COMP4161
Advanced Topics in Software Verification

$$\{P\} \ldots \{Q\}$$

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Content

➜ Foundations & Principles

- Intro, Lambda calculus, natural deduction [1,2]
- Higher Order Logic, Isar (part 1) [2,3[a]]
- Term rewriting [3,4]

➜ Proof & Specification Techniques

- Inductively defined sets, rule induction [4,5]
- Datatype induction, primitive recursion [5,7]
- General recursive functions, termination proofs [7]
- Proof automation, Isar (part 2) [8[b]]
- Hoare logic, proofs about programs, invariants [8,9]
- C verification [9,10]
- Practice, questions, exam prep [10[c]]

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[a] a1 due; [b] a2 due; [c] a3 due

A Crash Course in Semantics

(For more, see Concrete Semantics)

IMP - a small Imperative Language

Commands:

datatype com = SKIP
| Assign vname aexp (_ := _)
| Semi com com (_; _)
| Cond bexp com com (IF _ THEN _ ELSE _)
| While bexp com (WHILE _ DO _ OD)

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type_synonym state	=	vname $\Rightarrow$ nat

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type_synonym vname = string
type_synonym state = vname ⇒ nat

type_synonym aexp = state ⇒ nat
type_synonym bexp = state ⇒ bool

Example Program

Usual syntax:

$$
\begin{array}{l}
B := 1; \\
\mathrm{WHILE}\ A \neq 0\ \mathrm{DO} \\
\quad B := B * A; \\
\quad A := A - 1 \\
\mathrm{OD}
\end{array}
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Expressions are functions from state to bool or nat:

$$
\begin{array}{l}
B := (\lambda\sigma.\ 1); \\
\text{WHILE } (\lambda\sigma.\ \sigma\ A \neq 0) \text{ DO} \\
\quad B := (\lambda\sigma.\ \sigma\ B * \sigma\ A); \\
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$$

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Now we need: the meaning (semantics) of programs

How to define execution of a program?

- ➜ A wide field of its own
- ➜ Some choices:
  - Operational (inductive relations, big step, small step)
  - Denotational (map program syntax to e.g. state transformers)
  - Axiomatic (e.g. Hoare logic later this lecture)

Structural Operational Semantics

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Demo: Syntax and Semantics

Proofs about Programs

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- ➜ On what they work: State
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Example:
Show that example program from slide 6 implements the factorial.

$$\textbf{lemma}\ \langle \text{factorial}, \sigma \rangle \rightarrow \sigma' \Longrightarrow \sigma' B = \text{fac}\ (\sigma A)$$
$$(\text{where}\quad \text{fac}\ 0 = 1, \quad \text{fac}\ (\text{Suc}\ n) = (\text{Suc}\ n) * \text{fac}\ n)$$

Demo: Proof about Semantics

Too tedious

Induction needed for each loop

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Is there something easier?

Floyd/Hoare

Idea: describe meaning of program by pre/post conditions

Examples:

Floyd/Hoare

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$\{\text{True}\} \quad x := 2 \quad \{x = 2\}$

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$\{y = 2\} \quad x := 21 * y \quad \{x = 42\}$

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$\{x = n\} \quad \text{IF } y < 0 \text{ THEN } x := x + y \text{ ELSE } x := x - y \quad \{x = n - |y|\}$

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Proofs: have rules that directly work on such triples

Meaning of a Hoare-Triple

$$\{P\} \quad c \quad \{Q\}$$

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$$\models \{P\}\ c\ \{Q\} \quad \equiv \quad \forall \sigma\ \sigma'.\ P\ \sigma \wedge \langle c, \sigma\rangle \rightarrow \sigma' \longrightarrow Q\ \sigma'$$

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Total Correctness:

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$$(\forall \sigma.\ P\ \sigma \longrightarrow \exists \sigma'.\ \langle c, \sigma\rangle \rightarrow \sigma')$$

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This lecture: partial correctness only (easier)

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Are the Rules Correct?

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Demo: Hoare Logic in Isabelle

