COMP 4161

NICTA Advanced Course

Advanced Topics in Software Verification

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C

Last Time

- ➜ Deep and shallow embeddings
- ➜ Isabelle records
- ➜ Nondeterministic State Monad with Failure
- ➜ Monadic Weakest Predondition Rules

Content

➜ Intro & motivation, getting started [1]

➜ Foundations & Principles

- Lambda Calculus, natural deduction [1,2]
- Higher Order Logic [3[a]]
- Term rewriting [4]

➜ Proof & Specification Techniques

- Inductively defined sets, rule induction [5]
- Datatypes, recursion, induction [6, 7]
- Hoare logic, proofs about programs, C verification [8[b],9]
- (mid-semester break)
- Writing Automated Proof Methods [10]
- Isar, codegen, typeclasses, locales [11[c],12]

[a] a1 due; [b] a2 due; [c] a3 due

wp

apply (wp *extra_wp_rules*)

Tactic for automatic application of **weakest precondition rules**

- ➜ Originally developed by Thomas Sewell, NICTA, for the seL4 proofs
- ➜ Knows about a huge collection of existing wp rules for monads
- ➜ Works best when precondition is a schematic variable
- ➜ related tool: **wpc** for Hoare reasoning over **case** statements

When used with **AutoCorres**, allows automated reasoning about C programs.

Today we will learn about AutoCorres and C verification.

DEMO: INTRODUCTION TO AUTOCORRES AND WP

A Brief Overview of C and Simpl

Main new problems in verifying C programs:

- ➜ expressions with side effects
- ➜ more control flow (do/while, for, break, continue, return)
- ➜ local variables and blocks
- ➜ functions & procedures
- ➜ concrete C data types
- ➜ C memory model and C pointers

C is not a nice language for reasoning.

Things are going to get ugly.

AutoCorres will help, later.

C Parser: translates C into Simpl

Simpl: deeply embedded imperative langugae in Isabelle.

- ➜ generic imperative language by Norbert Schirmer, TU Munich
- ➜ state space and basic expressions/statements can be instantiated
- ➜ has operational semantics
- ➜ has its own Hoare logic with soundness and completeness proof, plus automated vcg

C Parser: parses C, produces Simpl definitions in Isabelle

- ➜ written by Michael Norrish, NICTA and ANU
- ➜ Handles a non-trivial subset of C
- ➜ Originally written to verify seL4's C implementation
- ➜ AutoCorres is built on top of the C Parser

Commands in Simpl

```
datatype ('s, 'p, 'f) com =
      Skip
    | Basic "'s ⇒ 's"
    | Spec "('s ∗ 's) set"
    | Seq "('s, 'p, 'f) com" "('s, 'p, 'f) com"
    | Cond "'s set" "('s, 'p, 'f) com"  "('s, 'p, 'f) com"
    | While "'s set" "('s, 'p, 'f) com"
    | Call 'p
    | DynCom "'s ⇒ ('s, 'p, 'f) com"
    | Guard 'f "'s set" "('s, 'p, 'f) com"
    | Throw
    | Catch "('s, 'p, 'f) com" "('s, 'p, 'f) com"
```

’s = state, **’p** = procedure names, **’f** = faults

Expressions with side effects

```
a = a * b;    x = f(h);    i = ++i - i++;    x = f(h) + g(x);
```

- **a = a * b** — Fine: easy to translate into Isabelle
- **x = f(h)** — Fine: may have side effects, but can be translated sanely.
- **i = ++i - i++** — Seriously? What does that even mean?
  Make this an error, force programmer to write instead:
  i0 = i; i++; i = i - i0; (or just **i = 1**)
- **x = f(h) + g(x)** — Ok if **g** and **h** do not have any side effects
  $\Longrightarrow$ Prove all functions in expressions are side-effect free

Alternative: explicitly model nondeterministic order of execution in expressions.

Control flow

```
do { c } while (condition);
```

Already can treat normal while-loops! Automatically translate into:

```
c; while (condition) { c }
```

Similarly:

```
for (init; condition; increment) { c }
```

becomes

```
init; while (condition) { c; increment; }
```

More control flow: break/continue

```
while (condition) {
    foo;
    if (Q) continue;
    bar;
    if (P) break;
}
```

Non-local control flow: **continue** goes to condition, **break** goes to end.

Can be modelled with exceptions:

➜ throw exception **’continue’**, catch at end of body.

➜ throw exception **’break’**, catch after loop.

Break/continue example becomes:

```
try {
    while (condition) {
        try {
            foo;
            if (Q) { exception = 'continue'; throw; }
            bar;
            if (P) { exception = 'break'; throw; }
        } catch { if (exception == 'continue') SKIP else throw; }
    }
} catch { if (exception == 'break') SKIP else throw; }
```

This is not C any more. But it models C behaviour!

Need to be careful that only the translation has access to exception state.

Return

```
if (P) return x;
foo;
return y;
```

Similar non-local control flow. **Similar solution:** use throw/try/catch

```
try {
    if (P) { return_val = x; exception = 'return'; throw; }
    foo;
    return_val = y; exception = 'return'; throw;
} catch {
    SKIP
}
```

Formal procedure parameters and local variables

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Simpl only has one global state space.

Basic idea:

- ➜ separate all locals and all globals
- ➜ keep both in one state space record
- ➜ on procedure entry, set formal parameters to actual values
- ➜ on procedure exit, restore previous values of all locals

Implemented using DynCom:

call init body restore result =

DynCom (λs. init; body; DynCom (λt. restore s t; result t))

Example: for procedure f(x) = { r = x + 2 }

y = CALL f(7) $\equiv$ call (x = 7) (r = x + 2) (λs t. s (| globals := globals t |)) (λt. y = r t)

AutoCorres

AutoCorres

AutoCorres: reduces the pain in reasoning about C code

➜ Written by David Greenaway, NICTA and UNSW

➜ Converts C/Simpl into (monadic) shallow embedding in Isabelle

➜ Shallow embedding easier to reason about than Simpl

Is self-certifying: produces Isabelle theorems proving its own correctness

➜ For each Simpl definition C and its shallow embedding A generated by AutoCorres

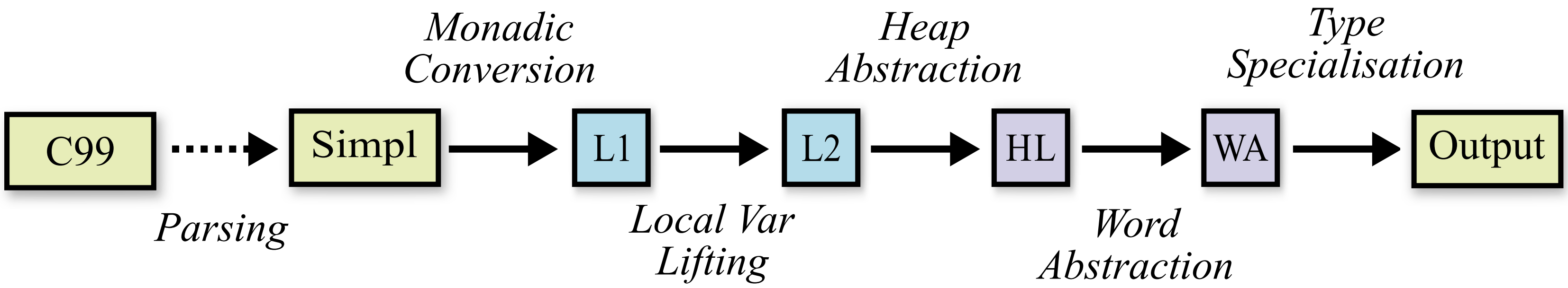
➜ AutoCorres proves an Isabelle theorem stating that C **refines** A

➜ Every behaviour of C has a corresponding behaviour of A

➜ Refinement guarantees that properties proved about A will also hold for C.

➜ (Provided that A never fails. c.f. Total Correctness)

AutoCorres Process



L1: initial monadic shallow embedding

L2: local variables introduced by λ-bindings

HL: heap state abstracted into a set of **typed heaps**

WA: machine words abstracted to idealised integers or nats

Output: human-readable output with **type strengthening**, polish

On-the-fly proof: Simpl refines **L1** refines **L2** refines **HL** refines **WA** refines **Output**

Example: C99

We will use the following example program to illustrate each of the phases.

```
unsigned some_func(unsigned *a, unsigned *b, unsigned c) {
  unsigned *p = NULL;

  if (c > 10u){
    p = a;
  } else {
    p = b;
  }

  return *p;
}
```

Example: Simpl

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```
some_func_body ≡
TRY
  ´p :== ptr_coerce (Ptr (scast 0));;
  IF 0xA < ´c THEN
    ´p :== ´a
  ELSE
    ´p :== ´b
  FI;;
  Guard C_Guard {|c_guard ´p|}
   (creturn global_exn_var_'_update ret__unsigned_'_update
     (λs. h_val (hrs_mem (t_hrs_' (globals s))) (p_' s)));;
  Guard DontReach {} SKIP
CATCH SKIP END
```

Example: L1 (monadic shallow embedding)

```
l1_some_func  ≡  L1_seq (L1_init ret__unsigned_'_update)
 (L1_seq (L1_modify (p_'_update (λ_. ptr_coerce (Ptr (scast 0)))))
   (L1_seq (L1_condition (λs. 0xA < c_' s)
                         (L1_modify (λs. s(|p_' := a_' s|)))
                         (L1_modify (λs. s(|p_' := b_' s|))))
     (L1_seq (L1_guard (λs. c_guard (p_' s)))
       (L1_seq (L1_modify (λs. s(|ret__unsigned_' :=
                   h_val (hrs_mem (t_hrs_' (globals s))) (p_' s)|)))
         (L1_modify (global_exn_var_'_update (λ_. Return)))))))
```

State type is the same as Simpl, namely a record with fields:

- ➜ **globals**: heap and type information
- ➜ **a_'**, **b_'**, **c_'**, **p_'** (parameters and local variables)
- ➜ **ret__unsigned_'**, **global_exn_var_'** (return value, exception type)

Example: L2 (local variables lifted)

```
l2_some_func a b c ≡
L2_seq (L2_condition (λs. 0xA < c)
                     (L2_gets (λs. a) [''p''])
                     (L2_gets (λs. b) [''p'']))
  (λp. L2_seq (L2_guard (λs. c_guard p))
     (λ_. L2_gets (λs. h_val (hrs_mem (t_hrs_' s)) p) [''ret'']))
```

State is a record with just the **globals** field

- ➜ function now takes its parameters as arguments
- ➜ local variable **p** now passed via λ-binding
- ➜ **L2_gets** annotated with local variable names
- ➜ This ensures preservation by later AutoCorres phases

Example: HL (heap abstracted into typed heaps)

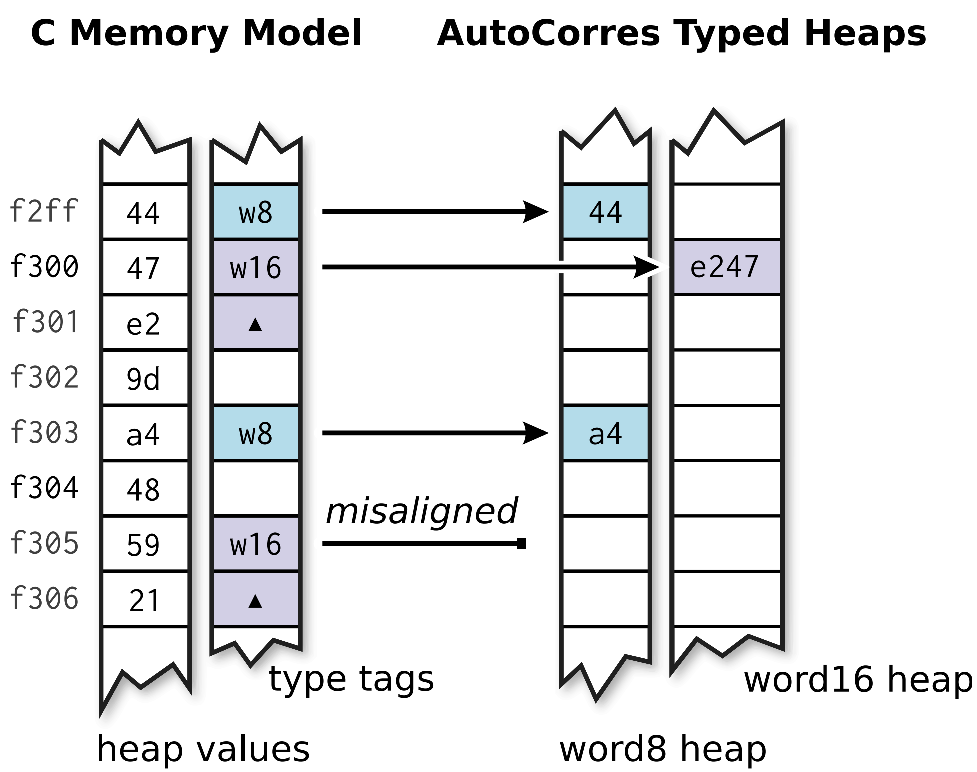
```
hl_some_func a b c ≡
L2_seq (L2_condition (λs. 0xA < c)
                     (L2_gets (λs. a) [''p''])
                     (L2_gets (λs. b) [''p'']))
  (λr. L2_seq (L2_guard (λs. is_valid_w32 s r))
    (λ_. L2_gets (λs. heap_w32 s r) [''ret'']))
```

State is a record with a set of **is_valid_** and **heap_** fields:

➜ These store **pointer validity** and **heap contents** respectively, per type

➜ above example has only 32-bit word pointers

Heap Abstraction



C Memory Model: by Harvey Tuch

- ➜ **Heap** is a mapping from 32-bit addresses to bytes: 32 word $\Rightarrow$ 8 word
- ➜ **Heap Type Description** stores type information for each heap location

AutoCorres Typed Heap Model

Abstracts the single C heap to a set of **typed heaps**:

- ➜ One **typed heap** for each type used in the program: *'a* ptr ⇒ *'a*
- ➜ Associated **validity information** for each type: *'a* ptr ⇒ bool

Aliasing Restrictions: (see C's **strict aliasing rule**)

```
struct us {
  unsigned u;
};

void test(struct us *us){
  unsigned *up = &(us->u);
  *up++;
}
```

```
test' us ≡
do guard (λs. is_valid_us_C s us);
  up ← return (Ptr &(us→[''u_C'']));
  guard (λs. is_valid_w32 s up);
  modify (λs. heap_w32_update
   (λa. a(up := heap_w32 s up + 1)) s)
od
```

Example: WA (words abstracted to ints and nats)

```
wa_some_func a b c ≡
L2_seq (L2_condition (λs. 10 < c)
                     (L2_gets (λs. a) [''p''])
                     (L2_gets (λs. b) [''p'']))
 (λr. L2_seq (L2_guard (λs. is_valid_w32 s r))
        (λ_. L2_gets (λs. unat (heap_w32 s r)) [''ret'']))
```

Word abstraction: C **int** → Isabelle int, C **unsigned** → Isabelle nat

- ➜ Guards inserted to ensure absence of unsigned underflow and overflow
- ➜ Signed under/overflow already has guards, because is undefined behaviour

In the example, the **unsigned** argument **c** is now of type **nat**

- ➜ The function also returns a nat result
- ➜ The heap is not abstracted, hence the call to **unat**

Example: Output (type strengthening and polish)

```
some_func' a b c ≡
DO p ← oreturn (if 10 < c then a else b);
   oguard (λs. is_valid_w32 s p);
   ogets (λs. unat (heap_w32 s p))
OD
```

Type Strengthening:

- ➜ Tries to converts output to a more restricted monad
- ➜ The above is in the **option** monad because it doesn't modify the state, but might fail
- ➜ The **type** of the option monad implies it cannot modify state

Polish:

- ➜ Simplify output as much as possible
- ➜ The **condition** has been rewritten to a **return** because the condition $\mathbf{10} < \mathbf{c}$ doesn't depend on the state

Type Strengthening

Example:

```
unsigned zero(void){ return 0u; }
```

Monad Type	**Kind**	**Type**	**Example**
pure	Pure function	*'a*	0
gets	Read-only, non-failing	*'s* $\Rightarrow$ *'a*	λs. 0
option	Read-only function	*'s* $\Rightarrow$ *'a* option	oreturn 0

Effect information now encoded in function **types**

Later proofs get this information for free!

Can be controlled by the **ts_force** option of AutoCorres

Option Monad

Another standard monad, familiar from e.g. Haskell

Return:

$$\text{oreturn } x \equiv \lambda s.\ \text{Some } x$$

Bind:

$$\text{obind } a\ b \equiv \lambda s.\ \text{case } a\ s \text{ of None} \Rightarrow \text{None} \mid \text{Some } r \Rightarrow b\ r\ s$$

- ➜ **Infix notation:** $|\gg$
- ➜ **Do notation:** DO ... OD

Hoare Logic:

$$\text{ovalid } P\ f\ Q \equiv \forall\ s\ r.\ P\ s \wedge f\ s = \text{Some } r \longrightarrow Q\ r\ s$$

$$\text{ovalid } (P\ x)\ (\text{oreturn } x)\ P \qquad \frac{\bigwedge r.\ \text{ovalid } (R\ r)\ (g\ r)\ Q \quad \text{ovalid } P\ f\ R}{\text{ovalid } P\ (f\ |\gg g)\ Q}$$

Exception Monad

Exceptions used to model early return, break and continue.

Exception Monad: $'s \Rightarrow ((\underline{'e + 'a}) \times 's)\ \text{set} \times \text{bool}$

- ➜ Instance of the nondeterministic state monad: return-value type is **sum type** $'e + 'a$
- ➜ Sum Type Constructors: **Inl** :: $'e \Rightarrow 'e + 'a$ **Inr** :: $'a \Rightarrow 'e + 'a$
- ➜ **Convention:** Inl used for exceptions, Inr used for ordinary return-values

Basic Monadic Operations

$$\text{returnOk}\ x \equiv \text{return (Inr}\ x) \qquad \text{throwError}\ e \equiv \text{return (Inl}\ e)$$

$$\text{lift}\ b \equiv (\lambda x.\ \text{case}\ x\ \text{of Inl}\ e \Rightarrow \text{throwError}\ e \mid \text{Inr}\ r \Rightarrow b\ r)$$

bindE: $a \gg\!= \text{E}\ b \equiv a \gg\!= (\text{lift b})$ **Do notation:** doE ... odE

Hoare Rules for Exceptions

New kind of Hoare triples to model normal and exceptional cases:

$$\{\!|P|\!\}\ f\ \{\!|Q|\!\},\ \{\!|E|\!\}$$

$$\equiv$$

$$\{\!|P|\!\}\ f\ \{\!|\lambda x\ s.\ \text{case}\ x\ \text{of Inl}\ e\ \Rightarrow E\ e\ s\ |\ \text{Inr}\ r\ \Rightarrow Q\ r\ s|\!\}$$

Weakest Precondition Rules:

$$\frac{}{\{\!|P\ x|\!\}\ \text{returnOk}\ x\ \{\!|P|\!\},\{\!|E|\!\}} \qquad \frac{}{\{\!|E\ e|\!\}\ \text{throwError}\ e\ \{\!|P|\!\},\{\!|E|\!\}}$$

$$\frac{\bigwedge x.\ \{\!|R\ x|\!\}\ b\ x\ \{\!|Q|\!\},\{\!|E|\!\} \quad \{\!|P|\!\}\ a\ \{\!|R|\!\},\{\!|E|\!\}}{\{\!|P|\!\}\ a\ \gg\!=\text{E}\ b\ \{\!|Q|\!\},\{\!|E|\!\}}$$

(other rules analogous)

Today we have seen

- ➜ The automated proof method **wp**
- ➜ The C Parser and translating C into Simpl
- ➜ AutoCorres and translating Simpl into monadic form
- ➜ The option and exception monads