COMP4161: Advanced Topics in Software Verification

based on slides by J. Blanchette, L. Bulwahn and T. Nipkow

Gerwin Klein, June Andronick, Ramana Kumar

S2/2016

Content

- Intro & motivation, getting started [1]
- Foundations & Principles
  - Lambda Calculus, natural deduction [1,2]
  - Higher Order Logic [3[a]]
  - Term rewriting [4]
- Proof & Specification Techniques
  - Inductively defined sets, rule induction [5]
  - Datatypes, recursion, induction [6, 7]
  - Hoare logic, proofs about programs, C verification [8[b],9]
  - (mid-semester break)
  - Writing Automated Proof Methods [10]
  - Isar, codegen, typeclasses, locales [11[c],12]

[a] a1 due; [b] a2 due; [c] a3 due

Overview

Automatic Proof and Disproof

➜ Sledgehammer: automatic proofs

Overview

Automatic Proof and Disproof

- ➜ Sledgehammer: automatic proofs
- ➜ Quickcheck: counter example by testing

Overview

Automatic Proof and Disproof

- ➜ Sledgehammer: automatic proofs
- ➜ Quickcheck: counter example by testing
- ➜ Nipick: counter example by SAT

Overview

Automatic Proof and Disproof

- ➜ Sledgehammer: automatic proofs
- ➜ Quickcheck: counter example by testing
- ➜ Nipick: counter example by SAT

Based on slides by Jasmin Blanchette, Lukas Bulwahn, and Tobias Nipkow (TUM).

Automation

Dramatic improvements in fully automated proofs in the last 2 decades.

Automation

Dramatic improvements in fully automated proofs in the last 2 decades.

➜ First-order logic (ATP): Otter, Vampire, E, SPASS

Automation

Dramatic improvements in fully automated proofs in the last 2 decades.

➜ First-order logic (ATP): Otter, Vampire, E, SPASS
➜ Propositional logic (SAT): MiniSAT, Chaff, RSat

Automation

Dramatic improvements in fully automated proofs in the last 2 decades.

➜ First-order logic (ATP): Otter, Vampire, E, SPASS
➜ Propositional logic (SAT): MiniSAT, Chaff, RSat
➜ SAT modulo theory (SMT): CVC3, Yices, Z3

Automation

Dramatic improvements in fully automated proofs in the last 2 decades.

- ➜ First-order logic (ATP): Otter, Vampire, E, SPASS
- ➜ Propositional logic (SAT): MiniSAT, Chaff, RSat
- ➜ SAT modulo theory (SMT): CVC3, Yices, Z3

The key:

Efficient reasoning engines, and restricted logics.

Automation in Isabelle

1980s *rule applications, write ML code*

Automation in Isabelle

1980s *rule applications, write ML code*

1990s *simplifier, automatic provers (blast, auto), arithmetic*

Automation in Isabelle

1980s *rule applications, write ML code*

1990s *simplifier, automatic provers (blast, auto), arithmetic*

2000s *embrace external tools, but don't trust them (ATP/SMT/SAT)*

Sledgehammer

Sledgehammer:

➜ *Connects Isabelle with ATPs and SMT solvers:*
E, SPASS, Vampire, CVC3, Yices, Z3

Sledgehammer

Sledgehammer:

- *Connects Isabelle with ATPs and SMT solvers:*
  E, SPASS, Vampire, CVC3, Yices, Z3

- *Simple invocation:*
  - *Users don't need to select or know facts*
  - *or ensure the problem is first-order*
  - *or know anything about the automated prover*

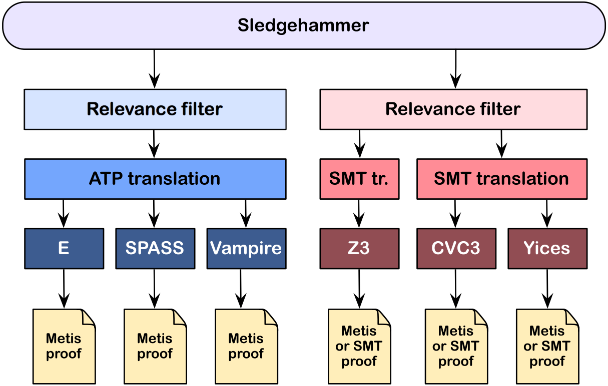
Sledgehammer

Sledgehammer:

- *Connects Isabelle with ATPs and SMT solvers:*
  E, SPASS, Vampire, CVC3, Yices, Z3
- *Simple invocation:*
  - *Users don't need to select or know facts*
  - *or ensure the problem is first-order*
  - *or know anything about the automated prover*
- *Exploits local parallelism and remote servers*

Demo: Sledgehammer

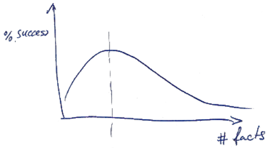
Sledgehammer Architecture



Fact Selection

Provers perform poorly if given 1000s of facts.

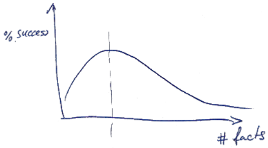
- ➜ *Best number of facts depends on the prover*
- ➜ *Need to take care which facts we give them*
- ➜ *Idea: order facts by relevance, give top n to prover ($n = 250, 1000, \ldots$)*



Fact Selection

Provers perform poorly if given 1000s of facts.

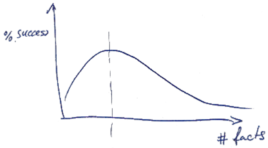
- ➜ *Best number of facts depends on the prover*
- ➜ *Need to take care which facts we give them*
- ➜ *Idea: order facts by relevance, give top n to prover ($n = 250, 1000, \ldots$)*
- ➜ *Meng & Paulson method: lightweight, symbol-based filter*



Fact Selection

Provers perform poorly if given 1000s of facts.

- ➜ *Best number of facts depends on the prover*
- ➜ *Need to take care which facts we give them*
- ➜ *Idea: order facts by relevance, give top n to prover* $(n = 250, 1000, \ldots)$
- ➜ *Meng & Paulson method: lightweight, symbol-based filter*
- ➜ *Machine learning method:*
  look at previous proofs to get a probability of relevance



From HOL to FOL

Source: *higher-order, polymorphism, type classes*
Target: *first-order, untyped or simply-typed*

From HOL to FOL

Source: *higher-order, polymorphism, type classes*
Target: *first-order, untyped or simply-typed*

- **First-order:**
  - *SK combinators, λ-lifting*
  - *Explicit function application operator*

From HOL to FOL

Source: *higher-order, polymorphism, type classes*
Target: *first-order, untyped or simply-typed*

- **First-order:**
  - *SK combinators, λ-lifting*
  - *Explicit function application operator*
- **Encode types:**
  - *Monomorphise (generate multiple instances), or*
  - *Encode polymorphism on term level*

Reconstruction

We don't want to trust the external provers.

Reconstruction

We don't want to trust the external provers.
Need to check/reconstruct proof.

Reconstruction

We don't want to trust the external provers.
Need to check/reconstruct proof.

➜ *Re-find using Metis*
Usually fast and reliable (sometimes too slow)

Reconstruction

We don't want to trust the external provers.
Need to check/reconstruct proof.

- ➜ *Re-find using Metis*
  Usually fast and reliable (sometimes too slow)
- ➜ *Rerun external prover for trusted replay*
  Used for SMT. Re-runs prover each time!

Reconstruction

We don't want to trust the external provers.
Need to check/reconstruct proof.

- ➜ *Re-find using Metis*
  Usually fast and reliable (sometimes too slow)
- ➜ *Rerun external prover for trusted replay*
  Used for SMT. Re-runs prover each time!
- ➜ *Recheck stored explicit external representation of proof*
  Used for SMT, no need to re-run. Fragile.

Reconstruction

We don't want to trust the external provers.
Need to check/reconstruct proof.

- ➜ *Re-find using Metis*
  Usually fast and reliable (sometimes too slow)
- ➜ *Rerun external prover for trusted replay*
  Used for SMT. Re-runs prover each time!
- ➜ *Recheck stored explicit external representation of proof*
  Used for SMT, no need to re-run. Fragile.
- ➜ *Recast into structured Isar proof*
  Fast, experimental.

Judgement Day

Evaluating Sledgehammer:

- *1240 goals out of 7 existing theories.*
- *How many can sledgehammer solve?*

Judgement Day

Evaluating Sledgehammer:

- *1240 goals out of 7 existing theories.*
- *How many can sledgehammer solve?*

- **2010:** *E, SPASS, Vampire (for 5-120s). 46%*
  $ESV \times 5s \approx V \times 120s$

Judgement Day

Evaluating Sledgehammer:

- *1240 goals out of 7 existing theories.*
- *How many can sledgehammer solve?*

- **2010:** *E, SPASS, Vampire (for 5-120s). 46%*
  $ESV \times 5s \approx V \times 120s$
- **2011:** *Add E-SInE, CVC2, Yices, Z3 (30s).*
  $Z3 > V$

Judgement Day

Evaluating Sledgehammer:

- *1240 goals out of 7 existing theories.*
- *How many can sledgehammer solve?*

- **2010:** *E, SPASS, Vampire (for 5-120s). 46%*
  $ESV \times 5s \approx V \times 120s$
- **2011:** *Add E-SInE, CVC2, Yices, Z3 (30s).*
  $Z3 > V$
- **2012:** *Better integration with SPASS. 64%*
  SPASS best (small margin)

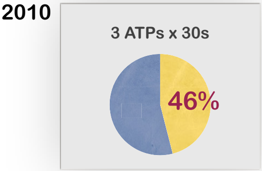
Judgement Day

Evaluating Sledgehammer:

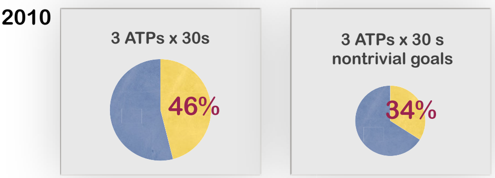
- ➜ *1240 goals out of 7 existing theories.*
- ➜ *How many can sledgehammer solve?*

- ➜ **2010:** *E, SPASS, Vampire (for 5-120s). 46%*
  $ESV \times 5s \approx V \times 120s$
- ➜ **2011:** *Add E-SInE, CVC2, Yices, Z3 (30s).*
  $Z3 > V$
- ➜ **2012:** *Better integration with SPASS. 64%*
  SPASS best (small margin)
- ➜ **2013:** *Machine learning for fact selection. 69%*
  Improves a few percent across provers.

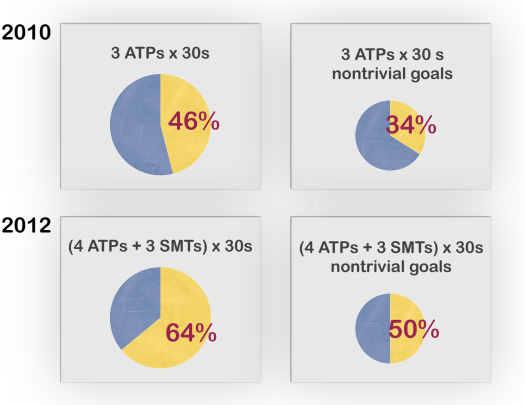
Evaluation



Evaluation



Evaluation


2010
3 ATPs x 30s
46%
3 ATPs x 30 s
nontrivial goals
34%
2012
(4 ATPs + 3 SMTs) x 30s
64%
(4 ATPs + 3 SMTs) x 30s
nontrivial goals
50%

Sledgehammer rules!

Example application:

- *Large Isabelle/HOL repository of algebras for modelling imperative programs (Kleene Algebra, Hoare logic, ..., ≈ 1000 lemmas)*
- *Intricate refinement and termination theorems*
- *Sledgehammer and Z3 automate algebraic proofs at textbook level.*

Sledgehammer rules!

Example application:

- ➜ *Large Isabelle/HOL repository of algebras for modelling imperative programs (Kleene Algebra, Hoare logic, . . ., ≈ 1000 lemmas)*
- ➜ *Intricate refinement and termination theorems*
- ➜ *Sledgehammer and Z3 automate algebraic proofs at textbook level.*

"The integration of ATP, SMT, and Nitpick is for our purposes very very helpful." – G. Struth

Disproof

Theorem proving and testing

Testing can show only the presence of errors, but not their absence. *(Dijkstra)*

Testing cannot prove theorems

Theorem proving and testing

Testing can show only the presence of errors,
but not their absence. *(Dijkstra)*

Testing cannot prove theorems, but it can refute conjectures!

Theorem proving and testing

Testing can show only the presence of errors,
but not their absence. *(Dijkstra)*

Testing cannot prove theorems, but it can refute conjectures!

Sad facts of life:

- ➜ *Most lemma statements are wrong the first time.*
- ➜ *Theorem proving is expensive as a debugging technique.*

Theorem proving and testing

**Testing can show only the presence of errors,
but not their absence.** *(Dijkstra)*

Testing cannot prove theorems, but it can refute conjectures!

Sad facts of life:

- ➜ *Most lemma statements are wrong the first time.*
- ➜ *Theorem proving is expensive as a debugging technique.*

Find counter examples automatically!

Quickcheck

Lightweight validation by testing.

Quickcheck

Lightweight validation by testing.

- *Motivated by Haskell's QuickCheck*
- *Uses Isabelle's code generator*
- *Fast*
- *Runs in background, proves you wrong as you type.*

Quickcheck

Covers a number of testing approaches:

- ➜ *Random and exhausting testing.*
- ➜ *Smart test data generators.*
- ➜ *Narrowing-based (symbolic) testing.*

Creates test data generators automatically.

Demo: Quickcheck

Test generators for datatypes

Fast iteration in continuation-passing-style

$$\textbf{datatype}\ \alpha\ \text{list} = \text{Nil} \mid \text{Cons}\ \alpha\ (\alpha\ \text{list})$$

Test function:

$$\text{test}_{\alpha\ \textit{list}}\ \text{P}\ \ =\ \ \text{P Nil}\ \textit{andalso}\ \text{test}_{\alpha}\ (\lambda \text{x.}\ \text{test}_{\alpha\ \textit{list}}\ (\lambda \text{xs.}\ \text{P (Cons x xs)}))$$

Test generators for predicates

$$\text{distinct xs} \Longrightarrow \text{distinct (remove1 x xs)}$$

Problem:
Exhaustive testing creates many useless test cases.

Test generators for predicates

$$\text{distinct xs} \Longrightarrow \text{distinct (remove1 x xs)}$$

Problem:
Exhaustive testing creates many useless test cases.

Solution:
Use definitions in precondition for smarter generator.
Only generate cases where distinct xs is true.

Test generators for predicates

$$\text{distinct xs} \Longrightarrow \text{distinct (remove1 x xs)}$$

Problem:
Exhaustive testing creates many useless test cases.

Solution:
Use definitions in precondition for smarter generator.
Only generate cases where distinct xs is true.

$\textit{test-distinct}_{\alpha\ \textit{list}}\ P\ =\ P\ \textit{Nil andalso}$
$\textit{test}_{\alpha}\ (\lambda x.\ \textit{test-distinct}_{\alpha\ \textit{list}}\ (\textit{if}\ x \notin \textit{xs then}\ (\lambda \textit{xs}.\ P\ (\textit{Cons x xs}))$
$\textit{else True}))$

Test generators for predicates

$$\text{distinct xs} \Longrightarrow \text{distinct (remove1 x xs)}$$

Problem:
Exhaustive testing creates many useless test cases.

Solution:
Use definitions in precondition for smarter generator.
Only generate cases where distinct xs is true.

$\textit{test-distinct}_{\alpha\ \textit{list}}\ P\ =\ P\ \textit{Nil andalso}$
$\textit{test}_{\alpha}\ (\lambda x.\ \textit{test-distinct}_{\alpha\ \textit{list}}\ (\textit{if}\ x \notin \textit{xs then}\ (\lambda xs.\ P\ (\textit{Cons}\ x\ xs))$
$\textit{else True}))$

Use data flow analysis to figure out which variables
must be computed and which generated.

Narrowing

Symbolic execution with demand-driven refinement

- ➜ *Test cases can contain variables*
- ➜ *If execution cannot proceed: instantiate with further symbolic terms*

Narrowing

Symbolic execution with demand-driven refinement

- ➜ *Test cases can contain variables*
- ➜ *If execution cannot proceed: instantiate with further symbolic terms*

Pays off if large search spaces can be discarded:

$$distinct\ (Cons\ 1\ (Cons\ 1\ x))$$

False for any x, no further instantiations for x necessary.

Narrowing

Symbolic execution with demand-driven refinement

- ➜ *Test cases can contain variables*
- ➜ *If execution cannot proceed: instantiate with further symbolic terms*

Pays off if large search spaces can be discarded:

distinct (Cons 1 (Cons 1 x))

False for any x, no further instantiations for x necessary.

Implementation:
Lazy execution with outer refinement loop.
Many re-computations, but fast.

Quickcheck Limitations

Only executable specifications!

➜ *No equality on functions with infinite domain*

➜ *No axiomatic specifications*

Nitpick

Nitpick

Finite model finder

- ➜ *Based on SAT via Kodkod (backend of Alloy prover)*
- ➜ *Soundly approximates infinite types*

Nitpick Successes

- *Algebraic methods*
- *C++ memory model*
- *Found soundness bugs in TPS and LEO-II*

Nitpick Successes

- *Algebraic methods*
- *C++ memory model*
- *Found soundness bugs in TPS and LEO-II*

Fan mail:

"Last night I got stuck on a goal I was sure was a theorem. After 5–10 minutes I gave Nitpick a try, and within a few secs it had found a splendid counterexample—despite the mess of locales and type classes in the context!"

Demo: Nitpick

We have seen today ...

➜ Proof: Sledgehammer

We have see today ...

➜ Proof: Sledgehammer
➜ Counter examples: Quickcheck

We have seen today ...

- ➜ Proof: Sledgehammer
- ➜ Counter examples: Quickcheck
- ➜ Counter examples: Nitpick