COMP4161
Advanced Topics in Software Verification

C

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T3/2024

Last Time

- ➜ Deep and shallow embeddings
- ➜ Isabelle records
- ➜ Nondeterministic State Monad with Failure
- ➜ Monadic Weakest Precondition Rules

Content

- ➜ Foundations & Principles
  - Intro, Lambda calculus, natural deduction [1,2]
  - Higher Order Logic, Isar (part 1) [2,3[a]]
  - Term rewriting [3,4]

- ➜ Proof & Specification Techniques
  - Inductively defined sets, rule induction [4,5]
  - Datatype induction, primitive recursion [5,7]
  - General recursive functions, termination proofs [7]
  - Proof automation, Isar (part 2) [8[b]]
  - Hoare logic, proofs about programs, invariants [8,9]
  - C verification [9,10]
  - Practice, questions, exam prep [10[c]]

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[a] a1 due; [b] a2 due; [c] a3 due

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apply (wp *extra_wp_rules*)

Tactic for automatic application of **weakest precondition rules**

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- ➜ Developed for the seL4 verification proofs
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- ➜ Works best when precondition is a schematic variable
- ➜ related tool: **wpc** for Hoare reasoning over **case** statements

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When used with **AutoCorres**, allows automated reasoning about C programs.

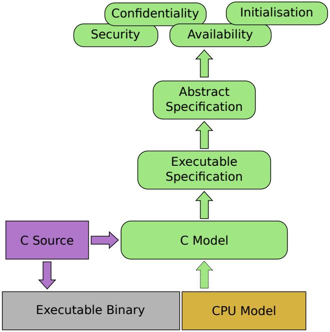
Today we will learn about AutoCorres and C verification.

DEMO

INTRODUCTION TO AUTOCORRES AND WP

Some Context: The L4.verified Project

The seL4 microkernel was verified *functionally correct* using Isabelle/HOL.



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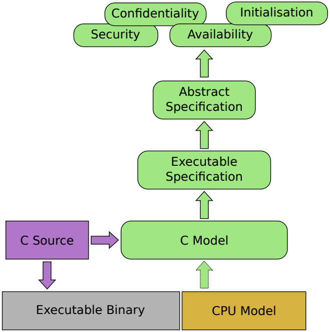

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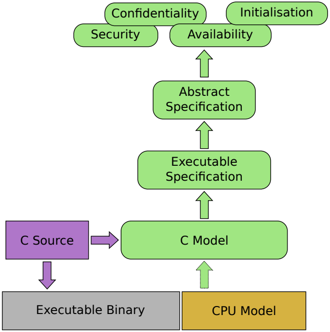
Initial proof (completed 2009).

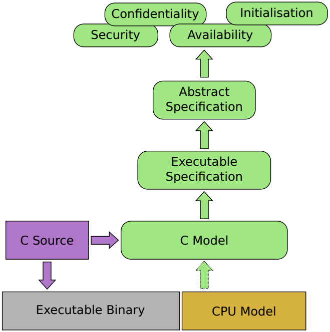
- 3 encodings of seL4 in Isabelle.
- 2 proofs of simulation/refinement.
    - Involve `corres` & `ccorres`.



Relating L4.verified to Best Practice

The L4.verified project is a bit specialised, but some of these concepts are universal:

- *abstract spec*: describes the system as seen by an external party.



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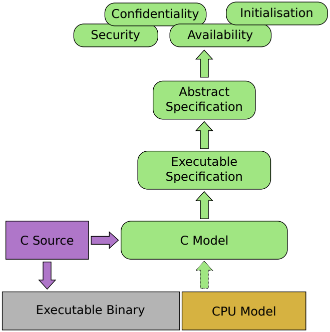

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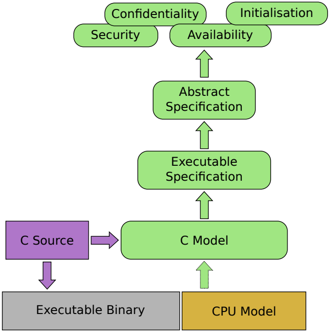
- *implementation*: ground model: encodes the semantics of the real system in detail.



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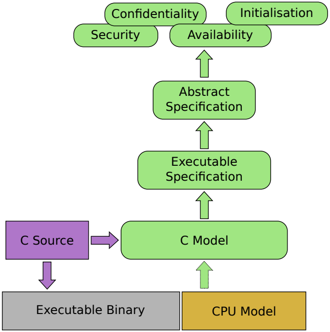
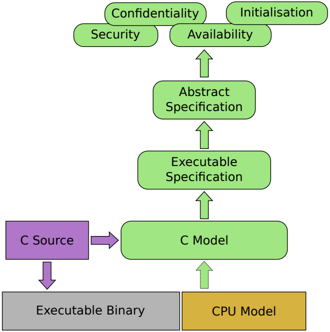
Think about a binary search tree. Where would its invariants go?



The AutoCorres Idea

The big idea of the AutoCorres project:

- Can we handle one layer *automatically*?



A BRIEF OVERVIEW OF C AND SIMPL

C

Main new problems in verifying C programs:

- ➜ expressions with side effects
- ➜ more control flow (do/while, for, break, continue, return)
- ➜ local variables and blocks
- ➜ functions & procedures
- ➜ concrete C data types
- ➜ C memory model and C pointers

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C is not a nice language for reasoning.

Things are going to get ugly.

AutoCorres will help.

C Parser: translates C into Simpl

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- ➜ state space and basic expressions/statements can be instantiated
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C Parser: parses C, produces Simpl definitions in Isabelle

- ➜ written by Michael Norrish, NICTA and ANU
- ➜ Handles a non-trivial subset of C
- ➜ Originally written to verify seL4's C implementation
- ➜ AutoCorres is built on top of the C Parser

Commands in Simpl

```
datatype ('s, 'p, 'f) com =
      Skip
    | Basic "'s ⇒ 's"
    | Spec "('s * 's) set"
    | Seq "('s, 'p, 'f) com" "('s, 'p, 'f) com"
    | Cond "'s set" "('s, 'p, 'f) com"  "('s, 'p, 'f) com"
    | While "'s set" "('s, 'p, 'f) com"
    | Call 'p
    | DynCom "'s ⇒ ('s, 'p, 'f) com"
    | Guard 'f "'s set" "('s, 'p, 'f) com"
    | Throw
    | Catch "('s, 'p, 'f) com" "('s, 'p, 'f) com"
```

's = state, **'p** = procedure names, **'f** = faults

Expressions with side effects

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a = a * b;  x = f(h);  i = ++i - i++;  x = f(h) + g(x);
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Alternative:
Explicitly model nondeterministic order of execution in expressions.

Control flow

```
do { c } while (condition);
```

automatically translates into:

```
c; while (condition) { c }
```

Similarly:

```
for (init; condition; increment) { c }
```

becomes

```
init; while (condition) { c; increment; }
```

More control flow: break/continue

```
while (condition) {
    foo;
    if (Q) continue;
    bar;
    if (P) break;
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Can be modelled with exceptions:

- ➜ throw exception **'continue'**, catch at end of body.

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Non-local control flow: **continue** goes to condition, **break** goes to end.
Can be modelled with exceptions:

- throw exception '**continue**', catch at end of body.
- throw exception '**break**', catch after loop.

Break/continue

Break/continue example becomes:

```
try {
    while (condition) {
        try {
            foo;
            if (Q) { exception = 'continue'; throw; }
            bar;
            if (P) { exception = 'break'; throw; }
        } catch { if (exception == 'continue') SKIP else throw; }
    }
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This is not C any more. But it models C behaviour!

Need to be careful that only the translation has access to exception state.

Return

```
if (P) return x;
foo;
return y;
```

Similar non-local control flow.

Return

```
if (P) return x;
foo;
return y;
```

Similar non-local control flow. **Similar solution:** use throw/try/catch

```
try {
    if (P) { return_val = x; exception = 'return'; throw; }
    foo;
    return_val = y; exception = 'return'; throw;
} catch {
    SKIP
}
```

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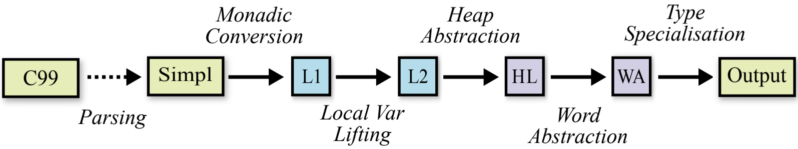
For each Simpl definition *C* and its generated shallow embedding *A*:

- ➜ AutoCorres proves an Isabelle theorem stating that *C* **refines** *A*
- ➜ Every behaviour of *C* has a corresponding behaviour of *A*
- ➜ Refinement guarantees that properties proved about *A* will also hold for *C*.
- ➜ (Provided that *A* never fails. c.f. Total Correctness)

AutoCorres Process



L1: initial monadic shallow embedding

L2: local variables introduced by λ-bindings

HL: heap state abstracted into a set of **typed heaps**

WA: machine words abstracted to idealised integers or nats

Output: human-readable output with **type strengthening**, polish

On-the-fly proof:
Simpl refines **L1** refines **L2** refines **HL** refines **WA** refines **Output**

Example: C99

We will use the following example program to illustrate each of the phases.

```
unsigned some_func(unsigned *a, unsigned *b, unsigned c) {
  unsigned *p = NULL;

  if (c > 10u){
    p = a;
  } else {
    p = b;
  }

  return *p;
}
```

Example: Simpl

```
some_func_body ≡
TRY
  ´p :== ptr_coerce (Ptr (scast 0));;
  IF 0xA < ´c THEN
    ´p :== ´a
  ELSE
    ´p :== ´b
  FI;;
  Guard C_Guard ⦃c_guard ´p⦄
   (creturn global_exn_var_'_update ret__unsigned_'_update
     (λs. h_val (hrs_mem (t_hrs_' (globals s))) (p_' s)));;
  Guard DontReach {} SKIP
CATCH SKIP END
```

Example: L1 (monadic shallow embedding)

```
l1_some_func  ≡  L1_seq (L1_init ret__unsigned_'_update)
 (L1_seq (L1_modify (p_'_update (λ_. ptr_coerce (Ptr (scast 0)))))
   (L1_seq (L1_condition (λs. 0xA < c_' s)
                         (L1_modify (λs. s(|p_' := a_' s|)))
                         (L1_modify (λs. s(|p_' := b_' s|))))
     (L1_seq (L1_guard (λs. c_guard (p_' s)))
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```

State type is the same as Simpl, namely a record with fields:

- ➜ **globals**: heap and type information
- ➜ **a_'**, **b_'**, **c_'**, **p_'** (parameters and local variables)
- ➜ **ret__unsigned_'**, **global_exn_var_'** (return value, exception type)

Example: L2 (local variables lifted)

```
l2_some_func a b c ≡
L2_seq (L2_condition (λs. 0xA < c)
                     (L2_gets (λs. a) [''p''])
                     (L2_gets (λs. b) [''p'']))
  (λp. L2_seq (L2_guard (λs. c_guard p))
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State is a record with just the **globals** field

- function now takes its parameters as arguments
- local variable **p** now passed via λ-binding
- **L2_gets** annotated with local variable names
- This ensures preservation by later AutoCorres phases

Example: HL (heap abstracted into typed heaps)

```
hl_some_func a b c ≡
L2_seq (L2_condition (λs. 0xA < c)
                     (L2_gets (λs. a) [''p''])
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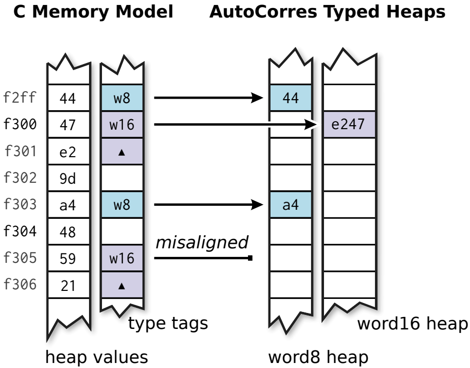
State is a record with a set of **is_valid_** and **heap_** fields:

- These store **pointer validity** and **heap contents** respectively, per type
- above example has only 32-bit word pointers

Heap Abstraction



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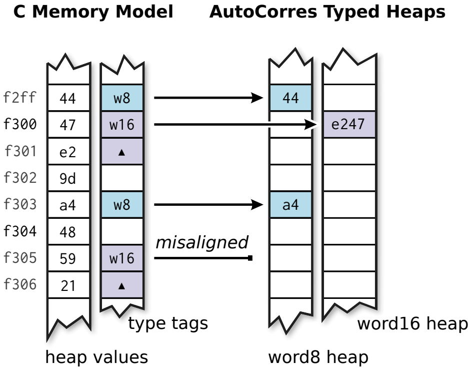

C Memory Model: by Harvey Tuch

- ➜ **Heap** is a mapping from 32-bit addresses to bytes: 32 word ⇒ 8 word
- ➜ **Heap Type Description** stores type information for each heap location

Example: WA (words abstracted to ints and nats)

```
wa_some_func a b c ≡
L2_seq (L2_condition (λs. 10 < c)
                     (L2_gets (λs. a) [''p''])
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- ➜ Guards inserted to ensure absence of unsigned underflow and overflow
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In the example, the **unsigned** argument **c** is now of type **nat**

- The function also returns a nat result
- The heap is not abstracted, hence the call to **unat**

Example: Output (type strengthening and polish)

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 some_func' a b c ≡
DO p ← oreturn (if 10 < c then a else b);
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- ➜ Tries to convert output to a more restricted monad
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Polish:

- Simplify output as much as possible
- The **condition** has been rewritten to a **return** because the condition **10** $<$ **c** doesn't depend on the state

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New kind of Hoare triples to model normal and exceptional cases:

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(other rules analogous)

Today we have seen

- ➜ The automated proof method **wp**
- ➜ The C Parser and translating C into Simpl
- ➜ AutoCorres and translating Simpl into monadic form
- ➜ The option and exception monads