XML and Databases

Lecture 5
XML Validation using Automata

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CSE@UNSW -- Semester 1, 2010

Outline

1. Recap: deterministic Reg Expr's
   / Glushkov Automaton
2. Complexity of DTD validation
3. Beyond DTDs: XML Schema and RELAX NG
4. Static Methods, based on Tree Automata

Previous Lecture

XML type definition languages

want to specify a certain subset of XML doc's = a "type" of XML documents

Remember
The specification/type definition should be **simple**, so that

→ a *validator* can be built automatically (and efficiently)
→ the *validator* runs efficient on any XML input

(similar demands as for a *parser*)

➔ Type def. language must be SIMPLE!

(similarly: parser generators use EBNF or smaller subclasses: LL / LR)

O(n^3) parsing

XML Type Definition Languages

DTD (Document Type Definition, W3C)
Originated from SGML. Now part of XML

→DTD may appear at the beginning of an XML document

Reg Exprs must be *deterministic* *(=1-unambiguous)*

same!!

"Unique Particle Attribution"

XML Schema (W3C)
Now at version 1.1
HUGE language, many built-in simple types

→Schemas themselves: written in XML

See the "Schema Primer" at http://www.w3.org/TR/xmlschema-0/

RELAX NG (Oasis)
For tree structure definition, more powerful than Schemas&DTDs

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XML Type Definition Languages

DTD (Document Type Definition)

```
<!DOCTYPE root-element [ doctype declaration …]>
```

```
<!ELEMENT element-name content-model>

content-models
• EMTPY
• ANY
• (#PCDATA | elem-name_1 | … | elem-name_n)*
• deterministic Reg Expr over element names
```

```
<!ATTLIST element-name attr-name attr-type attr-default ..>

Types: CDATA, (v1|..), ID, IDREFs
Defaults: #REQUIRED, #IMPLIED, "value", #FIXED
```

XML Type Definition Languages

DTD (Document Type Definition)

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• ANY
• (#PCDATA | elem-name_1 | … | elem-name_n)*
• deterministic Reg Expr
```

Most interesting / challenging aspect of DTDs

```
<!ATTLIST element-name attr-name attr-type attr-default ..>

Types: CDATA, (v1|..), ID, IDREFs
Defaults: #REQUIRED, #IMPLIED, "value", #FIXED
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Summary

In order to check whether a (large) document
is **valid** wrt to a given DTD ("it validates")
you need to

→ check if children lists match the given Reg Expr's

This can be done *efficiently*, using **finite-automata (FAs)**!

To check if a Reg Expr e is **allowed in a DTD**
we have to construct a particular finite automaton: the **Glushkov automaton**.
Glu(e)

Glu(e) must be *deterministic*.

Note If Glu(e) is *deterministic*, then its size (# transitions) is *linear* in size(e)!

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Question Can you explain why this is the case?

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Glu(e) must be *deterministic*.

Note If Glu(e) is *deterministic*, then its size (# transitions) is ~~*linear*~~ in size(e)!

Question Can you explain why this is the case?

not correct:
linear in **size(e) * #letters(e)**

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More Notes

(1) From a *deterministic* FA you **cannot** obtain a
deterministic (= 1-unambiguous) regular expression!!

Example: e = (a | b)* a (a | b) ← NO 1-unambiguous reg exp exists for e

Question
Can you build a **Deterministic Automaton**
for the expression e?

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More Notes

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Example: e = (a | b)* a (a | b) ← NO 1-unambiguous reg exp exists for e

Deterministic Automaton for e:
E.g., first nondeterministic FA, then determinize (subset construction)
CAVE: can cause exponential size blow-up!

Other important constructions on Finite Automata:

→ Union (easy)
→ Intersection (product construction)
→ Complementation

Question
Size blow-up for these?

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More Notes

*For more details:
See paper by Brüggemann-Klein,
Linked from the course web-page.*

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deterministic (= 1-unambiguous) regular expression!!

Example: e = (a | b)* a (a | b) ← NO 1-unambiguous reg exp exists for e

(2) Glu(e) is closely related to → Thomson(e) [remove ε-transitions]
and to → Berry/Sethi(e) [same]
and → Brzozowski(e)

To check if a Reg Expr e is **allowed in a DTD**
we have to construct a particular finite automaton: the **Glushkov automaton**.
Glu(e)

Note If Glu(e) is *deterministic*, then its size is *linear* in **size(e) * #letters(e)**

Glushkov automaton Glu(e)

Each letter-position in the Reg Expr e becomes one state of Glu;
plus, Glu has one extra begin state.

FIRST(e) = all possible begin positions of words matching e

e.g. FIRST(R (E | G) (EX)*) = { R_1 }

Glushkov's automaton

R (E | G) (E X) *

Following slides from: http://www.cs.ut.ee/~varmo/tday-rouge/tammeoja-slides.pdf

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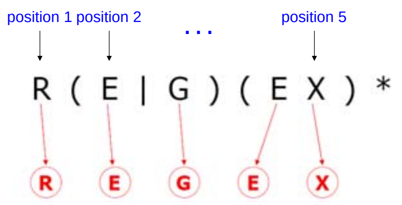
Glushkov's automaton

- Character in RE = **state** in automaton



Glushkov's automaton

- Character in RE = **state** in automaton
  + one state for the beginning of the RE

R (E | G) (E X) *

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- **Transitions** show which characters/positions can precede each other

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FOLLOW(e, x) = all possible positions following position x in e

e.g. FOLLOW(R (E | G) (EX)*, R_1) = { E_2, G_3 }

→ From state "R_1": add E-transition to E_2
G-transition to G_3

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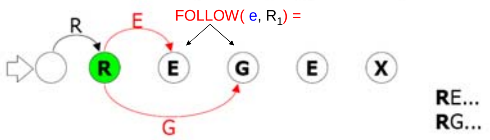

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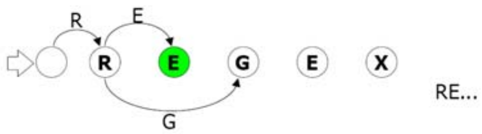

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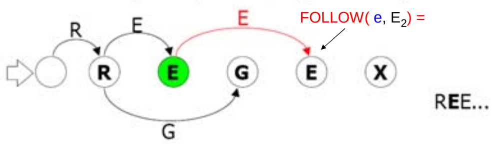

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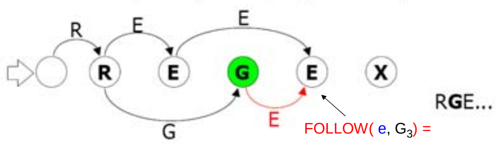

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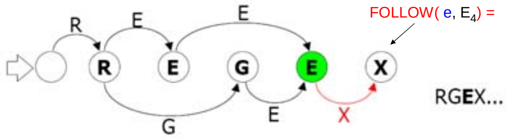

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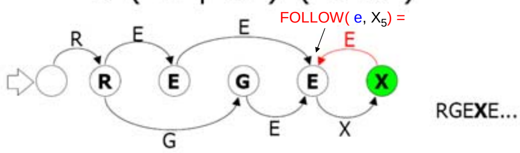

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Glushkov automaton G(e)

Each position in the Reg Expr e becomes one state of G;
plus, G has one extra begin state.

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e.g. FIRST(R (E | G) (EX)*) = { R_1 }

FOLLOW(e, x) = all possible positions following position x in e

e.g. FOLLOW(R (E | G) (EX)*, R_1) = { E_2, G_3 }

→ From state "R_1": add E-transition to E_2
G-transition to G_3

LAST(e) = all possible *end* positions of words matching e

e.g. LAST(R (E | G) (EX)*) = { E_2, G_3, X_5 }

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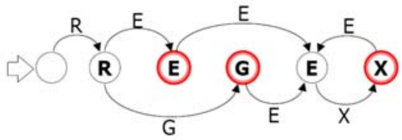

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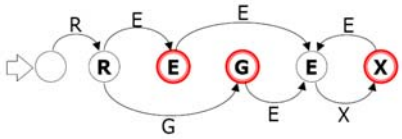

Is this automaton *deterministic* ??

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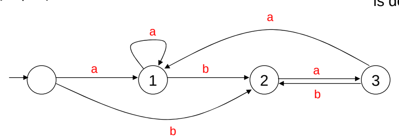
Glushkov automaton G(e)

Another example

(a* | ba)*

This FA
is deterministic.



Which of these is deterministic?

→ (ab) | (ac)
→ a (b | c)
→ a(a | b)*ac

Glushkov automaton G(e)

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plus, G has one extra begin state.

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e.g. FIRST(R (E | G) (EX)*) = { R_1 }

FOLLOW(e, x) = all possible positions *following* position x in e

LAST(e) = all possible *end* positions of words matching e

Naïve implementation: O(m^3) time, where m = size(e)

(for each position: computing FOLLOW goes through every position
at each step, needs to compute *union* → O(m*m*m)

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Not really needed. Can be improved to O(m^2)

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Naïve implementation: O(m^3) time, where m = size(e)

(for each position: computing FOLLOW goes through every position
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Can be improved to
O(size(e) + size(G(e))) ⇐ Not really needed. Can be improved to O(m^2)

Glushkov automaton G(e)

Note If G(e) is *deterministic*, then its size (# transitions) is *quadratic* in size(e)!

Linear in size(e) * #letters(e), if G(e) is deterministic!

→ O(size(e) * #letters(e))

Can this be improved? → E.g., to O(size(e) * log(#letters(e)) ?
→ Are there known *lower bounds*?

Naïve implementation: O(m^3) time, where m = size(e)

(for each position: computing FOLLOW goes through every position
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O(size(e) + **size(G(e))**) ⇐ Not really needed. Can be improved to O(m^2)

To avoid these expensive running times

DTD requires that `FA=G(e)` must be *deterministic*!

m = size(e)
n = length(w)

Total Running time O(m + n)

If s = #letters(e) is assumed fixed
(not part of the input)

Otherwise: O(ms + n)

How can you **implement** a regular expression?

Input: Reg Expr e, string w
Question: Does w match e?

deterministic FA: run on w takes
time **linear** in length(w)

Algorithm

```
FA = BuildFA(e);
DFA = BuildDFA(FA);
```

Size of FA is linear in size(e)=m
Size of DFA is exponential in m

Total Running time O(2^m + n)

Unrestricted Reg Expr e → Other alternative: O(mn)

Summary

Deterministic (1-unambiguous) content models give rise to
efficient matching algorithms.

(they avoid O(mn)
or O(2^m + n) complexities)

Disadvantages

→ Hard to know whether given reg expr is OK (deterministic)

→ Det. reg exprs are NOT closed under union. (not so nice.)

Question Can you see why?

Maybe, can find det reg exprs, such that
their union equals (a | b)* a (a | b) ?

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Now that we know how the check all the different content-models (in particular det. Reg Expr's) how to build a full validator for a given DTD?

elem-name_1 → RegExpr_1
elem-name_2 → RegExpr_2
…
elem-name_k → RegExpr_k

} Automata A_1, A_2, …, A_k

The Validation Problem
Given a DTD T and a document D, is D valid wrt T?

Top-Down Implementation
→ at element node w. label elem-name_i, run automaton A_i

→ check attribute constraints
→ check ID/IDREF constraints

Total Running time (Given A_1, A_2, …, A_k) *linear in the sum of size(DTD) * #letters(DTD) and size of document*: O(size(D) * #letters(D) + size(T))

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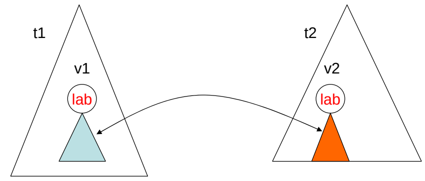
DTDs have the

"**label-guarded subtree exchange**" property:

t1, t2 trees in a DTD language T
v1 node in t1, labeled "lab"
v2 node in t2, labeled "lab"

aka "local"
→ content model only depends on label of parent

trees obtained by exchanging the subtrees rooted at v1 and v2 are also in T



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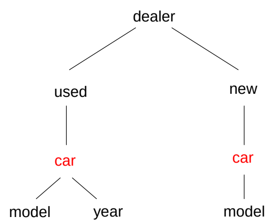
Beyond DTDs

Often, the expressive power of DTDs is *not sufficient*.
Problem each element name has precisely one content-model in a DTD.
Would like to distuingish, depending on the context (parent).



car has different structure, in different contexts.

(content model depends on the parent node)

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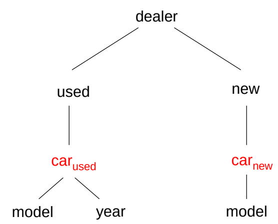

"specialization"

car has different structure, in different contexts.

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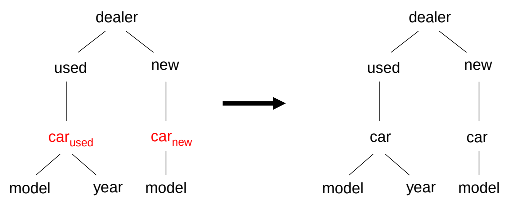
Specialized DTDs

dealer → used, new
used → $(car_{used})^*$
new → $(car_{new})^*$
car_{used} → model, year
car_{new} → model



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Specialized DTDs

dealer → used, new
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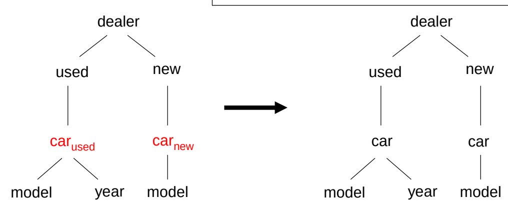
New notation. Use *capitalized* TYPE Names

Dealer → dealer [Used, New]
Used → used [$(Car_{used})^*$]
New → new [$(Car_{new})^*$]
Car_{used} → car [Model, Year]
Car_{new} → car [Model]



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Not local

Let us call this new concept a "grammar".

the "*local*" restriction

A grammar G is **local**, if
for any label[RegExpr_1], label[RegExpr_2] present in G
it holds that RegExpr_1 = RegExpr_2.

By definition: Every DTD is a local grammar, and vice versa.

New notation. Use *capitalized* TYPE Names

Dealer → dealer [Used, New]
Used → used [(Car_{used})*]
New → new [(Car_{new})*]
competing Car_{used} → **car** [Model, Year]
Car_{new} → **car** [Model]

Alternatively:

Call two TYPE Names T1 and T2 "competing"
if they have the same element name (but not identical rules)

Classes of Grammars

local no competing TYPE names! (DTDs)

single-type TYPE names in the *same content model* do not compete! (XML Schema's)

regular no restriction. (RELAX NG)

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Question Are there single-type grammars (XML Schemas) which cannot be expressed by local grammars (DTDs).

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New notation. Use *capitalized* TYPE Names

competing

Person → person [PersonName, Gender, Spouse?, Pet*]
PersonName → name [First, Last]
Pet → pet [Kind, PetName]
PetName → name [#PCDATA]
…

but are not in same content model!

Question Are there single-type grammars (XML Schemas) which cannot be expressed by local grammars (DTDs).
YES!

Classes of Grammars

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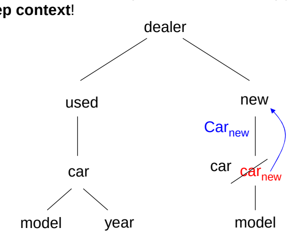
Through the use of TYPE Names (nonterminals / states) you can distinguish **deep context**!



"specialization" through parent

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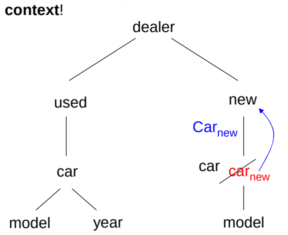
Can we model context that is far away from the specialized node?



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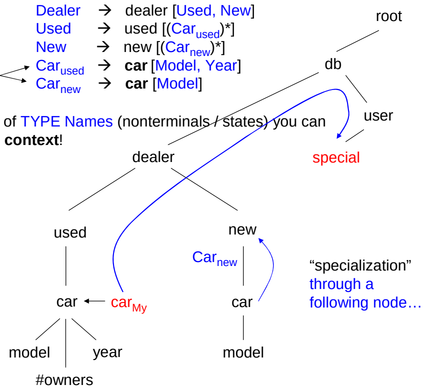

DB → db [Dealer, User]
Dealer → dealer [Used, New]
Used → used [(Car$_{used}$)*]
New → new [(Car$_{new}$)*]
Car$_{used}$ → car [Model, Year]
Car$_{new}$ → car [Model]

Doc → root [DB | sDB]
sDB → db [sDealer, sUser]
sDealer → dealer [sUsed, New]
sUsed → used [(sCar$_{used}$)*]
sCar$_{used}$ → car [Model, Own, Year]

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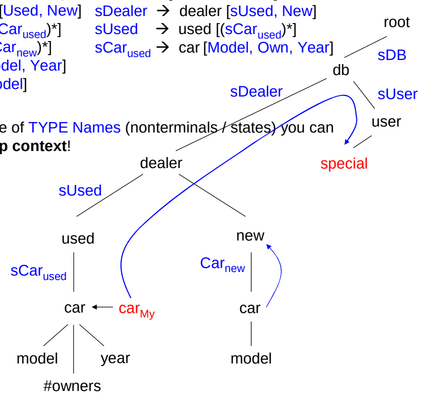
Sure!



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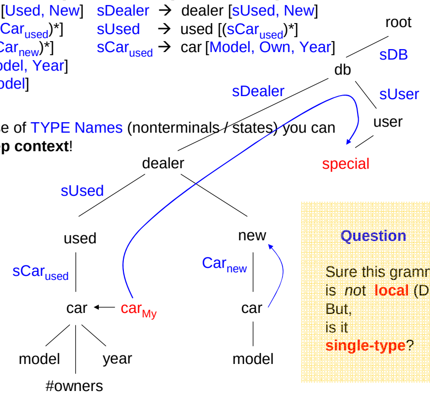
sDealer → dealer [sUsed, New]
sUsed → used [(sCar$_{used}$)*]
sCar$_{used}$ → car [Model, Own, Year]

Through the use of TYPE Names (nonterminals / states) you can distinguish **deep context**!

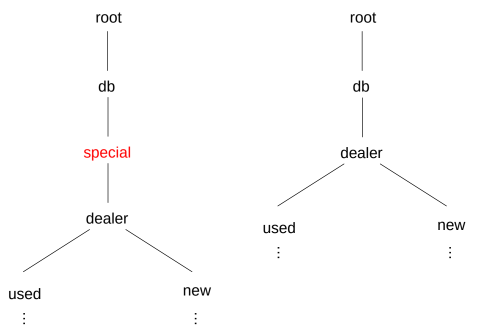
Can we model context that is far away from the specialized node?

Sure!



Question

Sure this grammar is *not* **local** (DTD).
But, is it **single-type**?



Question Is this grammar **single-type**?

Classes XML Type Formalisms

local no competing TYPE names! (**DTDs**)

single-type TYPE names in the *same content model* do not compete! (**XML Schema's**)

regular no restriction… (**RELAX NG**)

Increasing Expressivness
of defining sets of trees ("tree languages")

Questions

Given two DTDs D1 and D2 can we check if
→ all documents valid for D1 are also valid for D2? (DTD inclusion problem)
→ D1 and D2 describe the same set of documents? (DTD equality problem)

Given a Relax NG grammar G, can we check if
→ there exists any document that is valid for G? (emptiness problem)
→ there is a document valid for G *and* valid for G2? (intersection & emptiness)

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If we can do it for **regular tree** grammars, then also works for single-type/local!!

55

All of the checks can be done automatically, for **regular tree** grammars!

↑ equivalent to tree **automata**

Tree Automata: very powerful framework,

→ Have all the good properties of string automata!
→ Yet, they are more expressive!

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→ Have all the good properties of string automata!
→ Yet, they are more expressive!

Note

String automata are **not** sufficient to check DTDs / Schemas!
Even if we only consider well-bracketed strings!

Example 1	Example 2
c → c[a, c, b]	a → a[c, a]
a → empty	a → a[a, b]
b → empty	a / b / c → empty
c → empty	

57

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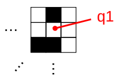
↑ equivalent to tree **automata**

constant memory computation

Finite-state automata are important:

→ Think you are in a maze, with only fixed memory and you can only read the maze (cannot mark anything).

Model by finite automaton. In state q1, (to [N|S|E|W], ■ (wall)) → (q2, [N|S|E|W])
q2, (to [N|S|E|W], □ (empty)) → (q3, [N|S|E|W])



Can an automaton search the maze?

58

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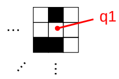


Can an automaton search the maze?

No!! → need markers ("pebbles").
How many? 5? 2?

59

All of the checks can be done automatically, for **regular tree** grammars!

↑ equivalent to tree **automata**

constant memory computation

Finite-state automata are important:

In our context, e.g., for

→ KMP (efficient string matching) [Knuth/Morris/Pratt]
generalization using automata. Used, e.g., in `grep`

→ Compression

→ Static analysis of schemas & queries
(= "everything you can do *before* running over the actual data")

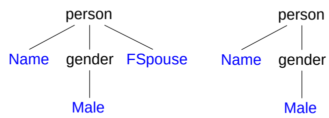
60

4. Static Methods, based on Tree Automata

Person → MPerson | FPerson
MPerson → person [Name, gender[Male], FSpouse?]
FPerson → person [Name, gender[Female], MSpouse?]

Regular Tree Grammar

Rules of the form TypeName → Tree ← Leaves may be labeled by TypeNames



Alternatively, regular tree languages are defined by **Tree Automata**.

state, element-name → state1, state2 ← conventionally, defined for *binary/ranked* trees.

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Given grammars D1 and D2 can we check if
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→ does there exists any document that is valid for D1? (emptiness problem)
→ there is a document valid for D1 *and* valid for D2? (intersection & emptiness)

ALL these checks are possible for ***regular tree grammars***!!

→ hence, they are also solvable for DTDs / XML Schemas / RELAX NG's

(1) use binary tree encodings
(2) translate XML Type Definition to a Tree Grammar (easy)

Alternatively, regular tree languages are defined by **Tree Automata**.

state, element-name → state1, state2 ← conventionally, defined for *binary/ranked* trees.

4. Static Methods, based on Tree Automata

Given grammars D1 and D2 can we check if
→ all documents valid for D1 are also valid for D2? (*inclusion problem*)
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→ there is a document valid for D1 *and* valid for D2? (intersection & emptiness)

ALL these checks are possible for ***regular tree grammars***!!

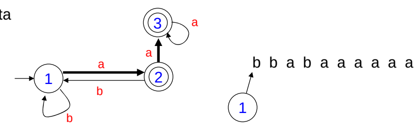
→ The checks above give rise to
very powerful optimization procedures for XML Databases!

For example:
documents d_1, d_2, …, d_n are valid for your schema “Small_xhtml”.

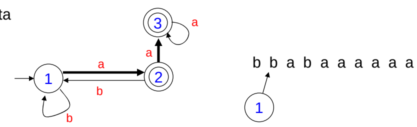
Are they also valid for schema XHTML?

→ Check *inclusion problem* for Small_html and XHTML!

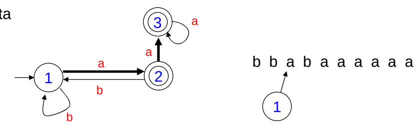
Automata on words



Automata on words



Automata on words



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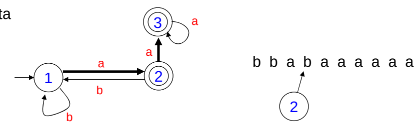
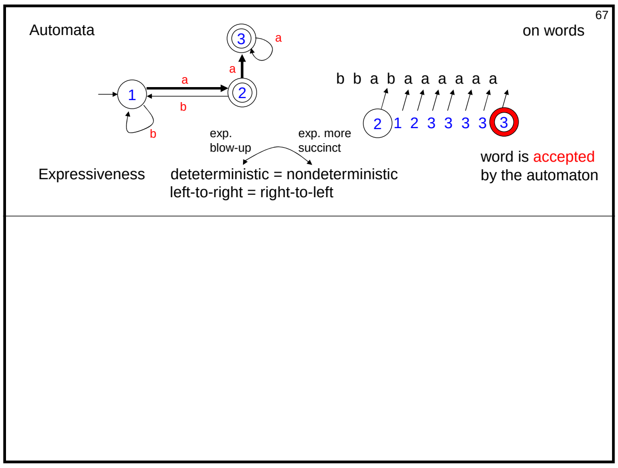
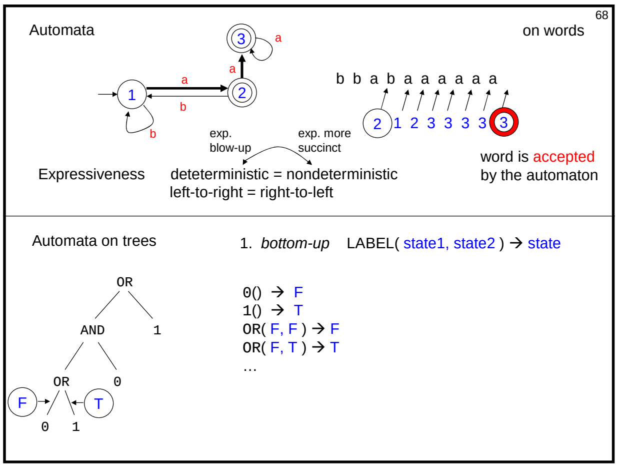



67
Automata
on words
a
a
a
b
b
1
2
3
b b a b a a a a a a
2 1 2 3 3 3 3 3
exp. blow-up
exp. more succinct
Expressiveness
deteterministic = nondeterministic
left-to-right = right-to-left
word is accepted by the automaton


68
Automata
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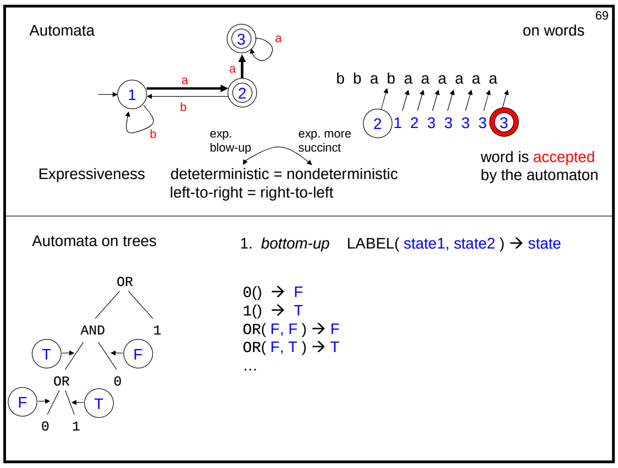
Automata on trees
1. bottom-up LABEL(state1, state2) → state
0() → F
1() → T
OR(F, F) → F
OR(F, T) → T
…
OR
AND
1
OR
0
F
T
0
1


69
Automata
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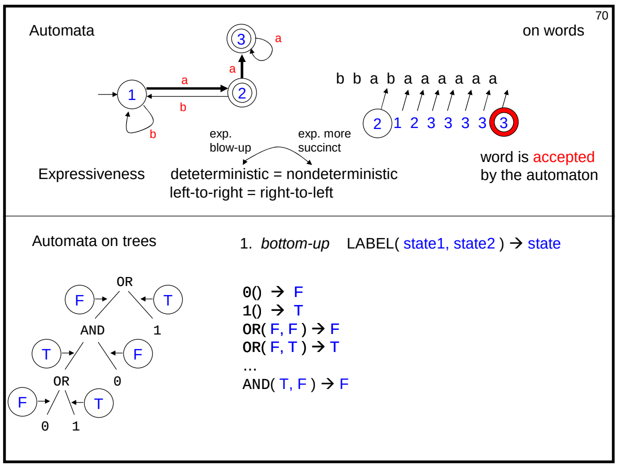
70
Automata
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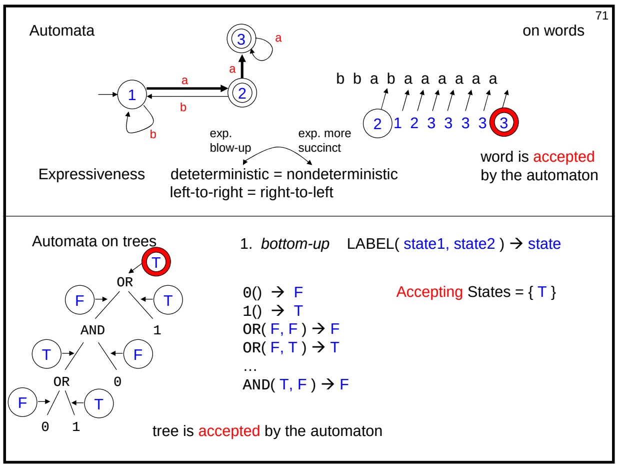
71
Automata
on words
b b a b a a a a a a
2 1 2 3 3 3 3 3
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1() → T
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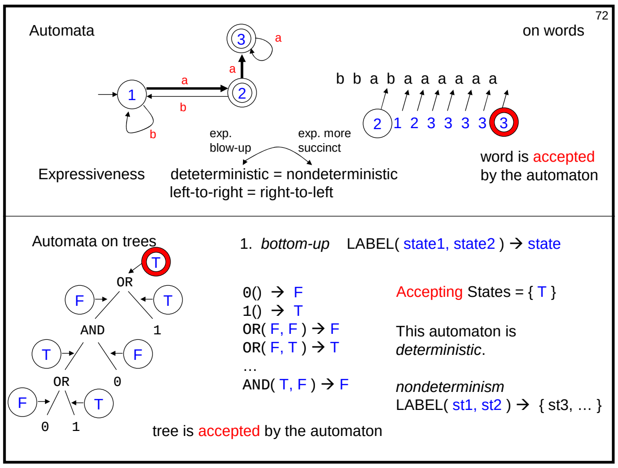
…
AND(T, F) → F
Accepting States = { T }
tree is accepted by the automaton


72
Automata
on words
b b a b a a a a a a
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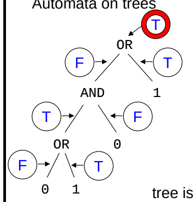
1() → T
OR(F, F) → F
OR(F, T) → T
…
AND(T, F) → F
Accepting States = { T }
This automaton is deterministic.
nondeterminism
LABEL(st1, st2) → { st3, … }
tree is accepted by the automaton

73

Question

How much memory do you need exactly, to run such a bottom-up tree automaton?

Automata on trees



1. *bottom-up* LABEL(state1, state2) → state

0() → F
1() → T
OR(F, F) → F
OR(F, T) → T
...
AND(T, F) → F

Accepting States = { T }

This automaton is *deterministic*.

nondeterminism
LABEL(st1, st2) → { st3, … }

tree is accepted by the automaton

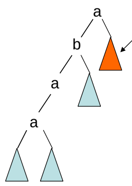
74

Similarly as for word automata:

> For every nondeterministic bottom-up tree automaton there is an equivalent deterministic bottom-up tree automaton.

Again, the construction can cause exponential size blow-up.

2. *top-down* state, LABEL → (state1, state2)



must contain a $-leaf

a,b = binary node labels
e,$ = leaf node labels

"top-most a-node on the left-most path must have a right-subtree which contains a $-node."

75

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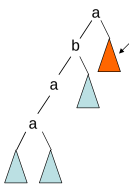

must contain a $-leaf

a,b = binary node labels
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"top-most a-node on the left-most path must have a right-subtree which contains a $-node."

begin, a → (any, find$)
begin, b → (begin, any)
find$, a/b → { (find$, any), (any, find$) } ← nondeterministic
find$, $ → ACC
find$, e → REJ
any, a/b → (any,any)
any, $/e → ACC

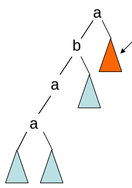
→ accept tree if there exists an accepting run (= all leaves go to ACC)

76

> For every nondeterministic bottom-up tree automaton there is an equivalent deterministic bottom-up tree automaton.

Question
Can you find an equivalent *bottom-up* automaton for this example?

2. *top-down* state, LABEL → (state1, state2)



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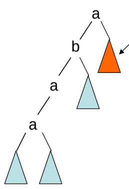
→ accept tree if there exists an accepting run

77

> For every nondeterministic bottom-up tree automaton
> → there is an equivalent deterministic bottom-up tree automaton, and
> → there is an equivalent nondeterministic *top-down* tree automaton.

→ Yes! you can… ☺

2. *top-down* state, LABEL → (state1, state2)



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Question
Is there an equivalent deterministic top-down automaton??

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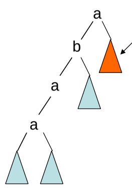

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79

For every nondeterministic bottom-up tree automaton
→ there is an equivalent deterministic bottom-up tree automaton, and
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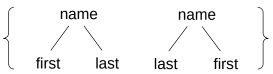
Question
Is there an equivalent deterministic top-down automaton??

→ NO! ☹ -- not a good model..



This set of two trees canNOT be recognized
by any determinstic top-down tree automaton!!

Why?

80

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Questions

What about **local** tree languages (defined by DTDs).
→ Can they be accepted by deterministic top-down automata?

What about **single-type** tree languages (defined by XML Schema's)
→ Can they be accepted by deterministic top-down automata?

81

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→ Can they be accepted by deterministic top-down automata?

What about **single-type** tree languages (defined by XML Schema's)
→ Can they be accepted by deterministic top-down automata?

Yes!
Hence, there is **no DTD / Schema** for { name[first,last], name[last,first] }

82

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→ there is an equivalent deterministic bottom-up tree automaton, and
→ there is an equivalent nondeterministic *top-down* tree automaton.

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Is there an equivalent deterministic top-down automaton??

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Nevertheless, **XML Schemas** are a subclass of
deterministic top-down automata.

Questions
What is the reasoning behind this?
Similarity to restriction to deterministic reg. expressions?

Recall: Deterministic (1-unambiguous) content models give rise to *efficient matching algorithms*.

(they avoid O(mn)
O(2^m + n) complexities)

83

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Similarity to restriction to deterministic reg. expressions?

In total, given a DTD D, we can build one
deterministic top-down automaton of size O(size(D) * #letters(D))

Thus, matching time is inside **O(m^2 + n)**

84

For every deterministic bottom-up tree automaton
there exists a minimal unique equivalent one!

→ Equivalence is decidable

In fact, YOU have already produced
minimal bottom-up tree automata!

The minimal DAG of a tree t can be seen as the minimal unique
deterministic BU tree automaton that only accepts the tree t.

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→ Equivalence is decidable

In fact, YOU have already produced
minimal bottom-up tree automata!

The minimal DAG of a tree t can be seen as the minimal unique
tree automaton that only accepts the tree t.

Question

Given deterministic BU tree automaton.
How expensive (complexity) to find unique mininmal one?

As for DFAs: merge equivalent states
Typically: quadratic running time. → Hopcroft's algorithm, O(n log n)

More generally, once we have partitioned on a block

and an input symbol, we need never partition on that

block and input symbol again until the block is

split and then we need only partition on one of

the two subblocks. Since the time needed to

partition on a block is proportional to the

transitions into the block and since we can always

select the half with fewer transitions, the total

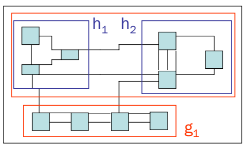
number of steps in the algorithm is bounded by

n log n.

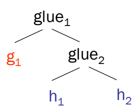
See
ftp://db.stanford.edu/pub/cstr/reports/cs/tr/71/190/CS-TR-71-190.pdf

Tree Automata are a very useful concept in CS!

→ Heavily used in verification
"Derive a property of a complex object
from the properties of its constituents…"



$g = glue_1(g_1, glue_2(h_1, h_2))$



→ Do all graphs / chip-layouts produced in this way, have property P?

Use the hierarchical construction history of an object, in order to
work on a "parse" tree instead of a complex graph.
From there, use tree automata. ☺

Many NP-complete graph problems become
tractable on "bounded-treewidth " graphs!

XML Tree Automata play crucial rule for

→ Efficient validators against XML Types

→ Optimizations If doc1 is of TYPE1, then no need to validate
against TYPE2, if we know TYPE2 included in TYPE1

- if only "slightly different" then only need to validate "there"
- incremental validation against updates
- etc, etc.

→ Efficient query evaluators, use richer automata which can
select nodes and produce query answers

→ Optimizations If answer of QUERY1 is in cache, then no need to
evaluate QUERY2, if "included" in QUERY1.

- if every possible answer set to QUERY1 (of TYPE X)
is EMPTY, then no need to evaluate on the real data!

→ XML Type Checking for Programming Languages

The Future

In 5-10 years from now: ☺

You can write a function in Programming Language X

```
Function foo(XML document D: TYPE1):  TYPE2
{
    traverse D
        & compute output;
    .
    .
    .
    return output
}
```

Compiler (XML Type Checker) will complain, if your
function does not compute documents of TYPE2.

→ If no complaint, then **guaranteed**:
ALL outputs are ALWAYS of correct type!!)

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Experimental PL's
In this direction:
→CDuce
→XDuce

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function does not compute documents of TYPE2.

→ If no complaint, then correct type **guaranteed**.

Compilers will **have** to be able to give *static guarantees* about input/output
behaviour of program!

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END
Lecture 5