COMP4161: Advanced Topics in Software Verification

based on slides by J. Blanchette, L. Bulwahn and T. Nipkow

Gerwin Klein, June Andronick, Christine Rizkallah, Miki Tanaka

S2/2018

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- Foundations & Principles
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  - Higher Order Logic [3[a]]
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- Proof & Specification Techniques
  - Inductively defined sets, rule induction [5]
  - Datatypes, recursion, induction [6, 7]
  - Hoare logic, proofs about programs, invariants [8[b],9]
  - (mid-semester break)
  - C verification [10]
  - CakeML, Isar [11[c]]
  - Concurrency [12]

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[a] a1 due; [b] a2 due; [c] a3 due

Overview

Automatic Proof and Disproof

➜ Sledgehammer: automatic proofs

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Based on slides by Jasmin Blanchette, Lukas Bulwahn, and Tobias Nipkow (TUM).

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The key:

Efficient reasoning engines, and restricted logics.

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2000s *embrace external tools, but don't trust them (ATP/SMT/SAT)*

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- *Simple invocation:*
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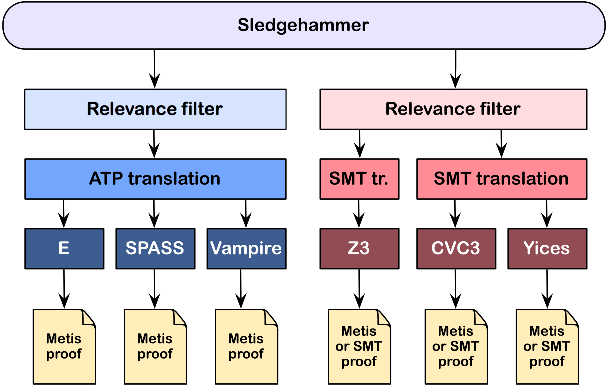
- *Exploits local parallelism and remote servers*

Demo: Sledgehammer

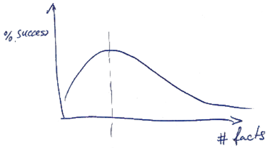
Sledgehammer Architecture



Fact Selection

Provers perform poorly if given 1000s of facts.

- ➜ *Best number of facts depends on the prover*
- ➜ *Need to take care which facts we give them*
- ➜ *Idea: order facts by relevance, give top n to prover ($n = 250, 1000, \ldots$)*



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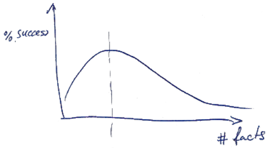

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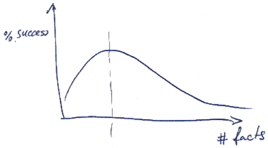
- ➜ *Machine learning method:*
  look at previous proofs to get a probability of relevance



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- **Encode types:**
  - *Monomorphise (generate multiple instances), or*
  - *Encode polymorphism on term level*

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- ➜ *Recast into structured Isar proof*
  Fast, experimental.

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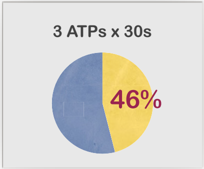
- **2012:** *Better integration with SPASS. 64%*
  SPASS best (small margin)
- **2013:** *Machine learning for fact selection. 69%*
  Improves a few percent across provers.

Evaluation

2010



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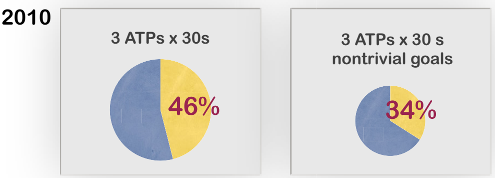

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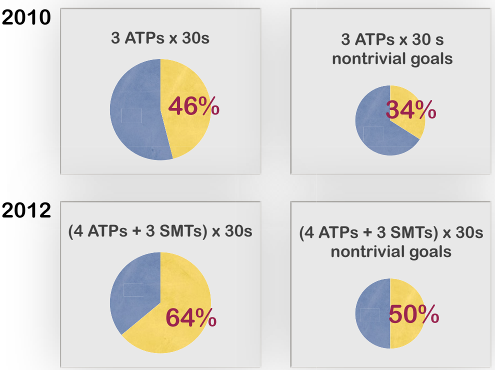
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3 ATPs x 30s
46%
3 ATPs x 30 s
nontrivial goals
34%
2012
(4 ATPs + 3 SMTs) x 30s
64%
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50%

Sledgehammer rules!

Example application:

- *Large Isabelle/HOL repository of algebras for modelling imperative programs (Kleene Algebra, Hoare logic, . . ., ≈ 1000 lemmas)*
- *Intricate refinement and termination theorems*
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"The integration of ATP, SMT, and Nitpick is for our purposes very very helpful." – G. Struth

Disproof

Theorem proving and testing

Testing can show only the presence of errors, but not their absence. *(Dijkstra)*

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Sad facts of life:

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Find counter examples automatically!

Quickcheck

Lightweight validation by testing.

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- *Motivated by Haskell's QuickCheck*
- *Uses Isabelle's code generator*
- *Fast*
- *Runs in background, proves you wrong as you type.*

Quickcheck

Covers a number of testing approaches:

- ➜ *Random and exhausting testing.*
- ➜ *Smart test data generators.*
- ➜ *Narrowing-based (symbolic) testing.*

Creates test data generators automatically.

Demo: Quickcheck

Test generators for datatypes

Fast iteration in continuation-passing-style

$$\textbf{datatype}\ \alpha\ \mathsf{list} = \mathsf{Nil} \mid \mathsf{Cons}\ \alpha\ (\alpha\ \mathsf{list})$$

Test function:

$$\mathsf{test}_{\alpha\ \mathit{list}}\ \mathsf{P}\ =\ \mathsf{P}\ \mathsf{Nil}\ \mathit{andalso}\ \mathsf{test}_{\alpha}\ (\lambda \mathsf{x}.\ \mathsf{test}_{\alpha\ \mathit{list}}\ (\lambda \mathsf{xs}.\ \mathsf{P}\ (\mathsf{Cons}\ \mathsf{x}\ \mathsf{xs})))$$

Test generators for predicates

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Use data flow analysis to figure out which variables
must be computed and which generated.

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Symbolic execution with demand-driven refinement

- ➜ *Test cases can contain variables*
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Implementation:
Lazy execution with outer refinement loop.
Many re-computations, but fast.

Quickcheck Limitations

Only executable specifications!

- ➜ *No equality on functions with infinite domain*
- ➜ *No axiomatic specifications*

Nitpick

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Finite model finder

- *Based on SAT via Kodkod (backend of Alloy prover)*
- *Soundly approximates infinite types*

Nitpick Successes

- ➜ *Algebraic methods*
- ➜ *C++ memory model*
- ➜ *Found soundness bugs in TPS and LEO-II*

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Fan mail:

> *"Last night I got stuck on a goal I was sure was a theorem. After 5–10 minutes I gave Nitpick a try, and within a few secs it had found a splendid counterexample—despite the mess of locales and type classes in the context!"*

Demo: Nitpick

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➜ Proof: Sledgehammer

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