2. No Looking Back

Interaction of **back=parent** with forward axes:

$$\text{descendant}::n/\text{parent}::m \equiv \text{descendant-or-self}::m[\text{child}::n] \quad (3)$$

$$\text{child}::n/\text{parent}::m \equiv \text{self}::m[\text{child}::n] \quad (4)$$

$$p/\text{self}::n/\text{parent}::m \equiv p[\text{self}::n]/\text{parent}::m \quad (5)$$

$$p/\text{following-sibling}::n/\text{parent}::m \equiv p[\text{following-sibling}::n]/\text{parent}::m \quad (6)$$

$$p/\text{following}::n/\text{parent}::m \equiv p/\text{following}::m[\text{child}::n] \quad (7)$$

$$\begin{aligned} &| \text{p/ancestor-or-self}::*[\text{following-sibling}::n] \\ &\quad / \text{parent}::m \end{aligned}$$

$$\text{descendant}::n [\text{parent}::m] \equiv \text{descendant-or-self}::m/\text{child}::n \quad (8)$$

$$\text{child}::n[\text{parent}::m] \equiv \text{self}::m/\text{child}::n \quad (9)$$

$$p/\text{self}::n[\text{parent}::m] \equiv p[\text{parent}::m]/\text{self}::n \quad (10)$$

$$p/\text{following-sibling}::n[\text{parent}::m] \equiv p[\text{parent}::m]/\text{following-sibling}::n \quad (11)$$

$$p/\text{following}::n[\text{parent}::m] \equiv p/\text{following}::m/\text{child}::n \quad (12)$$

$$\begin{aligned} &| \text{p/ancestor-or-self}::*[\text{parent}::m] \\ &\quad / \text{following-sibling}::n \end{aligned}$$

2. No Looking Back

Interaction of `back=ancestor` with forward axes:

$$\begin{aligned} p/\text{descendant}::n/\text{ancestor}::m &\equiv p[\text{descendant}::n]/\text{ancestor}::m \\ &\quad | p/\text{descendant-or-self}::m[\text{descendant}::n] \end{aligned} \tag{13}$$

2. No Looking Back

Interaction of **back=ancestor** with forward axes:

$$p/\text{descendant}::n/\text{ancestor}::m \equiv p[\text{descendant}::n]/\text{ancestor}::m \quad (13)$$

$$| p/\text{descendant-or-self}::m[\text{descendant}::n]$$

$$/\text{descendant}::n/\text{ancestor}::m \equiv /\text{descendant-or-self}::m[\text{descendant}::n] \quad (13a)$$

2. No Looking Back

Interaction of **back=ancestor** with forward axes:

$$p/\text{descendant}::n/\text{ancestor}::m \equiv p[\text{descendant}::n]/\text{ancestor}::m \quad (13)$$

$$| p/\text{descendant-or-self}::m[\text{descendant}::n]$$

$$/\text{descendant}::n/\text{ancestor}::m \equiv /\text{descendant-or-self}::m[\text{descendant}::n] \quad (13a)$$

$$p/\text{child}::n/\text{ancestor}::m \equiv p[\text{child}::n]/\text{ancestor-or-self}::m \quad (14)$$

2. No Looking Back

Interaction of **back=ancestor** with forward axes:

$$p/\text{descendant}::n/\text{ancestor}::m \equiv p[\text{descendant}::n]/\text{ancestor}::m \quad (13)$$

$$| p/\text{descendant-or-self}::m[\text{descendant}::n]$$

$$/\text{descendant}::n/\text{ancestor}::m \equiv /\text{descendant-or-self}::m[\text{descendant}::n] \quad (13a)$$

$$p/\text{child}::n/\text{ancestor}::m \equiv p[\text{child}::n]/\text{ancestor-or-self}::m \quad (14)$$

$$p/\text{self}::n/\text{ancestor}::m \equiv p[\text{self}::n]/\text{ancestor}::m \quad (15)$$

2. No Looking Back

Interaction of **back=ancestor** with forward axes:

$$p/\text{descendant}::n/\text{ancestor}::m \equiv p[\text{descendant}::n]/\text{ancestor}::m \quad (13)$$

$$| p/\text{descendant-or-self}::m[\text{descendant}::n]$$

$$/\text{descendant}::n/\text{ancestor}::m \equiv /\text{descendant-or-self}::m[\text{descendant}::n] \quad (13a)$$

$$p/\text{child}::n/\text{ancestor}::m \equiv p[\text{child}::n]/\text{ancestor-or-self}::m \quad (14)$$

$$p/\text{self}::n/\text{ancestor}::m \equiv p[\text{self}::n]/\text{ancestor}::m \quad (15)$$

$$p/\text{following-sibling}::n/\text{ancestor}::m \equiv p[\text{following-sibling}::n]/\text{ancestor}::m \quad (16)$$

2. No Looking Back

Interaction of **back=ancestor** with forward axes:

$$p/\text{descendant}::n/\text{ancestor}::m \equiv p[\text{descendant}::n]/\text{ancestor}::m \quad (13)$$

$$| p/\text{descendant-or-self}::m[\text{descendant}::n]$$

$$/\text{descendant}::n/\text{ancestor}::m \equiv /\text{descendant-or-self}::m[\text{descendant}::n] \quad (13a)$$

$$p/\text{child}::n/\text{ancestor}::m \equiv p[\text{child}::n]/\text{ancestor-or-self}::m \quad (14)$$

$$p/\text{self}::n/\text{ancestor}::m \equiv p[\text{self}::n]/\text{ancestor}::m \quad (15)$$

$$p/\text{following-sibling}::n/\text{ancestor}::m \equiv p[\text{following-sibling}::n]/\text{ancestor}::m \quad (16)$$

$$p/\text{following}::n/\text{ancestor}::m \equiv p/\text{following}::m[\text{descendant}::n] \quad (17)$$

$$| p/\text{ancestor-or-self}::*$$

$$[\text{following-sibling}::*/\text{descendant-or-self}::n]$$

$$/\text{ancestor}::m$$

Similar rules for **ancestor** in a filters.

2. No Looking Back

Interaction of **back=ancestor** with forward axes:

$$p/\text{descendant}::n/\text{ancestor}::m \equiv p[\text{descendant}::n]/\text{ancestor}::m \quad (13)$$
$$| p/\text{descendant-or-self}::m[\text{descendant}::n]$$

$$/\text{descendant}::n/\text{ancestor}::m \equiv /\text{descendant-or-self}::m[\text{descendant}::n] \quad (13a)$$

$$p/\text{child}::n/\text{ancestor}::m \equiv p[\text{child}::n]/\text{ancestor-or-self}::m \quad (14)$$

$$p/\text{self}::n/\text{ancestor}::m \equiv p[\text{self}::n]/\text{ancestor}::m \quad (15)$$

$$p/\text{following-sibling}::n/\text{ancestor}::m \equiv p[\text{following-sibling}::n]/\text{ancestor}::m \quad (16)$$

$$p/\text{following}::n/\text{ancestor}::m \equiv p/\text{following}::m[\text{descendant}::n] \quad (17)$$

$$| p/\text{ancestor-or-self}::*$$
$$[\text{following-sibling}::*/\text{descendant-or-self}::n]$$
$$/\text{ancestor}::m$$

Similar rules for **ancestor** in a filters.

E.g., what is the forward query for: `//*[ancestor::a]`

2. No Looking Back

Interaction of **back=preceding** with forward axes:

$$p/\text{descendant}::n/\text{preceding}::m \equiv p[\text{descendant}::n]/\text{preceding}::m \quad (33)$$

| $p/\text{child}::*$

| $[\text{following-sibling}::*/\text{descendant-or-self}::n]$

| $\text{descendant-or-self}::m$

$$/\text{descendant}::n/\text{preceding}::m \equiv /\text{descendant}::m[\text{following}::n] \quad (33a)$$

$$p/\text{child}::n/\text{preceding}::m \equiv p[\text{child}::n]/\text{preceding}::m \quad (34)$$

| $p/\text{child}::*[\text{following-sibling}::n]$

| $\text{descendant-or-self}::m$

$$p/\text{self}::n/\text{preceding}::m \equiv p[\text{self}::n]/\text{preceding}::m \quad (35)$$

$$p/\text{following-sibling}::n/\text{preceding}::m \equiv p[\text{following-sibling}::n]/\text{preceding}::m \quad (36)$$

| $p/\text{following-sibling}::*[\text{following-sibling}::n]$

| $\text{descendant-or-self}::m$

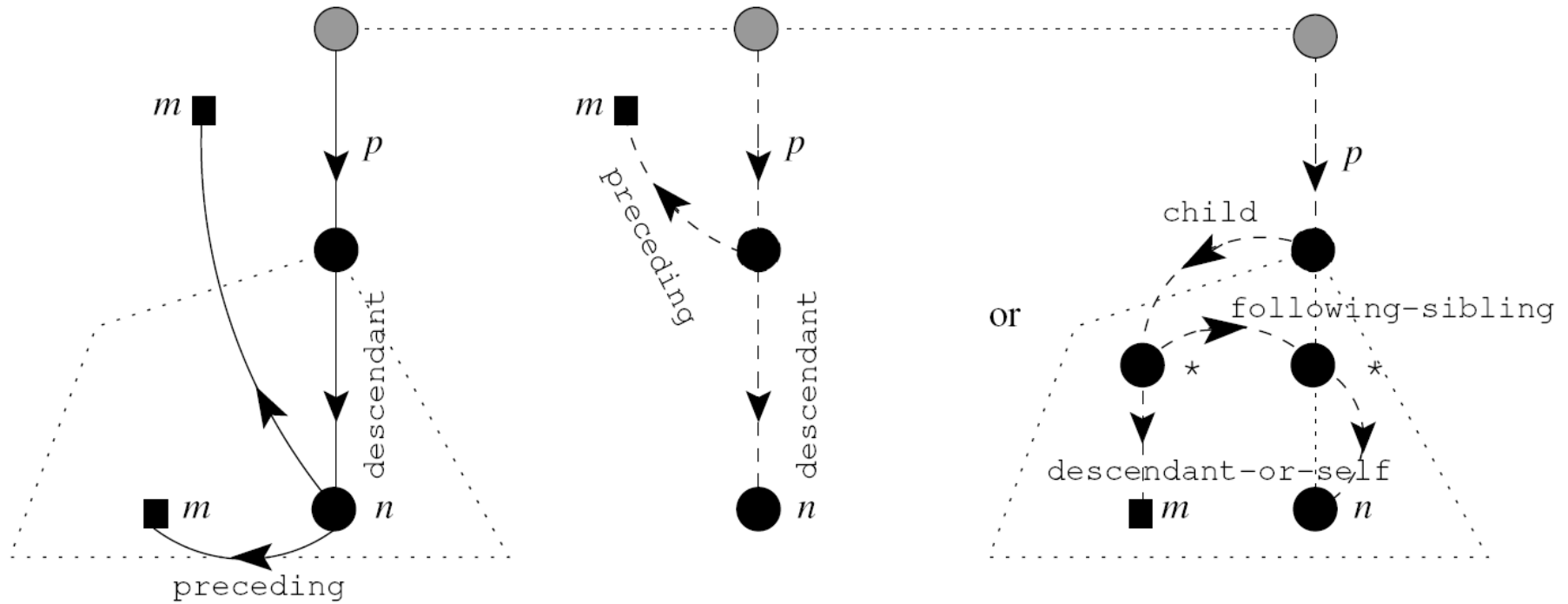
| $p[\text{following-sibling}::n]/\text{descendant-or-self}::m$

$$p/\text{following}::n/\text{preceding}::m \equiv p[\text{following}::n]/\text{preceding}::m \quad (37)$$

| $p/\text{following}::m[\text{following}::n]$

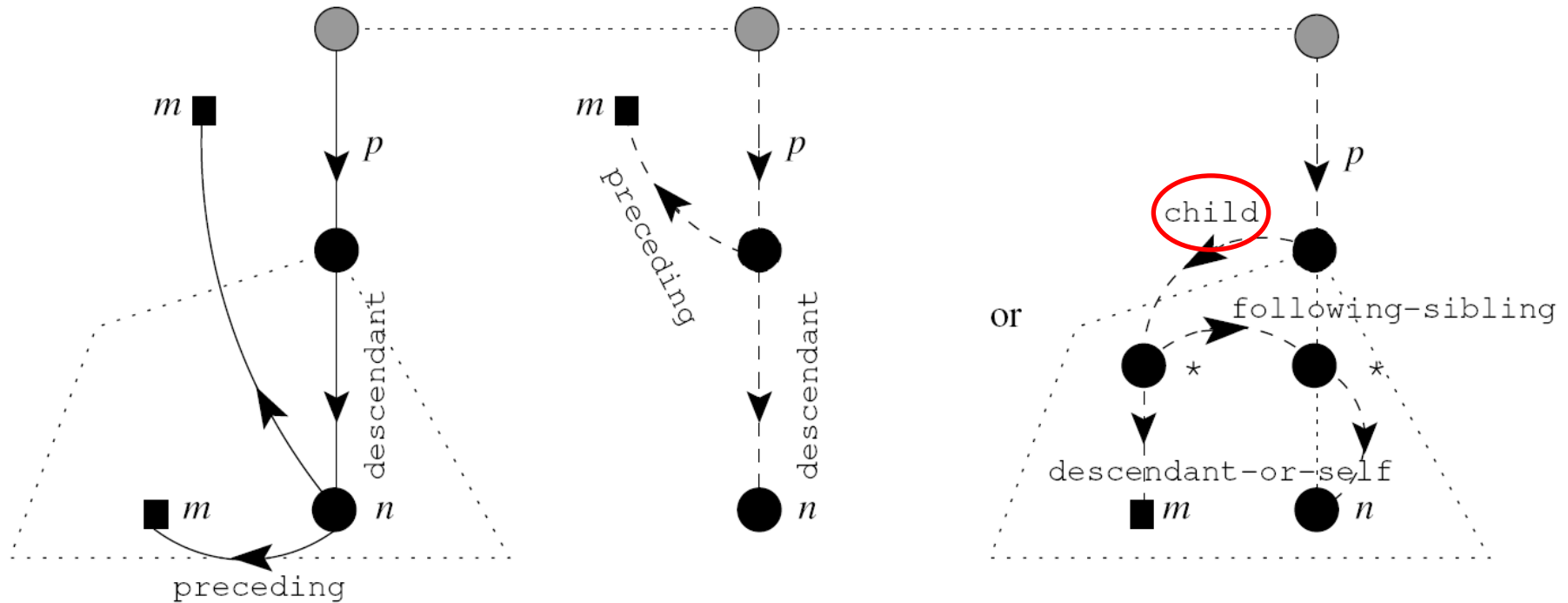
| $p[\text{following}::n]/\text{descendant-or-self}::m$

Rule 33



$$p/\text{descendant}::n/\text{preceding}::m \equiv p[\text{descendant}::n]/\text{preceding}::m \\ | p/\text{child}::*[\text{following-sibling}::*/\text{descendant-or-self}::n]/\text{descendant-or-self}::m$$

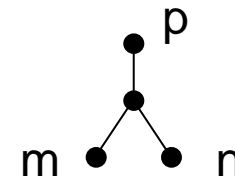
Rule 33



$p/\text{descendant}::n/\text{preceding}::m \equiv p[\text{descendant}::n]/\text{preceding}::m$

$| p/\text{child}::*[\text{following-sibling}::*/\text{descendant-or-self}::n]/\text{descendant-or-self}::m$

Wrong.
Should be **descendant** instead!



2. No Looking Back

/descendant: : pri ce/**precedi ng**: : name

is rewritten via Rule #2a into:

/descendant: : name[**fol l owi ng**: : pri ce==/descendant: : pri ce]

Now, let us use Rule (33a)

/descendant: : n/**precedi ng**: : m → /descendant: : m[**fol l owi ng**: : n]

We obtain

/descendant: : name[**fol l owi ng**: : pri ce]

p

/descendant::journal[child::title]/descendant::price/preceding::name

becomes

/descendant::name[following::price==
/descendant::journal[child::title]/descendant::price]

Rule (33a)

/descendant::n/preceding::m → /descendant::m[following::n]

doesn't work because descendant is absolute here.

Rule (33):

p/descendant::n/preceding::m → p[descendant::n]/preceding::m
| p[child::*[following-sibling::*/descendant-or-self::n]
/descendant-or-self::m

We obtain

p[descendant::price]/preceding::name
| p[child::*[following-sibling::*/descendant-or-self::price]
/descendant-or-self::name

p

/descendant::journal[chi ld::title]/descendant::pri ce/precedi ng::name

becomes

/descendant::name[fol lowi ng::pri ce==
/descendant::journal[chi ld::title]/descendant::pri ce]

Rule (33a)

/descendant::n/precedi ng::m → /descendant::m[fol lowi ng::n]

doesn't work because descendant is absolute here.

Rule (33):

p/descendant::n/precedi ng::m → p[descendant::n]/precedi ng::m
| p/chi ld::*[fol lowi ng-si bl i ng::*]/descendant-or-sel f::n]
/descendant-or-sel f::m

→ Rule (33a) with n = journal[chi ld::title][descendant::pri ce]

p[descendant::pri ce]/precedi ng::name
| p/chi ld::*[fol lowi ng-si bl i ng::*]/descendant-or-sel f::pri ce]
/descendant-or-sel f::name

p

/descendant::journal[chi ld::title]/descendant::pri ce/precedi ng::name

becomes

/descendant::name[fol lowi ng::pri ce==
/descendant::journal[chi ld::title]/descendant::pri ce]

Rule (33a)

/descendant::n/precedi ng::m → /descendant::m[fol lowi ng::n]
doesn't work because descendant is absolute here.

/descendant::name[fol lowi ng::journal[chi ld::title][descendant::pri ce]]
| p/chi ld::*[fol lowi ng-si bl i ng::*/descendant-or-sel f::pri ce]
/descendant-or-sel f::name

→ Rule (33a) with n = journal[chi ld::title][descendant::pri ce]

p[descendant::pri ce]/precedi ng::name
| p/chi ld::*[fol lowi ng-si bl i ng::*/descendant-or-sel f::pri ce]
/descendant-or-sel f::name

p

/descendant::journal[chi ld::ti tl e]/descendant::pri ce/precedi ng::name

becomes

=n

/descendant::name[fol l owi ng::pri ce==
/descendant::journal[chi ld::ti tl e]/descendant::pri ce]

Rule (33a)

/descendant::n/precedi ng::m → /descendant::m[fol l owi ng::n]

~~doesn't work because descendant is absolute here.~~ seems it does work! ☺

/descendant::name[fol l owi ng::journal[chi ld::ti tl e][descendant::pri ce]
| p/chi ld::*[fol l owi ng-si bl i ng::*]/descendant-or-sel f::pri ce]
/descendant-or-sel f::name

p[p1/p2]
= p[p1[p2]]

What about this one:

/descendant::name[fol l owi ng::journal[chi ld::ti tl e]/descendant::pri ce]

Theorem

(from D. Olteanu, H. Meuss, T. Furche, F. Bry
XPath: Looking Forward. [EDBT Workshops 2002](#): 109-127)

Given an **XPath expression** p that has no joins of the form $(p1 == p2)$ with both $p1, p2$ relative, an equivalent expression u **without reverse axes** can be computed.

Time needed: at most **exponential** in length of p

Length of u : at most **exponential** in length of p

(moreover: *no joins* are introduced when computing u)

Questions

- Why rewriting takes exponential time?
- Can you find a subclass for which *Time* to compute u is linear or polynomial?
- What is the problem with joins $(p1 == p2)$ for removal of reverse axes?

Theorem

Given an XPath expression p that has no joins of the form $(p1 == p2)$ with both $p1, p2$ relative, an equivalent expression u without reverse axes can be computed.

Time needed: at most **exponential** in length of p
(moreover: *no joins* are introduced when computing u)

More Questions

- Give an example of Core backward XPath with **negation**, for which there is no forward XPath query.
- Give an example of Core backward XPath with **data values**, for which there is no forward XPath query.
- Give an example of a Core backward XPath with **counting**, for which there is no forward XPath query.

3. XPath Containment Test

Given two XPath expressions p , q :

Are all nodes selected by p , also selected by q ? (on *any* document)
(p “contained in” q)

Has **many applications!**

Boolean query

Want to select documents that “match p ”.

→ If a document matches p , and p contained in q ,
then we know the document also matches q !

→ If a document does not match q , and p contained in q ,
then we know the document does not match p !

Applications

- Decrease online-time of publish/subscribe systems based on XPath
 - Decrease query-time by making use of materialized intermediate results
 - Optimization by ruling out queries with empty result set
- etc, etc

3. XPath Containment Test

Given two XPath expressions p, q

“0-containment” For every tree, if p selects a node then so does q .

$p \subseteq_0 q$

“1-containment” For every tree, all nodes selected by p are also selected by q .

$p \subseteq_1 q$

“2-containment” For every tree, and every context node N ,
all nodes selected by p starting from N ,
are also selected by q starting from N .

- 1. Inclusion on *Booleans*
 - 2. Inclusion on *Node Sets*
 - 3. Inclusion on *Node Relations*
- } start from root

(If only child and descendant axes are allowed
then \subseteq_1 and \subseteq_2 are the same! -- Why?)

3. XPath Containment Test

Given two XPath expressions p, q

“0-containment” For every tree, if p selects a node then so does q .

$p \subseteq_0 q$

“1-containment” For every tree, all nodes selected by p are also selected by q .

$p \subseteq_1 q$

Question

Given p, q and the fact $p \subseteq_1 q$,
how can you determine from a *result set of nodes* for q ,
the correct *result set of nodes* for p ?

3. XPath Containment Test

Given two XPath expressions p, q

Sometimes we want to test containment wrt a given DTD:

$p = /a/b//d$

$q = /a//c$

Boolean!

Want to check if $p \subseteq_0 q$.

NO!

a
|
b
|
d

But, what if documents are valid wrt to this DTD?

root	→	a^*
a	→	$b^* \mid c^*$
b	→	$d+c+$
c	→	$b?c?$

PTIME	XP(/, //, *) [21] XP(/, [], *) (see [19]) XP(/, //, []) [2], with fixed bounded SXICs [9] XP(/, //) + DTDs [22] XP(/, []) + DTDs [22]
coNP	XP(/, //, [], *) [19] XP(/, //, [], *,), XP(/,), XP(//,) [22] XP(/, []) + DTDs [22] XP(//, []) + DTDs [22]
Π_2^P	XP(/, //, [],) + existential variables + path equality + ancestor-or-self axis + fixed bounded SXICs [9] XP(/, //, [], *,) + existential variables + all backward axes + fixed bounded SXICs [9] XP(/, //, [],) + existential variables with inequality [22]
PSPACE	XP(/, //, [], *,) and XP(/, //,) if the alphabet is finite [22] XP(/, //, [], *,) + variables with XPath semantics [22]
EXPTIME	XP(/, //, [],) + existential variables + bounded SXICs [9] XP(/, //, [], *,) + DTDs [22] XP(/, //,) + DTDs [22] XP(/, //, [], *) + DTDs [22]
Undecidable	XP(/, //, [],) + existential variables + unbounded SXICs [9] XP(/, //, [],) + existential variables + bounded SXICs + DTDs [9] XP(/, //, [], *,) + nodeset equality + simple DTDs [22] XP(/, //, [], *,) + existential variables with inequality [22]

2. XPath Containment Test

from:

T. Schwentick

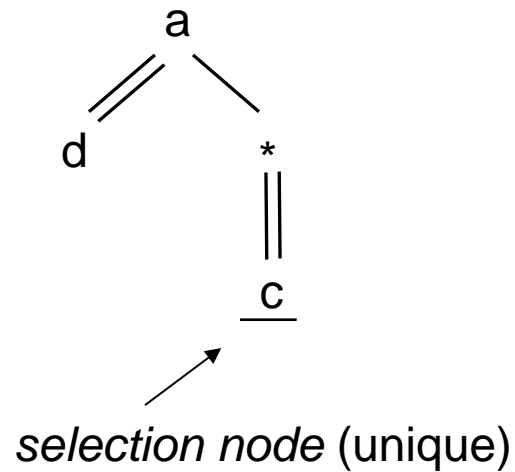
XPath query containment.

[SIGMOD Record 33](#)(1): 101-109 (2004)

Pattern trees

E.g. $p = a[. //d]/* //c$

Note: child order has no meaning in pattern trees!

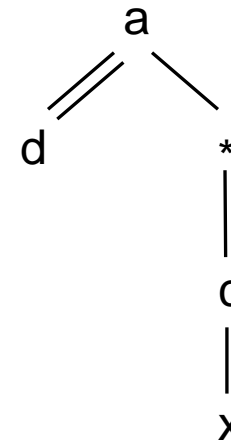


Test \subseteq_1 (node set inclusion) using \subseteq_0 (Boolean inclusion)

→ Simply add a new node below the *selection node*

New tree is Boolean (no selection node)

In a given XML tree:
pattern matches / does not match.



3. XPath Containment Test

4 techniques of testing XPath (Boolean) containment:

- (1) The Canonical Model Technique
- (2) The Homomorphism Technique
- (3) The Automaton Technique
- (4) The Chase Technique

3. XPath Containment Test

Canonical Model - XPath(/, //, [], *)

Idea: if there exists a tree that matches **p** but not **q**, then such a tree exists of **size polynomial in the size** of **p** and **q**.

Simple: remember, if you know that the XML document is only of height 5, then **//a/b/*/c** could be enumerated by **/a/b/*/c | /*/a/b/*/c | /**/a/b/*/c | /***/a...**

Similarly, we try to construct a counter example tree, by **replacing in p**

- every ***** by some new symbol **"z"**
- every **//** by **z/, z/z/, z/z/z/, ... z/z/..z/**

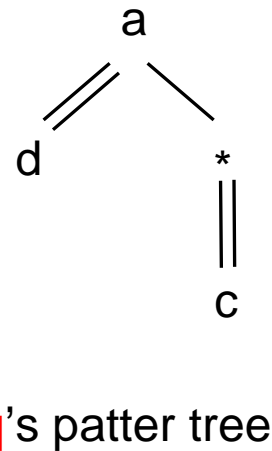
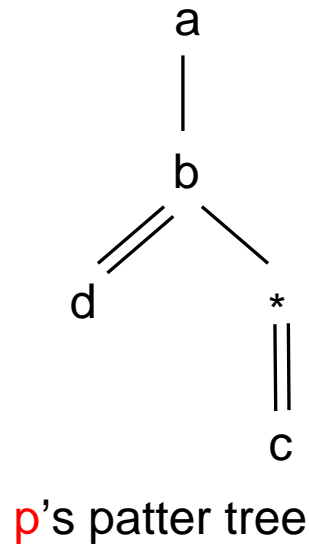
N+1 many z's

N = length of longest ***/../*** chain in **q**

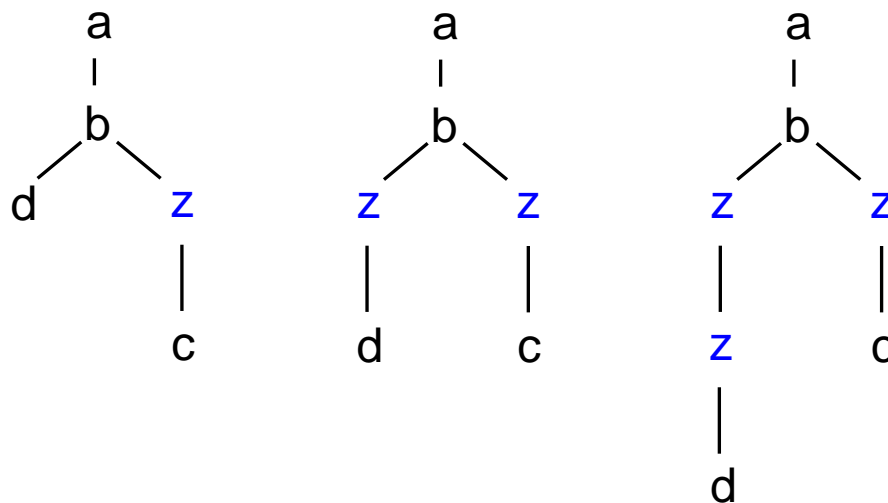
3. XPath Containment Test

Canonical Model - XPath(/, //, [], *)

Example



Test for
q-match:



Formally, must test
1 and 2 more **z**'s
at *right branch* of
each of the trees.

3. XPath Containment Test

Homomorphism h maps each node of q 's query tree Q
to a node of p 's query tree P such that

- (1) root of Q is mapped to root of P
- (2) if (u,v) is child-edge of Q then $(h(u),h(v))$ is child-edge of P
- (3) if (u,v) is descendant-edge of Q , then
 $h(v)$ is a “below” $h(u)$ in P
- (4) if u is labeled by “ e ” (not $*$), then $h(u)$ is also labeled by “ e ”.

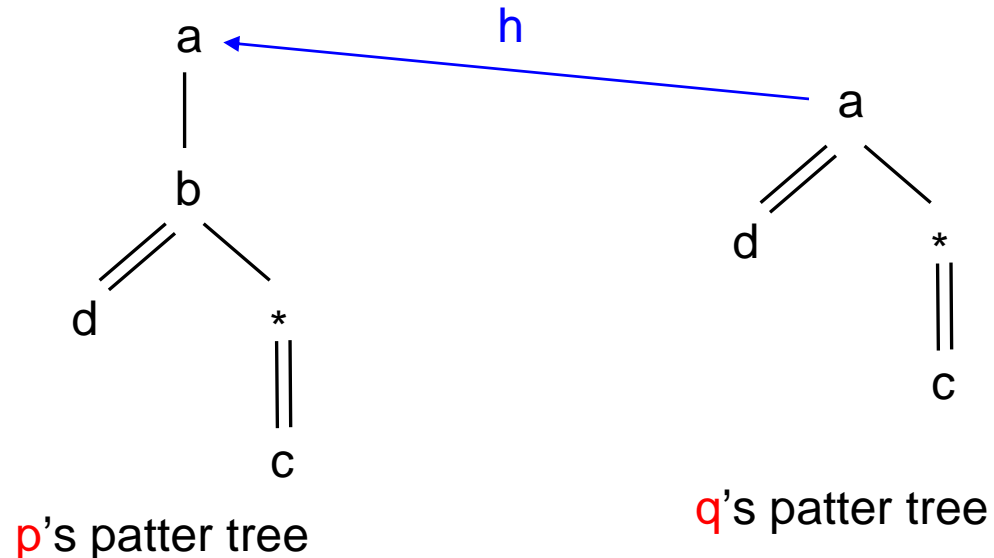
p, q expressions in XPath($/$, $//$, $[]$)

Theorem

$p \subseteq_0 q$ if and only if there is a homomorphism from Q to P .

3. XPath Containment Test

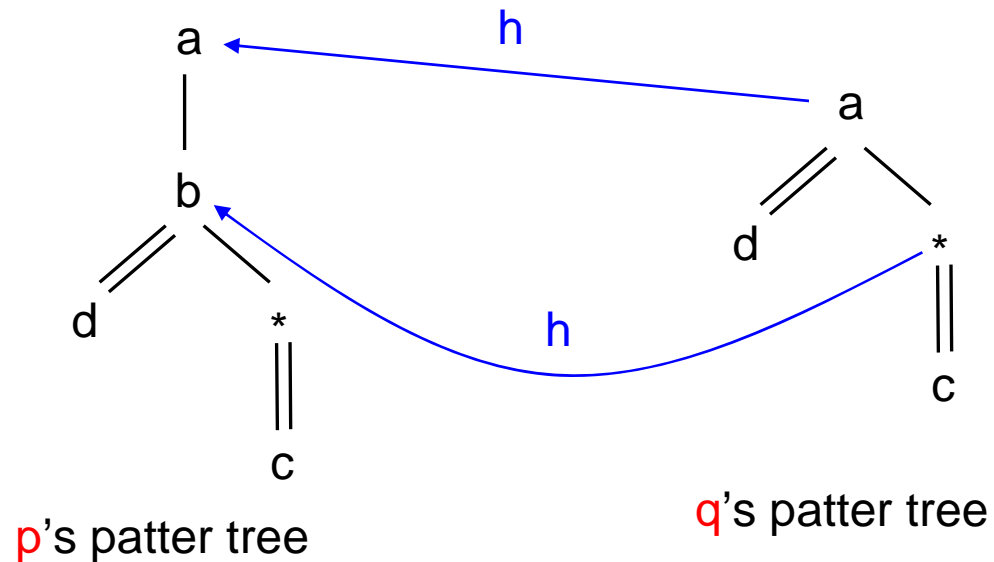
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3. XPath Containment Test

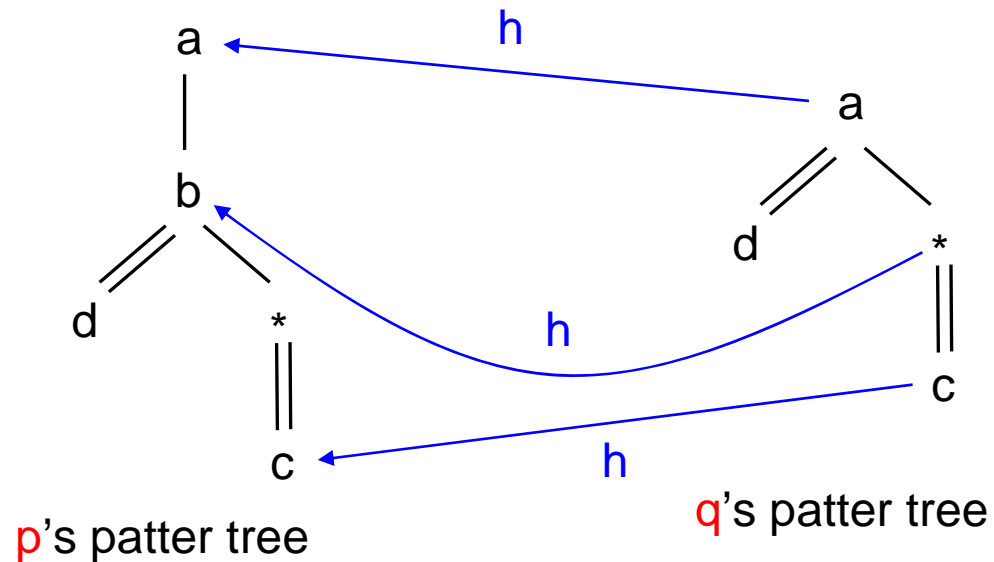
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3. XPath Containment Test

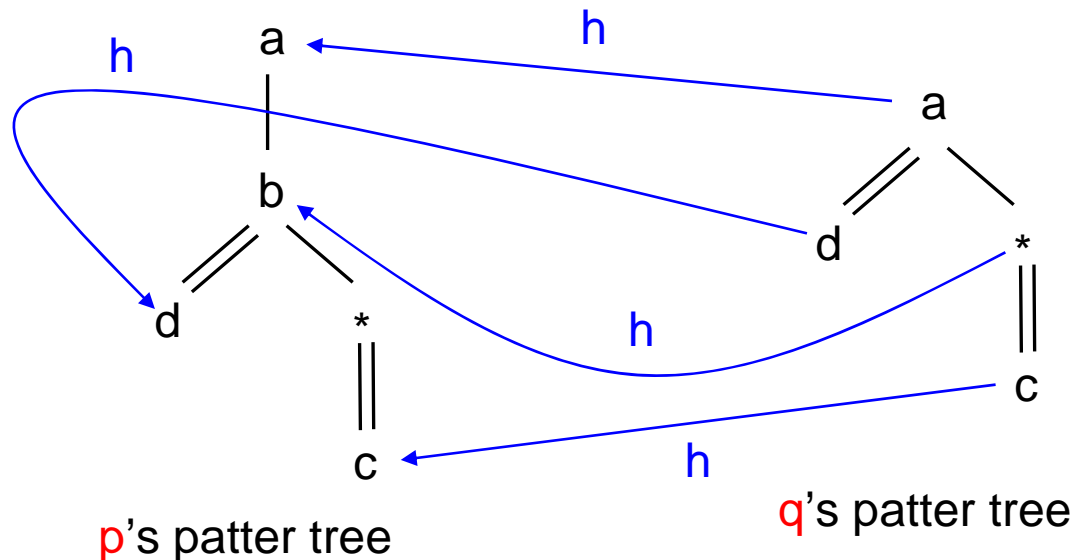
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- (4) if u is labeled by "e" (not $*$), then $h(u)$ is also labeled by "e".

3. XPath Containment Test

Homomorphism h maps each node of q 's query tree Q to a node of p 's query tree P such that



→ hom. h exists from Q to P , thus $p \subseteq_0 q$ must hold!

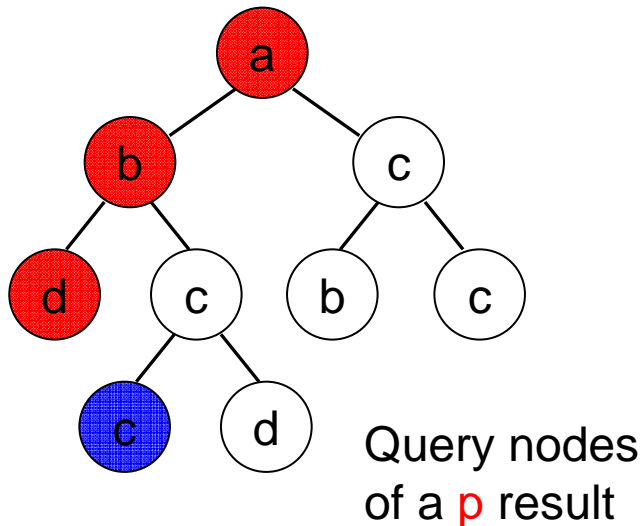
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- (4) if u is labeled by "e" (not *), then $h(u)$ is also labeled by "e".

3. XPath Containment Test

“1-containment” For every tree, all nodes selected by p are also selected by q .
 $p \subseteq_1 q$

Question

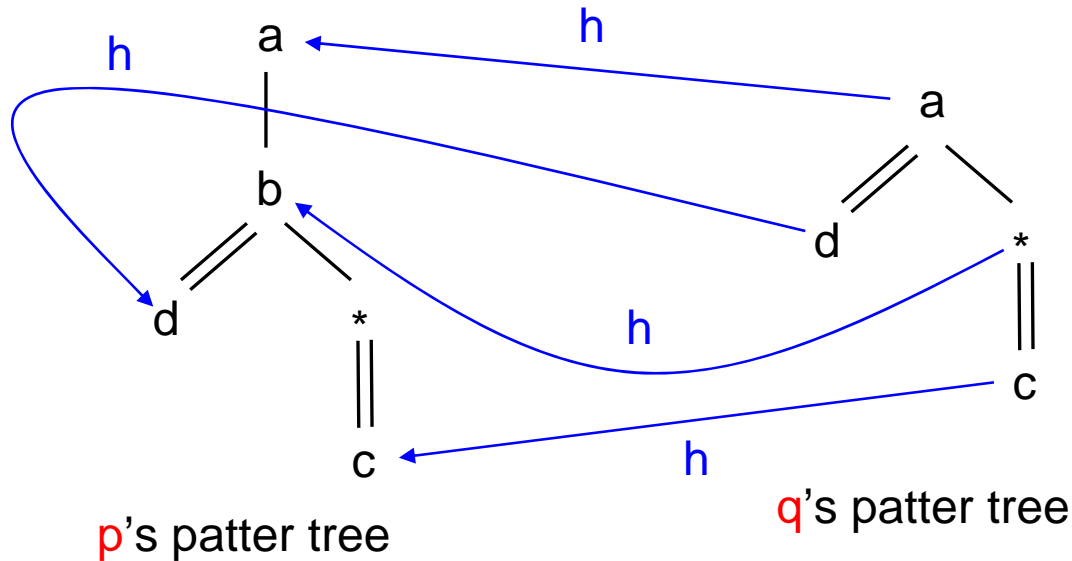
Given p, q and the fact $p \subseteq_1 q$,
how can you determine from a *result set of nodes* for q ,
the correct *result set of nodes* for p ?



→ With homomorphism technique:

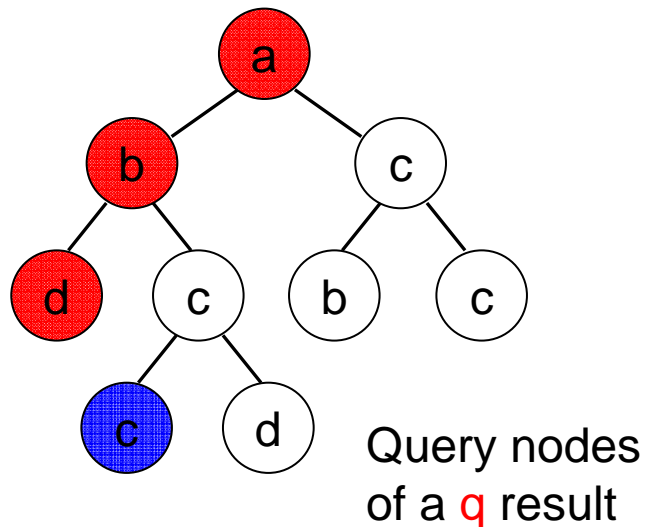
Use a result node of q together with
run-time info on pattern nodes.
Enables to search “inside”, only on paths
between pattern nodes.

3. XPath Containment Test



1. $x :=$ parent of q 's c-node
2. Check if q 's *-node is
 - (a) ancestor of x
 - (b) labeled b
 - (c) has a-parent
 - (d) is ancestor of q 's d-node

Cave: we will have to try all homomorphisms ...



→ With homomorphism technique:

Use a result node of q together with run-time info on pattern nodes.
Enables to search “inside”, only on paths between pattern nodes.

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p, q expressions in XPath($/, //, []$)

Theorem

$p \subseteq_0 q$ if and only if there is a homomorphism from Q to P .

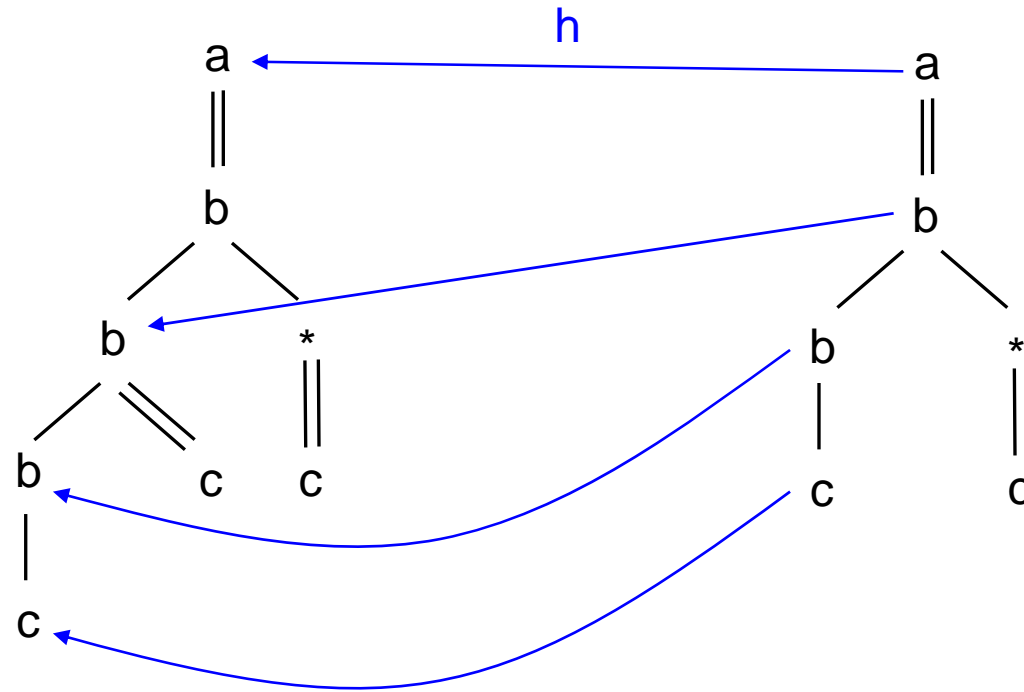
Cave If we add the star ($*$) then homomorphism need not exist!

→ there are $p, q \in \text{XPath}(/, //, [], *)$ such that $p \subseteq_0 q$ and
there is **no** homomorphism from Q to P ☹

3. XPath Containment Test

$[/a//b[./b[./b/c]//c]/*//c]$

$[/a//b[./b/c]/*//c]$



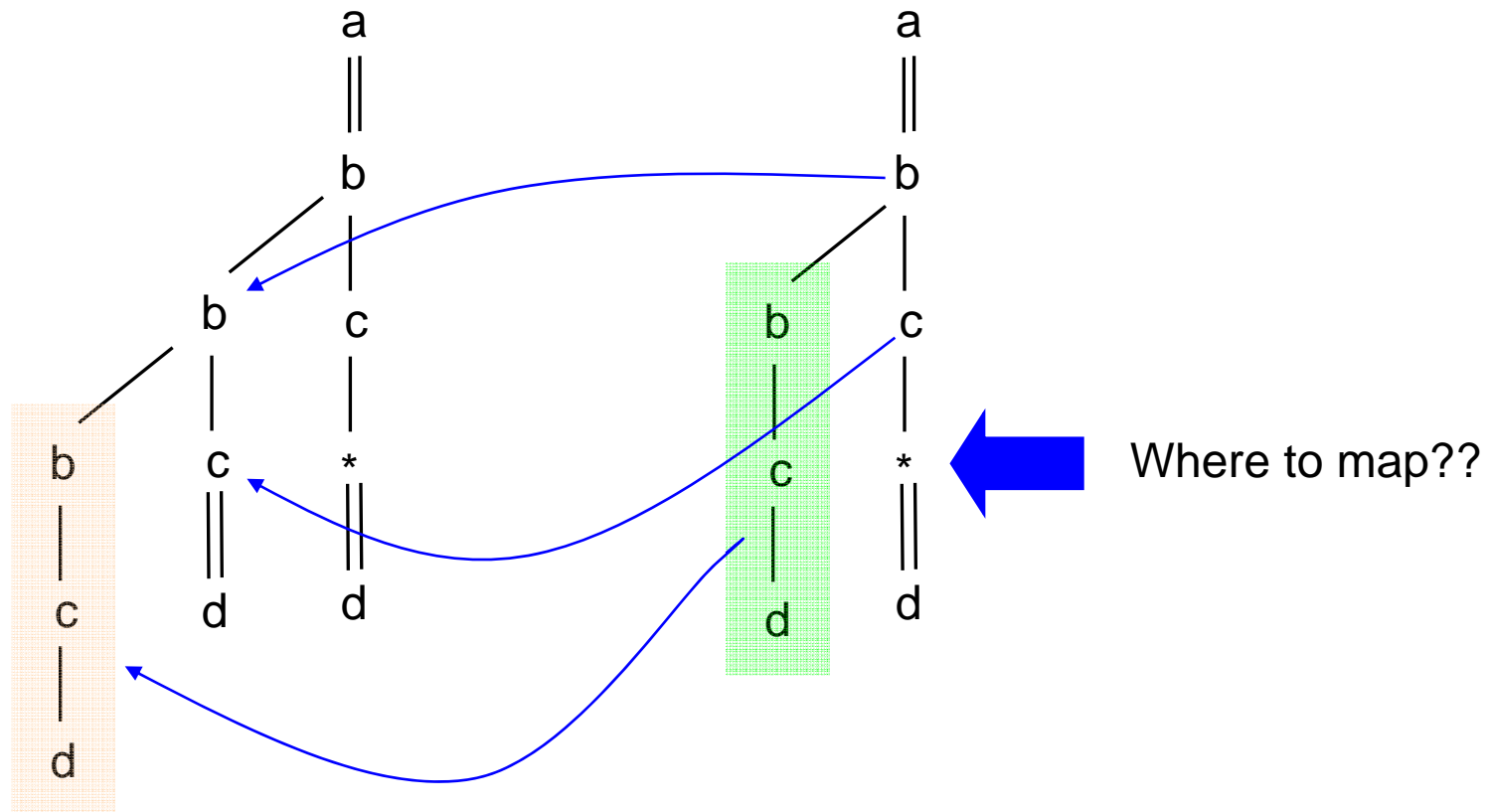
IS there a homomorphism??

Cave If we add the star (*) then homomorphism need not exist!

→ there are $p, q \in \text{XPath}(/, //, [], *)$ such that $p \subseteq_0 q$ and there is **no** homomorphism from Q to P ☹

$p = /a[. //b[c/*//d]/b[c//d]/b[c/d]]$

$q = /a[. //b[c/*//d]/b[c/d]]$

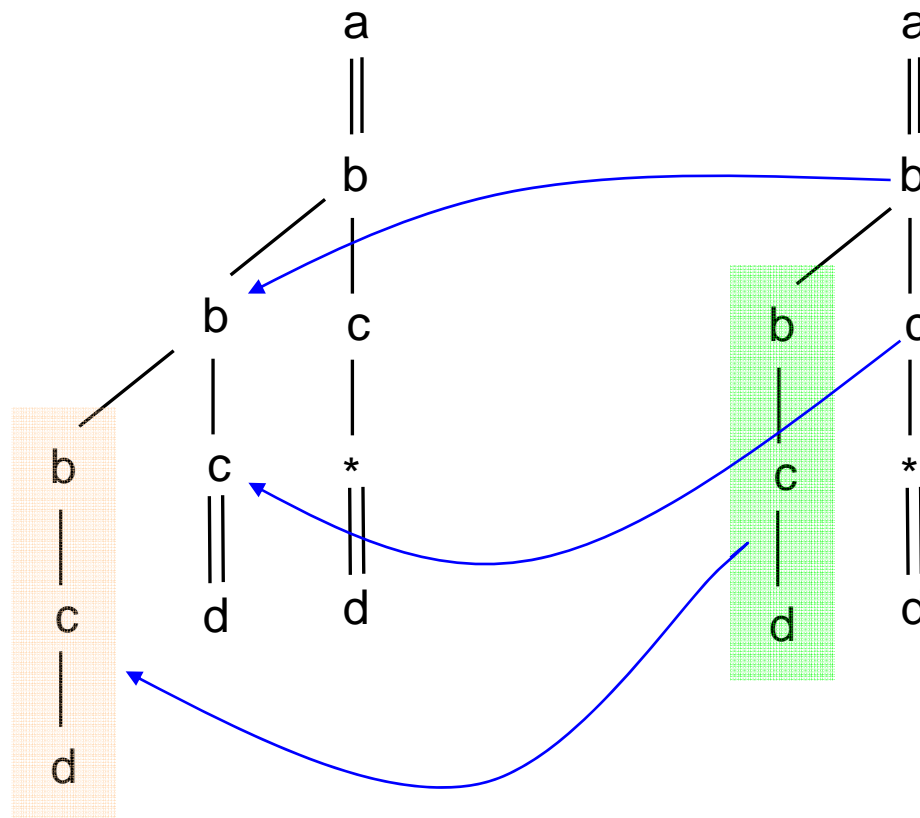


Cave If we add the star (*) then homomorphism need not exist!

→ there are $p, q \in \text{XPath}(/, //, [], *)$ such that $p \subseteq_0 q$ and there is **no** homomorphism from Q to P ☹

$p = /a[. //b[c/*//d]/b[c//d]/b[c/d]]$

$q = /a[. //b[c/*//d]/b[c/d]]$



Is p contained in q ?
 → Test this, using the
 canonical model!!

Where to map??

Cave If we add the star (*) then homomorphism need not exist!

→ there are $p, q \in \text{XPath}(/, //, [], *)$ such that $p \subseteq_0 q$ and
 there is **no** homomorphism from Q to P ☹

Let's check the web...

→ **YES** p contained in q!

University of Lübeck, Institute of Information Systems, www.ifis.uni-luebeck.de

XPath-Containment Checker

Implemented by Khaled Haj-Yahya ([khaled.h at gmx.de](mailto:khaled.h@gmx.de))
Supervised by B.C.Hammerschmidt ([former](#))

This is a Java implementation of the theoretical work of
Gerome Miklau and Dan Suciu ([Containment and Equivalence](#)
for a Fragment of XPath. J. ACM 51(1): 2-45 (2004) and [Containment](#)
and Equivalence for a Fragment of XPath. PODS 2002)

Instructions:

Enter two XPath expressions in the abbreviated syntax and press the
button.
For instance:
if $p = /a[b]$ and $p' = /a[*]$
the algorithm will detect that p is a subset of p' .

Or if $p = /a/*/b$ and $p' = /a/*/b$
the algorithm will detect that p is equal to p'
because the subset equation holds in both directions.

[Download the Java Source Code](#)

[Download Khaled's bachelor thesis \(in German\)](#)

If there is no application on the right side please contact
our system administrator: [webmaster at ifis.uni-luebeck.de](mailto:webmaster@ifis.uni-luebeck.de).

Query $p = /q[a[./b[c/*/d]/b[c/d]/b[c/d]]]$

Query $q = /q[a[./b[c/*/d]/b[c/d]]]$

$p \subseteq q$

XPath-Query p:

XPath-Query q:

$p = a/*//a$

$q = a//*/a$

a
|
*
||
a

a
||
*
|
a

Clearly, p is equivalent to q .
(containment holds in both directions)

But, no homomorphisms exist.

Cave If we add the star (*) then homomorphism need not exist!

→ there are $p, q \in \text{XPath}(/, //, [], *)$ such that $p \subseteq_0 q$ and there is **no** homomorphism from Q to P ☹

3. XPath Containment Test

Automaton Technique

Recall: for any DTD there is a tree automaton which recognized the corresponding trees.

Similarly, for any XPath(/, //, [], *, |) expression *ex* we can construct a (*non-deterministic* bottom-up) tree automaton *A* which accepts a tree if and only if *ex* matches the tree.

Theorem

Containment test of XPath(/, //, [], *, |) in the presence of DTDs can be solved in EXPTIME.



Exponential (deterministic) time

Blow-up due to non-determinism of tree automaton.

BUT: no hope for improvement:

The problem is actually **complete** for EXPTIME.

3. XPath Containment Test

Automaton Technique

Recall: for any DTD there is a tree automaton which recognized the corresponding trees.

Similarly, for any XPath($/$, $//$, $[]$, $*$, $|$) expression ex we can construct a (*non-deterministic* bottom-up) tree automaton A which accepts a tree if and only if ex matches the tree.

Theorem

Containment test of XPath($/$, $//$, $[]$, $*$, $|$) in the presence of DTDs can be solved in EXPTIME.

Union of automata

Intersection of automata
(“product construction”)

Proof Idea construct automaton for **all possible counter example trees**. Test if this automaton accepts any tree.

3. XPath Containment Test

Automaton Technique

Recall: for any DTD there is a tree automaton which recognized the corresponding trees.

Similarly, for any XPath($/$, $//$, $[]$, $*$, $|$) expression ex we can construct a (*non-deterministic* bottom-up) tree automaton A which accepts a tree if and only if ex matches the tree.

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Containment test of XPath($/$, $//$, $[]$, $*$, $|$) in the presence of DTDs can be solved in EXPTIME.

Emptiness test
for automata

Proof Idea construct automaton for **all possible counter example trees**. Test if this automaton accepts any tree.

→ Automata can also be Tested for **Finiteness**!

Is $p \subseteq_0 q$, for all trees but finitely many exceptions?

↑
solvable!

3. XPath Containment Test

Chase Technique -- 1979 relational DB's to check query containment
in the presence of *integrity constraints*.

Example

DTD $E =$

root	\rightarrow	a^*
a	\rightarrow	$b^* \mid c^*$
b	\rightarrow	d^+c^+
c	\rightarrow	$b?c?$

(“the chase”
extends the relational
homomorphism
technique)

$p = /a/b//d$

$q = /a//c$

Is p contained in q for E -conform documents?

First Possibility: use tree automata

- \rightarrow Construct automata A_p, A_q, A_E
- \rightarrow Construct B_q for the complement of A_q (=not q)
- \rightarrow Intersect B_q with A_p with A_E (gives automaton A)
- \rightarrow Check if A accepts any tree.

3. XPath Containment Test

Chase Technique -- 1979 relational DB's to check query containment
in the presence of *integrity constraints*.

Example

DTD $E =$

root	\rightarrow	a^*
a	\rightarrow	$b^* \mid c^*$
b	\rightarrow	d^+c^+
c	\rightarrow	$b?c?$

(“the chase”
extends the relational
homomorphism
technique)

$p = /a/b//d$

$q = /a//c$

Is p contained in q for E -conform documents?

Each b-element has a d-child and a c-child

\rightarrow *constraints*

c1: $b \rightarrow d$

c2: $b \rightarrow c$

a
|
b
||
d

p 's pattern tree

3. XPath Containment Test

Chase Technique -- 1979 relational DB's to check query containment
in the presence of *integrity constraints*.

Example

DTD $E =$

root	\rightarrow	a^*
a	\rightarrow	$b^* \mid c^*$
b	\rightarrow	d^+c^+
c	\rightarrow	$b?c?$

(“the chase”
extends the relational
homomorphism
technique)

$p = /a/b//d$

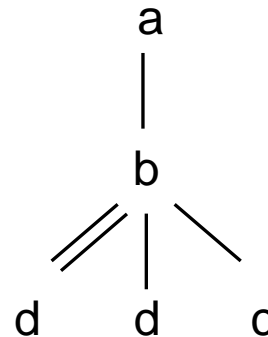
$q = /a//c$

Is p contained in q for E -conform documents?

Each b -element has a d -child and a c -child
 \rightarrow constraints

$c1: b \rightarrow d$

$c2: b \rightarrow c$



p 's pattern tree
after *chasing* with $c1, c2$

3. XPath Containment Test

Chase Technique -- 1979 relational DB's to check query containment
in the presence of *integrity constraints*.

Example

DTD $E =$

root	\rightarrow	a^*
a	\rightarrow	$b^* \mid c^*$
b	\rightarrow	d^+c^+
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(“the chase”
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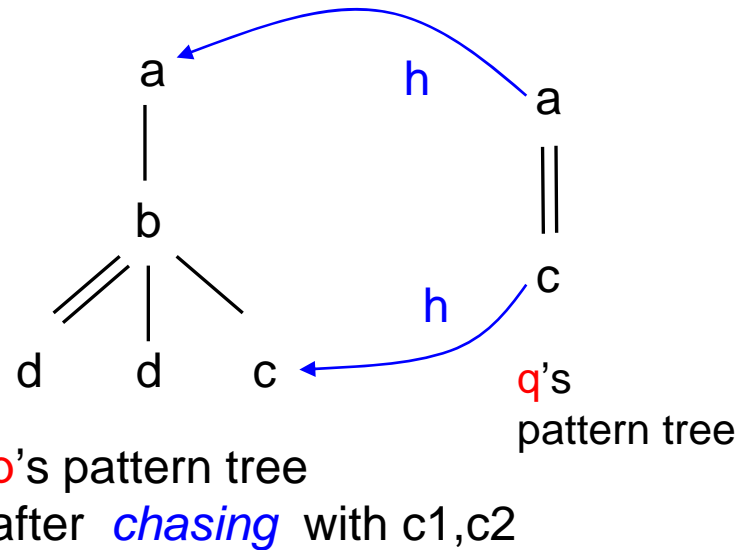
Is p contained in q for E -conform documents?

Each b -element has a d -child and a c -child
 \rightarrow constraints

$c1: b \rightarrow d$

$c2: b \rightarrow c$

p is contained in q
in the presence
of the DTD E



END
Lecture 8