COMP4161
Advanced Topics in Software Verification

INV & Exam Prep

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T3/2024

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Practice with invariants!

Practice with invariants!

Recall:

- ➜ invariants are needed to automate the application of hoare rules
- ➜ they are used by the weakest precondition calculus to deal with loops

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Recall:

➜ an invariant needs to be "enough" (to prove the postcondition)
➜ an invariant needs to be an invariant

Practice with invariants!

Recall:

- ➜ invariants are needed to automate the application of hoare rules
- ➜ they are used by the weakest precondition calculus to deal with loops

Recall:

- ➜ an invariant needs to be “enough” (to prove the postcondition)
- ➜ an invariant needs to be an invariant
  - ➜ “true before the loop”
  - ➜ “if true at the start of an iteration, still true after one iteration”

Weakest precondition - recall

$$\{\ P\ \}\ \ i_0;\ i_1;\ i_2;\ \ \{\ Q\ \}$$

Weakest precondition - recall

$$(P \Longrightarrow pre\ (i_0;\ i_1;\ i_2;)\ Q) \Longrightarrow \{\ P\ \}\ \ i_0;\ i_1;\ i_2;\ \ \{\ Q\ \}$$

Weakest precondition - recall

$(P \Longrightarrow pre\ (i_0;\ i_1;\ i_2;)\ Q) \Longrightarrow \{\ P\ \}\ i_0;\ i_1;\ i_2;\ \{\ Q\ \}$

$\{\ P\ \}$

$i_0;$

$i_1;$

$i_2;$

$\{\ Q\ \}$

Weakest precondition - recall

$(P \Longrightarrow pre\ (i_0;\ i_1;\ i_2;)\ Q) \Longrightarrow \{\ P\ \}\ i_0;\ i_1;\ i_2;\ \{\ Q\ \}$

$\{\ P\ \}$

$i_0;$

$i_1;$

$pre\ i_2\ Q$

$i_2;$

$\{\ Q\ \}$

Weakest precondition - recall

$(P \Longrightarrow pre\ (i_0;\ i_1;\ i_2;)\ Q) \Longrightarrow \{\ P\ \}\ i_0;\ i_1;\ i_2;\ \{\ Q\ \}$

$\{\ P\ \}$

$i_0;$

$pre\ i_1\ (pre\ i_2\ Q)$

$i_1;$

$pre\ i_2\ Q$

$i_2;$

$\{\ Q\ \}$

Weakest precondition - recall

$(P \Longrightarrow pre\ (i_0;\ i_1;\ i_2;)\ Q) \Longrightarrow \{\ P\ \}\ i_0;\ i_1;\ i_2;\ \{\ Q\ \}$

$\{\ P\ \}$

$pre\ i_0\ (pre\ i_1\ (pre\ i_2\ Q)) = pre\ i_1;\ i_2;\ i_3;\ Q$

$i_0;$

$pre\ i_1\ (pre\ i_2\ Q)$

$i_1;$

$pre\ i_2\ Q$

$i_2;$

$\{\ Q\ \}$

Invariant - recall

$\{ P \}$

WHILE b

DO

c

OD

$\{ Q \}$

Invariant - recall

$\{ P \}$

??

WHILE b

DO

c

OD

$\{ Q \}$

Invariant - recall

$\{ P \}$

$WHILE\ b\ INV\ I$

DO

c

OD

$\{ Q \}$

?? $pre\ (WHILE\ b\ INV\ I\ DO\ c\ OD) = I$

Invariant - recall

$\{ P \}$

$P \Longrightarrow I$ ("true before the loop")

?? $pre\ (WHILE\ b\ INV\ I\ DO\ c\ OD) = I$

$WHILE\ b\ INV\ I$

DO

c

OD

$\{ Q \}$

Invariant - recall

$\{ P \}$

$P \implies I$ ("true before the loop")

?? $pre\ (WHILE\ b\ INV\ I\ DO\ c\ OD) = I$

$WHILE\ b\ INV\ I$

$I \wedge b \implies pre\ c\ I$

("if true at the start of an iteration,")

DO ("still true after one iteration")

c

OD

$\{ Q \}$

Invariant - recall

$\{ P \}$

$P \implies I$ ("true before the loop")

?? $pre\ (WHILE\ b\ INV\ I\ DO\ c\ OD) = I$

$WHILE\ b\ INV\ I$

$I \wedge b \implies pre\ c\ I$

("if true at the start of an iteration,")

("still true after one iteration")

DO

c

OD

$I \wedge \neg b \implies Q$ ("enough")

$\{ Q \}$

Example 1

$\{ \; a \geq 0 \;\wedge\; b \geq 0 \; \}$
$A := 0;$
$B := 0;$

WHILE $A \neq a$
DO
  $B := B + b;$
  $A := A + 1$
OD

Example 1

$\{\ a \geq 0\ \wedge\ b \geq 0\ \}$

$A := 0;$

$B := 0;$

$A =$	0	1	2	3	4	...
$B =$	0	b	b+b	b+b+b	b+b+b+b	...

WHILE $A \neq a$

DO

  $B := B + b;$

  $A := A + 1$

OD

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$\{\ B = b * a\ \}$

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$A := 0;$
$B := 0;$
INV { B = b * A}
WHILE $A \neq a$
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OD
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$A =$	0	1	2	3	4	...
$B =$	0	b	b+b	b+b+b	b+b+b+b	...

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Example 1

$\{ a \geq 0 \ \wedge \ b \geq 0 \}$
$A := 0;$
$B := 0;$ $\quad 0 = b * 0 \quad \checkmark$
INV { B = b * A}
WHILE $A \neq a$
DO
$\quad B := B + b;$
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OD
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Example 1

$\{ a \geq 0 \ \wedge \ b \geq 0 \}$
$A := 0;$
$B := 0;$ $\qquad 0 = b * 0 \quad \checkmark$
INV { B = b * A}
WHILE $A \neq a$ $\qquad B = b * A \wedge A \neq a \ \longrightarrow \ B + b = b * (A + 1)$
DO
$\quad B := B + b;$
$\quad A := A + 1$
OD
$\{ B = b * a \}$

Example 1

$\{ a \geq 0 \ \wedge \ b \geq 0 \}$
$A := 0;$
$B := 0;$ $\qquad 0 = b * 0 \quad \checkmark$
INV { B = b * A}
WHILE $A \neq a$ $\qquad B = b * A \wedge A \neq a \longrightarrow B + b = b * (A + 1)$
DO $\qquad = b * A + b$
$\quad B := B + b;$ $\qquad = B + b \quad \checkmark$
$\quad A := A + 1$
OD
$\{ B = b * a \}$

Example 1

$\{\ a \geq 0\ \wedge\ b \geq 0\ \}$
$A := 0;$
$B := 0;$ $\quad 0 = b * 0 \quad \checkmark$
INV { B = b * A}
WHILE $A \neq a$ $\quad B = b * A \wedge A \neq a \longrightarrow B + b = b * (A + 1)$
DO $\quad = b * A + b$
$\quad B := B + b;$ $\quad = B + b \quad \checkmark$
$\quad A := A + 1$
OD $\quad B = b * A \wedge A = a \longrightarrow B = b * a \quad \checkmark$
$\{\ B = b * a\ \}$

Example 2

$\{\ a \geq 0\ \wedge\ b \geq 0\ \}$
$A := 0;$
$B := 0;$

WHILE $A < a$
DO
  $B := B + b;$
  $A := A + 1$
OD

Example 2

$\{ a \geq 0 \ \wedge \ b \geq 0 \}$
$A := 0;$
$B := 0;$

WHILE $A < a$
DO
    $B := B + b;$
    $A := A + 1$
OD
$\{ B = b * a \}$

Example 2

$\{\ a \geq 0\ \wedge\ b \geq 0\ \}$
$A := 0;$
$B := 0;$
INV { B = b * A}
WHILE $A < a$
DO
    $B := B + b;$
    $A := A + 1$
OD
$\{\ B = b * a\ \}$

Example 2

$\{ a \geq 0 \ \wedge \ b \geq 0 \}$
$A := 0;$
$B := 0;$ $\quad 0 = b * 0 \quad \checkmark$
INV { B = b * A}
WHILE $A < a$ $\quad B = b * A \wedge A < a \longrightarrow B + b = b * (A + 1)$
DO $\quad = b * A + b$
$\quad B := B + b;$ $\quad = B + b \quad \checkmark$
$\quad A := A + 1$
OD $\quad B = b * A \wedge A \geq a \longrightarrow B = b * a \quad ???$
$\{ B = b * a \}$

Example 2

$\{\ a \geq 0\ \wedge\ b \geq 0\ \}$
$A := 0;$
$B := 0;$ $\qquad 0 = b * 0$
INV { B = b * A ∧ A≤ a}
WHILE $A < a$ $\qquad B = b * A \wedge A < a \longrightarrow B + b = b * (A + 1)$
DO
$\quad B := B + b;$
$\quad A := A + 1$
OD $\qquad B = b * A \wedge A \geq a \longrightarrow B = b * a$
$\{\ B = b * a\ \}$

Example 2

$\{ a \geq 0 \ \wedge \ b \geq 0 \}$
$A := 0;$
$B := 0;$
INV { B = b * A ∧ A≤ a}
WHILE $A < a$
DO
 $B := B + b;$
 $A := A + 1$
OD
$\{ B = b * a \}$

$0 = b * 0 \wedge 0 \leq a$

$B = b * A \wedge A < a \wedge A \leq a \longrightarrow B + b = b * (A + 1) \wedge A + 1 \leq a$

$B = b * A \wedge A \geq a \wedge A \leq a \longrightarrow B = b * a$

Example 2

$\{ a \geq 0 \ \wedge \ b \geq 0 \}$
$A := 0;$
$B := 0;$ $\quad 0 = b * 0 \wedge 0 \leq a \quad \checkmark$
INV $\{ B = b * A \wedge A \leq a \}$
WHILE $A < a$ $\quad B = b * A \wedge A < a \longrightarrow B + b = b * (A + 1)$
DO $\quad \wedge A \leq a \qquad \wedge A + 1 \leq a \quad \checkmark$
$\quad B := B + b;$
$\quad A := A + 1$
OD $\quad B = b * A \wedge A \geq a \longrightarrow B = b * a$
$\{ B = b * a \}$ $\quad \wedge A \leq a \quad \checkmark$

Example 3

$\{ \, a \geq 0 \; \wedge \; b > 0 \, \}$

$A := a;$

$B := 1;$

WHILE $A \neq 0$

DO

$\quad B := B * b;$

$\quad A := A - 1$

OD

Example 3

$\{ a \geq 0 \ \wedge \ b > 0 \}$

$A := a;$	$A =$	a	a-1	a-2	a-3	...
$B := 1;$	$B =$	1	b	b*b	b*b*b	...

WHILE $A \neq 0$
DO
  $B := B * b;$
  $A := A - 1$
OD

Example 3

$\{\ a \geq 0\ \wedge\ b > 0\ \}$

$A := a$;	$A =$	a	a-1	a-2	a-3	...
$B := 1$;	$B =$	1	b	b*b	b*b*b	...

WHILE $A \neq 0$
DO
  $B := B * b$;
  $A := A - 1$
OD
$\{\ B = b^a\ \}$

Example 3

$\{\ a \geq 0\ \wedge\ b > 0\ \}$
$A := a;$
$B := 1;$

$A =$	a	a-1	a-2	a-3	...
$B =$	1	b	b*b	b*b*b	...
				$= b^3 = b^{a-A}$	

WHILE $A \neq 0$
DO
    $B := B * b;$
    $A := A - 1$
OD
$\{\ B = b^a\ \}$

Example 3

$\{ \ a \geq 0 \ \wedge \ b > 0 \ \}$

$A := a;$	$A =$	a	a-1	a-2	a-3	...
$B := 1;$	$B =$	1	b	b*b	b*b*b	...
					$= b^3 = b^{a-A}$	

$\text{INV } \{ \ B = b^{a-A} \}$

WHILE $A \neq 0$

DO

$\quad B := B * b;$

$\quad A := A - 1$

OD

$\{ \ B = b^a \ \}$

Example 3

$\{ \ a \geq 0 \ \wedge \ b > 0 \ \}$
$A := a;$
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$A =$	a	a-1	a-2	a-3	...
$B =$	1	b	b*b	b*b*b	...
				$= b^3 = b^{a-A}$	

$1 = b^{a-a}$

INV $\{ \ B = b^{a-A} \}$
WHILE $A \neq 0$
DO
  $B := B * b;$
  $A := A - 1$
OD
$\{ \ B = b^{a} \ \}$

$B = b^{a-A} \wedge A \neq 0 \longrightarrow B * b = b^{a-(A-1)}$

$B = b^{a-A} \wedge A = 0 \longrightarrow B = b^{a}$

Example 3

$\{ \ a \geq 0 \ \wedge \ b > 0 \ \}$
$A := a;$
$B := 1;$

$A =$	a	a-1	a-2	a-3	...
$B =$	1	b	b*b	b*b*b	...
				$= b^3 = b^{a-A}$	

$1 = b^{a-a}$

INV $\{ \ B = b^{a-A} \wedge A \leq a \}$
WHILE $A \neq 0$
DO
$\quad B := B * b;$
$\quad A := A - 1$
OD
$\{ \ B = b^a \ \}$

$B = b^{a-A} \wedge A \neq 0 \longrightarrow B * b = b^{a-(A-1)}$

$B = b^{a-A} \wedge A = 0 \longrightarrow B = b^a$

Example 4

$\{\ \mathit{True}\ \}$
$X := x;$
$Y := [];$

WHILE $X \neq []$

DO
  $Y := (hd\ X \# Y);$
  $X := tl\ X$
OD

Example 4

$\{ \ True \ \}$
$X := x;$
$Y := [];$

$X =$	$[x_0; x_1; x_2 \ldots]$	$[x_1; x_2 \ldots]$	$[x_2 \ldots]$	...
$Y =$	$[]$	$x_0 \# []$	$x_1 \# x_0 \# []$	...

WHILE $X \neq []$

DO
$\quad Y := (hd\ X \# Y);$
$\quad X := tl\ X$
OD

Example 4

$\{\ True\ \}$
$X := x;$
$Y := [];$

$X =$	$[x_0; x_1; x_2 \dots]$	$[x_1; x_2 \dots]$	$[x_2 \dots]$	...
$Y =$	$[]$	$x_0 \# []$	$x_1 \# x_0 \# []$	...

WHILE $X \neq []$

DO
$\quad Y := (hd\ X \# Y);$
$\quad X := tl\ X$
OD
$\{\ Y = rev\ x\ \}$

Example 4

$\{\ True\ \}$
$X := x;$ $\quad X = \quad [x_0; x_1; x_2 ...] \quad [x_1; x_2 ...] \quad [x_2 ...] \quad ...$
$Y := [];$ $\quad Y = \quad [] \quad x_0\#[] \quad x_1\#x_0\#[] \quad ...$

INV $\{\ (rev\ X)@Y = rev\ x\}$
WHILE $X \neq []$

DO
$\quad Y := (hd\ X\#Y);$
$\quad X := tl\ X$
OD
$\{\ Y = rev\ x\ \}$

Example 4

$\{\ \mathit{True}\ \}$
$X := x;$ $\quad X = \quad [x_0; x_1; x_2 \ldots] \quad [x_1; x_2 \ldots] \quad [x_2 \ldots] \quad \ldots$
$Y := [];$ $\quad Y = \quad [] \quad x_0 \# [] \quad x_1 \# x_0 \# [] \quad \ldots$

$(\mathit{rev}\ x)@[] = \mathit{rev}\ x$

INV $\{\ (\mathit{rev}\ X)@Y = \mathit{rev}\ x\}$
WHILE $X \neq []$

$(\mathit{rev}\ X)@Y = \mathit{rev}\ x\ \wedge X \neq [] \longrightarrow$
$(\mathit{rev}\ (\mathit{tl}\ X))@((\mathit{hd}\ X)\#Y) = \mathit{rev}\ x$

DO
$\quad Y := (\mathit{hd}\ X\#Y);$
$\quad X := \mathit{tl}\ X$
OD

$(\mathit{rev}\ X)@Y = \mathit{rev}\ x\ \wedge X = []\ \longrightarrow Y = \mathit{rev}\ x$

$\{\ Y = \mathit{rev}\ x\ \}$

Example 4

$\{ \; True \; \}$
$X := x;$ $\quad X = \; [x_0; x_1; x_2 \ldots] \quad [x_1; x_2 \ldots] \quad [x_2 \ldots] \quad \ldots$
$Y := [];$ $\quad Y = \; [] \quad x_0 \# [] \quad x_1 \# x_0 \# [] \quad \ldots$

$(rev\ x)@[] = rev\ x$

INV $\{ \; (rev\ X)@Y = rev\ x \}$
WHILE $X \neq []$

$(rev\ X)@Y = rev\ x \; \wedge X \neq [] \longrightarrow$
$(rev\ (tl\ X))@((hd\ X)\#Y) = rev\ x$

DO

$= (rev\ X)@Y$
$= (rev\ ((hd\ X)\#(tl\ X)))@Y$

$Y := (hd\ X\#Y);$
$X := tl\ X$
OD

$(rev\ X)@Y = rev\ x \; \wedge X = [] \; \longrightarrow Y = rev\ x$

$\{ \; Y = rev\ x \; \}$

Example 5

```
A := a; B := b; C := 1;

WHILE B ≠ 0
DO

    WHILE (B mod 2 = 0)

        DO
        A := A ∗ A;
        B := B div 2;
        OD
    C := C ∗ A;
    B := B − 1
OD
```

Example 5

Try with $b = 10 = 2^1 + 2^3$ or $b = 12 = 2^2 + 2^3$ (and e.g. a=3)

```
A := a; B := b; C := 1;

WHILE B ≠ 0
DO

  WHILE (B mod 2 = 0)

     DO
     A := A * A;
     B := B div 2;
     OD
  C := C * A;
  B := B − 1
OD
```

Example 5

Try with $b = 10 = 2^1 + 2^3$ or $b = 12 = 2^2 + 2^3$ (and e.g. a=3)

$\{ a \geq 0 \wedge b \geq 0 \}$
$A := a;\ B := b;\ C := 1;$

WHILE $B \neq 0$
DO

    WHILE (B mod 2 = 0)

        DO
        $A := A * A;$
        $B := B\ div\ 2;$
        OD
    $C := C * A;$
    $B := B - 1$
OD
$\{ C = a^b \}$

Example 5

Try with $b = 10 = 2^1 + 2^3$ or $b = 12 = 2^2 + 2^3$ (and e.g. a=3)

$\{ a \geq 0 \wedge b \geq 0 \}$
$A := a; B := b; C := 1;$
INV $\{ a^b = C * A^B \}$
WHILE $B \neq 0$
DO
INV $\{ a^b = C * A^B \}$
    WHILE (B mod 2 = 0)

        DO
        $A := A * A;$
        $B := B \; div \; 2;$
        OD
    $C := C * A;$
    $B := B - 1$
OD
$\{ C = a^b \}$

Example 5

Try with $b = 10 = 2^1 + 2^3$ or $b = 12 = 2^2 + 2^3$ (and e.g. a=3)

$\{\ a \geq 0 \wedge b \geq 0\ \}$

$A := a;\ B := b;\ C := 1;$ $\quad a^b = 1 * a^b$

INV $\{\ a^b = C * A^B \}$

WHILE $B \neq 0$ $\quad a^b = C * A^B \wedge B \neq 0 \longrightarrow\ a^b = (C * A) * A^{B-1}$

DO

INV $\{\ a^b = C * A^B \}$

WHILE (B mod 2 = 0)

$a^b = C * A^B \wedge B\ mod\ 2 = 0 \longrightarrow\ a^b = C * (A * A)^{B\ div\ 2}$

DO

$A := A * A;$

$B := B\ div\ 2;$

OD

$C := C * A;$

$B := B - 1$

OD $\quad a^b = C * A^B \wedge B = 0 \longrightarrow\ C = a^b$

$\{\ C = a^b\ \}$

Example 6

$l := 0; u := \mathit{length}\ A - 1; A := a$

WHILE $l \leq u$
DO

WHILE $l < \mathit{length}\ A \wedge A!l \leq \mathit{piv}$ DO $l := l + 1$ OD;

WHILE $0 < u \wedge \mathit{piv} \leq A!u$ DO $u := u - 1$ OD;

IF $l \leq u$ THEN $A := A[l := A!u, u := A!l]$ ELSE SKIP FI
OD

Example 6

$l := 0; u := \textit{length } A - 1; A := a$

WHILE $l \leq u$
DO

WHILE $l < \textit{length } A \wedge A!l \leq \textit{piv}$ DO $l := l + 1$ OD;

WHILE $0 < u \wedge \textit{piv} \leq A!u$ DO $u := u - 1$ OD;

IF $l \leq u$ THEN $A := A[l := A!u, u := A!l]$ ELSE SKIP FI
OD
$\{ \textit{LEQ } A\ u \wedge \textit{EQ } A\ u\ l \wedge \textit{GEQ } A\ l \wedge A \text{ permutes } a \}$

Example 6

$$LEQ\ A\ n = \forall k.\ k < n \longrightarrow\ A!k \leq piv$$
$$GEQ\ A\ n = \forall k.\ n < k < length\ A \longrightarrow A!k \geq piv$$
$$EQ\ A\ n\ m = \forall k.\ n \leq k \leq m \longrightarrow A!k = piv$$

$\{ 0 < length\ A \}$
$l := 0;\ u := length\ A - 1;\ A := a$

WHILE $l \leq u$
DO

WHILE $l < length\ A \wedge A!l \leq piv$ DO $l := l + 1$ OD;

WHILE $0 < u \wedge piv \leq A!u$ DO $u := u - 1$ OD;

IF $l \leq u$ THEN $A := A[l := A!u, u := A!l]$ ELSE SKIP FI
OD
$\{ LEQ\ A\ u \wedge EQ\ A\ u\ l \wedge GEQ\ A\ l \wedge A$ permutes $a \}$

Example 6

$LEQ\ A\ n = \forall k.\ k < n \longrightarrow\ A!k \leq piv$
$GEQ\ A\ n = \forall k.\ n < k < length\ A \longrightarrow A!k \geq piv$
$EQ\ A\ n\ m = \forall k.\ n \leq k \leq m \longrightarrow A!k = piv$

$\{\ 0 < length\ A\ \}$
$l := 0;\ u := length\ A - 1;\ A := a$
INV $\{\ LEQ\ A\ l \wedge GEQ\ A\ u \wedge u < length\ A \wedge l \leq length\ A \wedge A$ permutes $a\}$
WHILE $l \leq u$
DO
  INV $\{\ LEQ\ A\ l \wedge GEQ\ A\ u \wedge u < length\ A \wedge l \leq length\ A \wedge A$ permutes $a\}$
  WHILE $l < length\ A \wedge A!l \leq piv$ DO $l := l + 1$ OD;

  INV $\{\ LEQ\ A\ l \wedge GEQ\ A\ u \wedge u < length\ A \wedge l \leq length\ A \wedge A$ permutes $a\}$
  WHILE $0 < u \wedge piv \leq A!u$ DO $u := u - 1$ OD;

  IF $l \leq u$ THEN $A := A[l := A!u, u := A!l]$ ELSE SKIP FI
OD
$\{\ LEQ\ A\ u \wedge EQ\ A\ u\ l \wedge GEQ\ A\ l \wedge A$ permutes $a\ \}$

Example 7

Reminder:

datatype ref = Ref int | Null
Pointer access: $p \rightarrow$field
Pointer update: $p \rightarrow$field :== v

Definition:

"*List nxt p Ps*" is a linked list, starting at pointer *p* following the next pointer through the function *nxt*, and where *Ps* contains the list of the pointers of the linked list.

$\{ \; List \; nxt \; p \; Ps \wedge X \in Ps \; \}$

WHILE $p \neq Null \wedge p \neq Ref \; X$

DO
  $p := p \rightarrow nxt;$
OD

Example 7

Reminder:

datatype ref = Ref int | Null
Pointer access: p→field
Pointer update: p→field :== v

Definition:

"*List nxt p Ps*" is a linked list, starting at pointer *p* following the next pointer through the function *nxt*, and where *Ps* contains the list of the pointers of the linked list.

$\{ \mathit{List\ nxt\ p\ Ps} \wedge X \in \mathit{Ps} \}$

WHILE $p \neq \mathit{Null} \wedge p \neq \mathit{Ref}\ X$

DO
$p := p \rightarrow \mathit{nxt}$;
OD

$\{ p = \mathit{Ref}\ X \}$

Example 7

Reminder:

datatype ref = Ref int | Null
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Definition:

"*List nxt p Ps*" is a linked list, starting at pointer *p* following the next pointer through the function *nxt*, and where *Ps* contains the list of the pointers of the linked list.

$\{ \textit{List nxt p Ps} \wedge X \in \textit{Ps} \}$

WHILE $p \neq \textit{Null} \wedge p \neq \textit{Ref } X$

DO
    $p := p \rightarrow \textit{nxt}$;
OD

$\{ p = \textit{Ref } X \}$

Example 7

Reminder:

datatype ref = Ref int | Null
Pointer access: p→field
Pointer update: p→field :== v

Definition:

"*List nxt p Ps*" is a linked list, starting at pointer *p* following the next pointer through the function *nxt*, and where *Ps* contains the list of the pointers of the linked list.

$\{\ \mathit{List\ nxt\ p\ Ps} \wedge X \in \mathit{Ps}\ \}$
$\text{INV}\ \{\ \exists \mathit{Qs}.\ \mathit{List\ nxt\ p\ Qs} \wedge X \in \mathit{Qs}\}$
$\text{WHILE}\ p \neq \mathit{Null} \wedge p \neq \mathit{Ref}\ X$

DO
$p := p \rightarrow \mathit{nxt};$
OD

$\{\ p = \mathit{Ref}\ X\ \}$

Example 7

Reminder:

datatype ref = Ref int | Null
Pointer access: p→field
Pointer update: p→field :== v

Definition:

"*List nxt p Ps*" is a linked list, starting at pointer *p* following the next pointer through the function *nxt*, and where *Ps* contains the list of the pointers of the linked list.

$\{\ List\ nxt\ p\ Ps \wedge X \in Ps\ \}$
INV $\{\ \exists Qs.\ List\ nxt\ p\ Qs \wedge X \in Qs \}$
WHILE $p \neq Null \wedge p \neq Ref\ X$
DO
  $p := p \rightarrow nxt;$
OD
$\{\ p = Ref\ X\ \}$

$\exists Qs.\ List\ nxt\ p\ Qs \wedge X \in Qs$

$\exists Qs.\ List\ nxt\ p\ Qs \wedge X \in Qs$
$\wedge p \neq Null \wedge p \neq Ref\ X \longrightarrow$
$\exists Qs.\ List\ nxt\ (p \rightarrow nxt)\ Qs \wedge X \in Qs$

$\exists Qs.\ List\ nxt\ p\ Qs \wedge X \in Qs$
$\wedge (p = Null \vee p = Ref\ X) \longrightarrow\ p = Ref\ X$

Example 8

What is is Isabelle function doing?

$fun\ f\ ::\ 'a\ list \Rightarrow' a\ list \Rightarrow' a\ list\ where$
$f\ []\ ys = ys|$
$f\ xs\ []\ = xs|$
$f\ (x\#xs)\ (y\#ys) = x\#y\#\ f\ xs\ ys$

Example 8

What is is Isabelle function doing?

$fun\ splice\ ::\ 'a\ list \Rightarrow' a\ list \Rightarrow' a\ list\ where$
$splice\ []\ ys = ys|$
$splice\ xs\ [] = xs|$
$splice\ (x\#xs)\ (y\#ys) = x\#y\#\ f\ xs\ ys$

Example 8

What is is Isabelle function doing?

$$\textit{fun splice} :: {}'a\ list \Rightarrow {}'a\ list \Rightarrow {}'a\ list\ \textit{where}$$
$$\textit{splice}\ []\ ys = ys |$$
$$\textit{splice}\ xs\ []\ = xs |$$
$$\textit{splice}\ (x\#xs)\ (y\#ys) = x\#y\#\ f\ xs\ ys$$

Let's write it with linked lists!

Example 8

$\{\ \textit{List nxt p Ps} \wedge \textit{List nxt q Qs} \wedge (\textit{set Ps} \cap \textit{set Qs}) = \{\} \wedge \textit{size Qs} \leq \textit{size Ps}\ \}$

$\{\ \textit{List nxt p}\ (\textit{splice Ps Qs})\ \}$

Example 8

$\{$ *List nxt p Ps* $\wedge$ *List nxt q Qs* $\wedge$ (*set Ps* $\cap$ *set Qs*) $= \{\} \wedge$ *size Qs* $\leq$ *size Ps* $\}$

$pp := p;$

WHILE $q \neq Null$

DO

$qq := q \rightarrow nxt; q \rightarrow nxt := pp \rightarrow nxt; pp \rightarrow nxt = q; pp := q \rightarrow nxt; q := qq;$

OD

$\{$ *List nxt p* (*splice Ps Qs*) $\}$

Example 8

$List\ nxt\ p\ Ps = Path\ nxt\ p\ Ps\ Null$

$Path\ nxt\ p\ Ps\ Null$ is a linked list from p to q following function nxt and containing list of pointers Ps

$\{\ List\ nxt\ p\ Ps \wedge List\ nxt\ q\ Qs \wedge (set\ Ps \cap set\ Qs) = \{\} \wedge size\ Qs \leq size\ Ps\ \}$

$pp := p;$

INV $\{$

$\}$

WHILE $q \neq Null$

DO

$qq := q \rightarrow nxt; q \rightarrow nxt := pp \rightarrow nxt; pp \rightarrow nxt = q; pp := q \rightarrow nxt; q := qq;$

OD

$\{\ List\ nxt\ p\ (splice\ Ps\ Qs)\ \}$

Example 8

List nxt p Ps $=$ *Path nxt p Ps Null*
Path nxt p Ps Null is a linked list from *p* to *q* following function *nxt* and containing list of pointers *Ps*

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$pp := p;$
INV $\{ \exists PPs$
List nxt pp PPs

$\}$
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$pp := p;$

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$\}$

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$\{ List\ nxt\ p\ (splice\ Ps\ Qs) \}$

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$\}$

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$\{\ List\ nxt\ p\ (splice\ Ps\ Qs)\ \}$

Example 8

$List\ nxt\ p\ Ps = Path\ nxt\ p\ Ps\ Null$

Path nxt p Ps Null is a linked list from *p* to *q* following function *nxt* and containing list of pointers *Ps*

$\{\ List\ nxt\ p\ Ps \wedge List\ nxt\ q\ Qs \wedge (set\ Ps \cap set\ Qs) = \{\} \wedge size\ Qs \leq size\ Ps\ \}$

$pp := p;$

INV $\{\ \exists PPs\ QQs\ PPPs.$

$\quad List\ nxt\ pp\ PPs\ \wedge List\ nxt\ q\ QQs\ \wedge Path\ nxt\ p\ PPPs\ pp$

$\quad \wedge PPPs@splice\ PPs\ QQs\ =\ splice\ Ps\ Qs$

$\}$

WHILE $q \neq Null$

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$\quad qq := q \rightarrow nxt; q \rightarrow nxt := pp \rightarrow nxt; pp \rightarrow nxt = q; pp := q \rightarrow nxt; q := qq;$

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INV $\{\ \exists PPs\ QQs\ PPPs.\ \ size\ QQs \leq size\ PPs\ \wedge$

$\quad List\ nxt\ pp\ PPs\ \wedge List\ nxt\ q\ QQs\ \wedge Path\ nxt\ p\ PPPs\ pp$

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$\{\ List\ nxt\ p\ (splice\ Ps\ Qs)\ \}$

Example 8

List nxt p Ps $=$ *Path nxt p Ps Null*

Path nxt p Ps Null is a linked list from *p* to *q* following function *nxt* and containing list of pointers *Ps*

$\{$ *List nxt p Ps* $\wedge$ *List nxt q Qs* $\wedge$ (*set Ps* $\cap$ *set Qs*) $= \{\} \wedge$ *size Qs* $\leq$ *size Ps* $\}$

pp $:=$ *p*;

INV $\{$ $\exists$*PPs QQs PPPs.* *size QQs* $\leq$ *size PPs* $\wedge$

List nxt pp PPs $\wedge$ *List nxt q QQs* $\wedge$ *Path nxt p PPPs pp*

$\wedge$ *PPPs@splice PPs QQs* $=$ *splice Ps Qs* $\wedge$

set PPs $\cap$ *set QQs* $= \{\} \wedge$ *distinct PPPs* $\wedge$ *set PPPs* $\cap$ (*set PPs* $\cup$ *set QQs*) $= \{\}$

$\}$

WHILE $q \neq Null$

DO

$qq := q \rightarrow nxt; q \rightarrow nxt := pp \rightarrow nxt; pp \rightarrow nxt = q; pp := q \rightarrow nxt; q := qq;$

OD

$\{$ *List nxt p* (*splice Ps Qs*) $\}$

DEMO

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A2 Recap

Thomas Sewell, Miki Tanaka, Rob Sison

T3/2024

A2 Recap

Assignment 2 is now marked.

- ➜ Biggest frustration: *nat* arithmetic.

- ➜ Getting the right induction is *very* important.
  - ➜ A generalisation of the problem of the right invariant.
- ➜ The proofs about `to_nat` require helper lemmas.

- ➜ We will release a reference solution (or two) privately.

A2 Recap

Assignment 2 is now marked.

- ➜ Biggest frustration: *nat* arithmetic.
  - ➜ We know & sympathise.
  - ➜ Underflow conditions are tricky.
- ➜ Getting the right induction is *very* important.
  - ➜ A generalisation of the problem of the right invariant.
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A2 Recap

Assignment 2 is now marked.

- ➜ Biggest frustration: *nat* arithmetic.
  - ➜ We know & sympathise.
  - ➜ Underflow conditions are tricky.
- ➜ Getting the right induction is *very* important.
  - ➜ A generalisation of the problem of the right invariant.
- ➜ The proofs about `to_nat` require helper lemmas.
  - ➜ e.g. `to_nat` $(2 * n)$, `to_nat` $((2 * n) + 1)$
- ➜ We will release a reference solution (or two) privately.

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Exam Prep

Thomas Sewell, Miki Tanaka, Rob Sison

T3/2024

Last Time

- ➜ The automated proof method **wp**
- ➜ The C Parser and translating C into Simpl
- ➜ AutoCorres and translating Simpl into monadic form
- ➜ The option and exception monads

Exam

➜ 24h take-home exam (same as previous years)

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- Open book: can use any passive resource (books, slides, google, etc)
- **Not** allowed to ask for help from anyone
- **Not** allowed AI assistance for technical support (e.g. ChatGPT).
- starts 8am AEST, Thursday 28th Nov 2024, ends 7:59am AEST, Friday 29th Nov 2024

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- ➜ Should be doable in about 4-6 hours.
  The 24h are for flexibility not for you to stay awake actual 24 hours.
- ➜ Recommend to start early, finish the easy questions first.
- ➜ Take breaks. Don't forget to eat :-)
- ➜ If there are clarification questions, make **private** threads on Ed.

Content

➜ Foundations & Principles

- Intro, Lambda calculus, natural deduction [1,2]
- Higher Order Logic, Isar (part 1) [2,3[a]]
- Term rewriting [3,4]

➜ Proof & Specification Techniques

- Inductively defined sets, rule induction [4,5]
- Datatype induction, primitive recursion [5,7]
- General recursive functions, termination proofs [7]
- Proof automation, Isar (part 2) [8[b]]
- Hoare logic, proofs about programs, invariants [8,9]
- C verification [9,10]
- Practice, questions, exam prep [10[c]]

[a] a1 due; [b] a2 due; [c] a3 due

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λ

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We have learned so far...

- ➜ λ calculus syntax
- ➜ free variables, substitution
- ➜ β reduction
- ➜ α and η conversion
- ➜ β reduction is confluent
- ➜ λ calculus is very expressive (turing complete)
- ➜ λ calculus results in an inconsistent logic

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$$\lambda^{\rightarrow}$$

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We have learned so far...

- Simply typed lambda calculus: $\lambda^{\rightarrow}$
- Typing rules for $\lambda^{\rightarrow}$, type variables, type contexts
- β-reduction in $\lambda^{\rightarrow}$ satisfies subject reduction
- β-reduction in $\lambda^{\rightarrow}$ always terminates
- Types and terms in Isabelle

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$\lambda^{\rightarrow}$ and HOL

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What we have learned so far...

- ➜ natural deduction rules for $\wedge$, $\vee$, $\longrightarrow$, $\neg$, iff...
- ➜ proof by assumption, by intro rule, elim rule
- ➜ safe and unsafe rules

- ➜ indent your proofs! (one space per subgoal)
- ➜ prefer implicit backtracking (chaining) or *rule_tac*, instead of *back*
- ➜ *prefer* and *defer*
- ➜ *oops* and *sorry*

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HOL

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We have learned so far...

- ➜ Isar style proofs
- ➜ proof, qed
- ➜ assumes, shows
- ➜ fix, obtain
- ➜ moreover, ultimately
- ➜ forward, backward
- ➜ mixing proof styles

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HOL

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We have learned today ...

- ➜ Defining HOL
- ➜ Higher Order Abstract Syntax
- ➜ Deriving proof rules
- ➜ More automation
- ➜ Equations and Term Rewriting

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$\longrightarrow$

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We have seen today...

- ➜ Equations and Term Rewriting
- ➜ Confluence and Termination of reduction systems
- ➜ Term Rewriting in Isabelle

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$\longrightarrow$

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We have learned today ...

- ➜ Conditional term rewriting
- ➜ Congruence rules
- ➜ AC rules
- ➜ More on confluence

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{}

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We have learned today ...

- ➜ Sets
- ➜ Type Definitions
- ➜ Inductive Definitions

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{ }

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We have learned today ...

- ➜ Formal background of inductive definitions
- ➜ Definition by intersection
- ➜ Computation by iteration
- ➜ Formalisation in Isabelle

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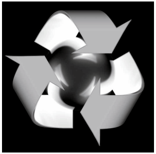

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We have seen today ...

- ➜ Datatypes
- ➜ Primitive recursion
- ➜ Case distinction
- ➜ Structural Induction

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fun

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We have seen today ...

- ➜ General recursion with **fun**/**function**
- ➜ Induction over recursive functions
- ➜ How **fun** works
- ➜ Termination, partial functions, congruence rules

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We have seen today ...

- ➜ sledgehammer
- ➜ nitpick
- ➜ quickcheck

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{P} . . . {Q}

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We have seen today ...

- ➜ Syntax of a simple imperative language
- ➜ Operational semantics
- ➜ Program proof on operational semantics
- ➜ Hoare logic rules
- ➜ Soundness of Hoare logic

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$\{P\} \ldots \{Q\}$

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We have seen today ...

- ➜ Weakest precondition
- ➜ Verification conditions
- ➜ Example program proofs
- ➜ Arrays, pointers

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$\gg=$

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We have seen today

- ➜ Deep and shallow embeddings
- ➜ Isabelle records
- ➜ Nondeterministic State Monad with Failure
- ➜ Monadic Weakest Precondition Rules

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C

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Today we have seen

- ➜ The automated proof method **wp**
- ➜ The C Parser and translating C into Simpl
- ➜ AutoCorres and translating Simpl into monadic form
- ➜ The option and exception monads

Two Final Announcements

Two last things to mention before the end of the course.

Thanks for being a great audience!

➜ It's great to see the level of engagement and interesting questions in this course.

➜ Good luck with the remaining assignment and exam.

➜ There may be more opportunities to use Isabelle and other theorem provers in research & industry.

Provisional: TS Hiring

Thomas, Miki & Rob work for/with the TS group at UNSW.

TS has funding for some Isabelle and seL4 related projects:

- seL4 specification gap
- seL4 WCET
- seL4 time-protection extensions