COMP 4161

Data61 Advanced Course

Advanced Topics in Software Verification

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Binary Search (java.util.Arrays)

```
1:     public static int binarySearch(int[] a, int key) {
2:         int low = 0;
3:         int high = a.length - 1;
4:
5:         while (low <= high) {
6:             int mid = (low + high) / 2;
7:             int midVal = a[mid];
8:
9:             if (midVal < key)
10:                 low = mid + 1
11:             else if (midVal > key)
12:                 high = mid - 1;
13:             else
14:                 return mid; // key found
15:         }
16:         return -(low + 1);  // key not found.
17:     }
```

```
6:             int mid = (low + high) / 2;
```

http://googleresearch.blogspot.com/2006/06/extra-extra-read-all-about-it-nearly.html

Organisatorials

When	Mon	10:00 – 11:30	
	Thu	09:00 – 10:30	
Where	Mon:	Colombo LG02	(B16-LG02)
	Thu:	Webster 256	(G14-256)

`http://www.cse.unsw.edu.au/~cs4161/`

About us

The trustworthy systems verification team

➜ Functional correctness and security of the seL4 microkernel
Security $\leftrightarrow$ Isabelle/HOL model $\leftrightarrow$ Haskell model $\leftrightarrow$ C code $\leftrightarrow$ Binary

➜ 10 000 LOC / 500 000 lines of proof script; about 25 person years of effort

➜ More: Cogent code/proof co-generation; CakeML verified compiler; etc.

Open Source

`http://sel4.systems`

`https://cakeml.org`

We are always embarking on exciting new projects.
We offer

➜ summer student scholarship projects

What you will learn

- ➜ how to use a theorem prover
- ➜ background, how it works
- ➜ how to prove and specify
- ➜ how to reason about programs

Health Warning

Theorem Proving is addictive

Prerequisites

This is an advanced course. It assumes knowledge in

- Functional programming
- First-order formal logic

The following program should make sense to you:

$$\begin{array}{lcl} \text{map f []} & = & \text{[]} \\ \text{map f (x:xs)} & = & \text{f x : map f xs} \end{array}$$

You should be able to read and understand this formula:

$$\exists x.\ (P(x) \longrightarrow \forall x.\ P(x))$$

Content — Using Theorem Provers

	Rough timeline
➜ Intro & motivation, getting started	[today]
➜ Foundations & Principles	
• Lambda Calculus, natural deduction	[1,2]
• Higher Order Logic	[3[a]]
• Term rewriting	[4]
➜ Proof & Specification Techniques	
• Inductively defined sets, rule induction	[5]
• Datatypes, recursion, induction	[6, 7]
• Hoare logic, proofs about programs, C verification	[8[b],9]
• (mid-semester break)	
• Writing Automated Proof Methods	[10]
• Isar, codegen, typeclasses, locales	[11[c],12]

[a] a1 due; [b] a2 due; [c] a3 due

What you should do to have a chance at succeeding

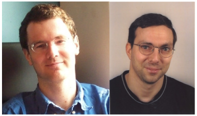
- ➜ attend lectures
- ➜ try Isabelle early
- ➜ redo all the demos alone
- ➜ try the exercises/homework we give, when we do give some
- ➜ **DO NOT CHEAT**
  - Assignments and exams are take-home. This does NOT mean you can work in groups. Each submission is personal.
  - For more info, see Plagiarism Policy[a]

[a] `https://student.unsw.edu.au/plagiarism`

Credits

some material (in using-theorem-provers part) shamelessly stolen from

Tobias Nipkow, Larry Paulson, Markus Wenzel



David Basin, Burkhardt Wolff

Don't blame them, errors are ours

What is a proof?

to prove (Merriam-Webster)

- ➜ from Latin probare (test, approve, prove)
- ➜ to learn or find out by experience (archaic)
- ➜ to establish the existence, truth, or validity of (by evidence or logic)
  prove a theorem, the charges were never proved in court

pops up everywhere

- ➜ politics (weapons of mass destruction)
- ➜ courts (beyond reasonable doubt)
- ➜ religion (god exists)
- ➜ science (cold fusion works)

What is a mathematical proof?

In mathematics, a proof is a demonstration that, given certain axioms, some statement of interest is necessarily true. (Wikipedia)

Example: $\sqrt{2}$ is not rational.

Proof: assume there is $r \in \mathbb{Q}$ such that $r^2 = 2$.
Hence there are mutually prime p and q with $r = \frac{p}{q}$.
Thus $2q^2 = p^2$, i.e. p^2 is divisible by 2.
2 is prime, hence it also divides p, i.e. $p = 2s$.
Substituting this into $2q^2 = p^2$ and dividing by 2 gives $q^2 = 2s^2$.
Hence, q is also divisible by 2. Contradiction. Qed.

Nice, but..

- ➜ still not rigorous enough for some
  - what are the rules?
  - what are the axioms?
  - how big can the steps be?
  - what is obvious or trivial?
- ➜ informal language, easy to get wrong
- ➜ easy to miss something, easy to cheat

Theorem. A cat has nine tails.
Proof. No cat has eight tails. Since one cat has one more tail than no cat, it must have nine tails.

What is a formal proof?

A derivation in a formal calculus

Example: $A \wedge B \longrightarrow B \wedge A$ derivable in the following system

Rules: $\dfrac{X \in S}{S \vdash X}$ (assumption) $\dfrac{S \cup \{X\} \vdash Y}{S \vdash X \longrightarrow Y}$ (impl)

$\dfrac{S \vdash X \quad S \vdash Y}{S \vdash X \wedge Y}$ (conjI) $\dfrac{S \cup \{X, Y\} \vdash Z}{S \cup \{X \wedge Y\} \vdash Z}$ (conjE)

Proof:

1.	$\{A, B\} \vdash B$	(by assumption)
2.	$\{A, B\} \vdash A$	(by assumption)
3.	$\{A, B\} \vdash B \wedge A$	(by conjI with 1 and 2)
4.	$\{A \wedge B\} \vdash B \wedge A$	(by conjE with 3)
5.	$\{\} \vdash A \wedge B \longrightarrow B \wedge A$	(by impl with 4)

What is a theorem prover?

Implementation of a formal logic on a computer.

- ➜ fully automated (propositional logic)
- ➜ automated, but not necessarily terminating (first order logic)
- ➜ with automation, but mainly interactive (higher order logic)

- ➜ based on rules and axioms
- ➜ can deliver proofs

There are other (algorithmic) verification tools:

- ➜ model checking, static analysis, ...
- ➜ usually do not deliver proofs
- ➜ See COMP3153: Algorithmic Verification

Why theorem proving?

- Analysing systems/programs thoroughly
- Finding design and specification errors early
- High assurance (mathematical, machine checked proof)
- it's not always easy
- it's fun

Main theorem proving system for this course

Isabelle

➜ used here for applications, learning how to prove

What is Isabelle?

A generic interactive proof assistant

➜ **generic:**
not specialised to one particular logic
(two large developments: HOL and ZF, will mainly use HOL)

➜ **interactive:**
more than just yes/no, you can interactively guide the system

➜ **proof assistant:**
helps to explore, find, and maintain proofs

Why Isabelle?

- free
- widely used systems
- active development
- high expressiveness and automation
- reasonably easy to use
- (and because we know it best ;-))

If I prove it on the computer, it is correct, right?

If I prove it on the computer, it is correct, right?

No, because:

① hardware could be faulty
② operating system could be faulty
③ implementation runtime system could be faulty
④ compiler could be faulty
⑤ implementation could be faulty
⑥ logic could be inconsistent
⑦ theorem could mean something else

If I prove it on the computer, it is correct, right?

No, but:
probability for

- ➜ OS and H/W issues reduced by using different systems
- ➜ runtime/compiler bugs reduced by using different compilers
- ➜ faulty implementation reduced by having the right prover architecture
- ➜ inconsistent logic reduced by implementing and analysing it
- ➜ wrong theorem reduced by expressive/intuitive logics

No guarantees, but assurance immensly higher than manual proof

If I prove it on the computer, it is correct, right?

Soundness architectures

careful implementation	PVS
LCF approach, small proof kernel	HOL4 Isabelle
explicit proofs + proof checker	Coq Twelf Isabelle HOL4

Meta Logic

Meta language:

The language used to talk about another language.

Examples:

English in a Spanish class, English in an English class

Meta logic:

The logic used to formalize another logic

Example:

Mathematics used to formalize derivations in formal logic

Meta Logic – Example

Syntax:

Formulae: $F ::= V \mid F \longrightarrow F \mid F \wedge F \mid \mathit{False}$

$V ::= [A - Z]$

Derivable: $S \vdash X$ X a formula, S a set of formulae

logic / meta logic

$$\frac{X \in S}{S \vdash X} \qquad \frac{S \cup \{X\} \vdash Y}{S \vdash X \longrightarrow Y}$$

$$\frac{S \vdash X \quad S \vdash Y}{S \vdash X \wedge Y} \qquad \frac{S \cup \{X, Y\} \vdash Z}{S \cup \{X \wedge Y\} \vdash Z}$$

Isabelle's Meta Logic

$$\bigwedge \qquad\qquad \Longrightarrow \qquad\qquad \lambda$$

$\bigwedge$

Syntax: $\bigwedge x.\ F$ (F another meta level formula)
in ASCII: `!!x. F`

- ➜ universal quantifier on the meta level
- ➜ used to denote parameters
- ➜ example and more later

$\Longrightarrow$

Syntax: $A \Longrightarrow B$ (A, B other meta level formulae)
in ASCII: `A ==> B`

Binds to the right:

$$A \Longrightarrow B \Longrightarrow C \quad = \quad A \Longrightarrow (B \Longrightarrow C)$$

Abbreviation:

$$[\![A; B]\!] \Longrightarrow C \quad = \quad A \Longrightarrow B \Longrightarrow C$$

- ➜ read: A and B implies C
- ➜ used to write down rules, theorems, and proof states

Example: a theorem

mathematics: if $x < 0$ and $y < 0$, then $x + y < 0$

formal logic: $\vdash \; x < 0 \wedge y < 0 \longrightarrow x + y < 0$

variation: $x < 0; y < 0 \;\vdash\; x + y < 0$

Isabelle: **lemma** "$x < 0 \wedge y < 0 \longrightarrow x + y < 0$"

variation: **lemma** "$[\![x < 0; y < 0]\!] \Longrightarrow x + y < 0$"

variation: **lemma**

assumes "$x < 0$" and "$y < 0$" shows "$x + y < 0$"

Example: a rule

logic: $$\frac{X \quad Y}{X \wedge Y}$$

variation: $$\frac{S \vdash X \quad S \vdash Y}{S \vdash X \wedge Y}$$

Isabelle: $[\![X; Y]\!] \Longrightarrow X \wedge Y$

Example: a rule with nested implication

logic:

$$\frac{X \lor Y \qquad \begin{array}{c} X \\ \vdots \\ Z \end{array} \qquad \begin{array}{c} Y \\ \vdots \\ Z \end{array}}{Z}$$

variation:

$$\frac{S \cup \{X\} \vdash Z \qquad S \cup \{Y\} \vdash Z}{S \cup \{X \lor Y\} \vdash Z}$$

Isabelle:

$$[\![X \lor Y; X \Longrightarrow Z; Y \Longrightarrow Z]\!] \Longrightarrow Z$$

λ

Syntax: $\lambda x.\ F$ (F another meta level formula)
in ASCII: `%x. F`

- lambda abstraction
- used for functions in object logics
- used to encode bound variables in object logics
- more about this in the next lecture

Enough Theory!

Getting started with Isabelle

System Architecture

Prover IDE (jEdit) – user interface

HOL, ZF – object-logics

Isabelle – generic, interactive theorem prover

Standard ML – logic implemented as ADT

User can access all layers!

System Requirements

- **Linux**, **Windows**, or **MacOS X (10.7 +)**
- **Standard ML**
  (PolyML fastest, SML/NJ supports more platforms)
- **Java** (for jEdit)

Premade packages for Linux, Mac, and Windows + info on:

`http://mirror.cse.unsw.edu.au/pub/isabelle/`

Documentation

Available from `http://isabelle.in.tum.de`

➜ Learning Isabelle

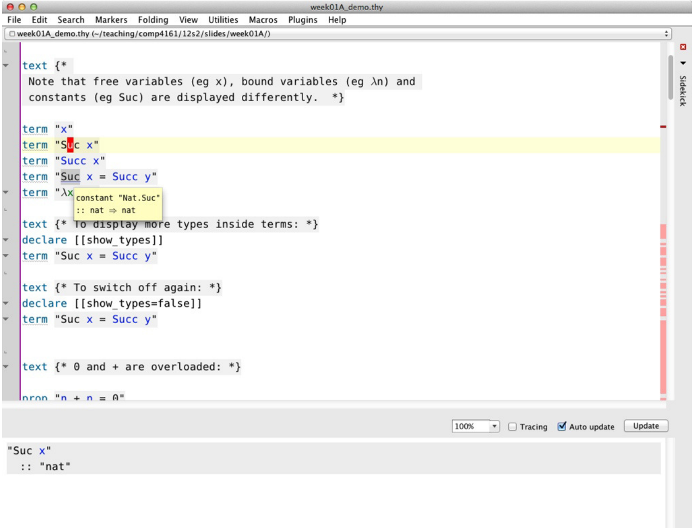
- Tutorial on Isabelle/HOL (LNCS 2283)
- Tutorial on Isar
- Tutorial on Locales

➜ Reference Manuals

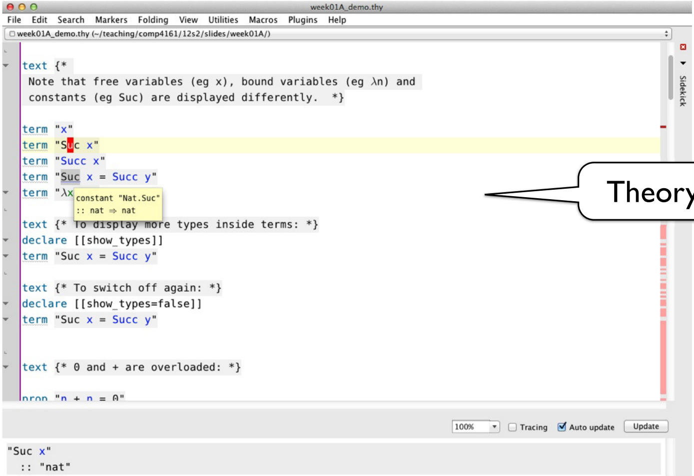
- Isabelle/Isar Reference Manual
- Isabelle Reference Manual
- Isabelle System Manual

➜ Reference Manuals for Object-Logics

jEdit/PIDE


week01A_demo.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
week01A_demo.thy (~/teaching/comp4161/12s2/slides/week01A/)
text {*
Note that free variables (eg x), bound variables (eg λn) and
constants (eg Suc) are displayed differently. *}
term "x"
term "Suc x"
term "Succ x"
term "Suc x = Succ y"
term "λx
constant "Nat.Suc"
:: nat ⇒ nat
text {* To display more types inside terms: *}
declare [[show_types]]
term "Suc x = Succ y"
text {* To switch off again: *}
declare [[show_types=false]]
term "Suc x = Succ y"
text {* 0 and + are overloaded: *}
prop "n + n = 0"
Sidekick
100%
Tracing
Auto update
Update
"Suc x"
:: "nat"

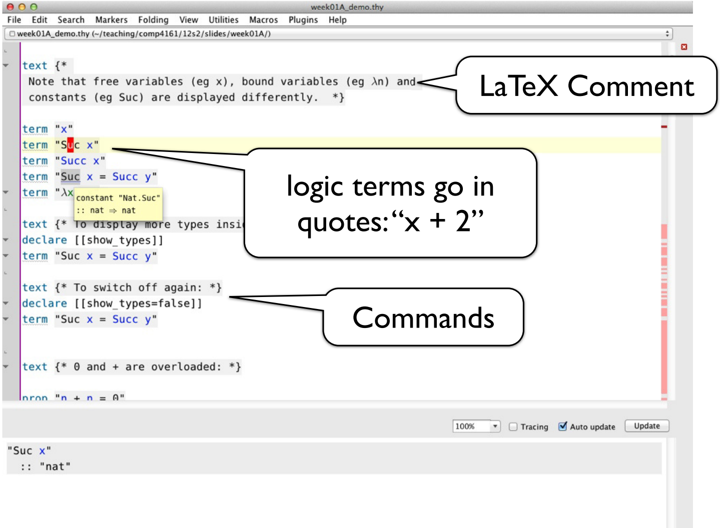
jEdit/PIDE



Theory File

Isabelle Output

jEdit/PIDE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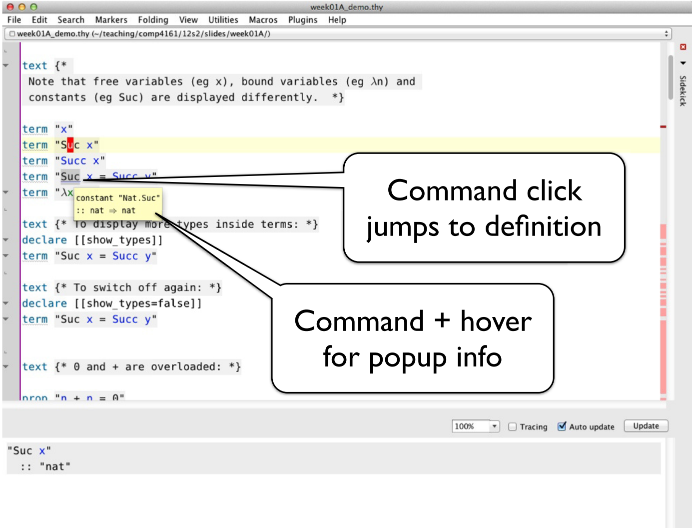
LaTeX Comment
logic terms go in quotes: “x + 2”
Commands

jEdit/PIDE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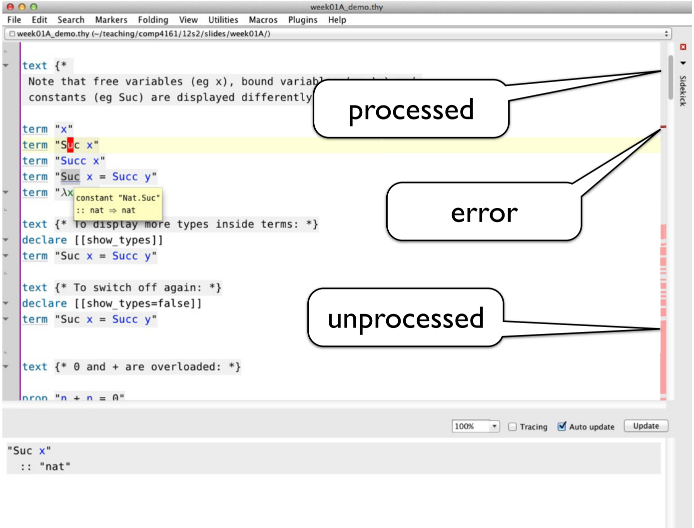
text {* 0 and + are overloaded: *}
prop "n + n = 0"
Sidekick
100%
Tracing
Auto update
Update
"Suc x"
:: "nat"
Command click jumps to definition
Command + hover for popup info

jEdit/PIDE



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text {* To switch off again: *}
declare [[show_types=false]]
term "Suc x = Succ y"
text {* 0 and + are overloaded: *}
prop "n + n = 0"
processed
error
unprocessed
Sidekick
100%
Tracing
Auto update
Update
"Suc x"
:: "nat"

Demo

Exercises

- ➜ Download and install Isabelle from `http://mirror.cse.unsw.edu.au/pub/isabelle/`
- ➜ Step through the demo files from the lecture web page
- ➜ Write your own theory file, look at some theorems in the library, try 'find_theorems'
- ➜ How many theorems can help you if you need to prove something containing the term "Suc(Suc x)"?
- ➜ What is the name of the theorem for associativity of addition of natural numbers in the library?