COMP 4161
NICTA Advanced Course

Advanced Topics in Software Verification

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$$\{P\} \ldots \{Q\}$$

Slide 1

Last Time

- Syntax of a simple imperative language
- Operational semantics
- Program proof on operational semantics
- Hoare logic rules
- Soundness of Hoare logic

Slide 2

Content

- Intro & motivation, getting started [1]
- Foundations & Principles
  - Lambda Calculus, natural deduction [1,2]
  - Higher Order Logic [3[a]]
  - Term rewriting [4]
- Proof & Specification Techniques
  - Inductively defined sets, rule induction [5]
  - Datatypes, recursion, induction [6, 7]
  - Hoare logic, proofs about programs, C verification [8[b],9]
  - (mid-semester break)
  - Writing Automated Proof Methods [10]
  - Isar, codegen, typeclasses, locales [11[c],12]

[a] a1 due; [b] a2 due; [c] a3 due

Slide 3

Automation?

Last time: Hoare rule application is nicer than using operational semantic.

BUT:

- it's still kind of tedious
- it seems boring & mechanical

Automation?

Slide 4

Invariant

Problem: While – need creativity to find right (invariant) P

Solution:

- ➜ annotate program with invariants
- ➜ then, Hoare rules can be applied automatically

Example:

$\{M = 0 \wedge N = 0\}$
WHILE $M \neq a$ INV $\{N = M * b\}$ DO $N := N + b; M := M + 1$ OD
$\{N = a * b\}$

Slide 5

Weakest Preconditions

pre c Q = weakest P such that $\{P\}\ c\ \{Q\}$

With annotated invariants, easy to get:

pre SKIP Q	$=$	Q
pre $(x := a)$ Q	$=$	$\lambda\sigma.\ Q(\sigma(x := a\sigma))$
pre $(c_1; c_2)$ Q	$=$	pre c_1 (pre c_2 Q)
pre (IF b THEN c_1 ELSE c_2) Q	$=$	$\lambda\sigma.\ (b \longrightarrow$ pre c_1 Q $\sigma) \wedge$ $(\neg b \longrightarrow$ pre c_2 Q $\sigma)$
pre (WHILE b INV I DO c OD) Q	$=$	I

Slide 6

Verification Conditions

$\{$pre c $Q\}\ c\ \{Q\}$ **only true under certain conditions**

These are called **verification conditions** vc c Q:

vc SKIP Q	$=$	True
vc $(x := a)$ Q	$=$	True
vc $(c_1; c_2)$ Q	$=$	vc c_2 Q $\wedge$ (vc c_1 (pre c_2 Q))
vc (IF b THEN c_1 ELSE c_2) Q	$=$	vc c_1 Q $\wedge$ vc c_2 Q
vc (WHILE b INV I DO c OD) Q	$=$	$(\forall\sigma.\ I\sigma \wedge b\sigma \longrightarrow$ pre c I $\sigma) \wedge$ $(\forall\sigma.\ I\sigma \wedge \neg b\sigma \longrightarrow Q\ \sigma) \wedge$ vc c I

vc c Q $\wedge$ $(P \Longrightarrow$ pre c $Q) \Longrightarrow \{P\}\ c\ \{Q\}$

Slide 7

Syntax Tricks

- ➜ $x := \lambda\sigma.\ 1$ instead of $x := 1$ sucks
- ➜ $\{\lambda\sigma.\ \sigma\ x = n\}$ instead of $\{x = n\}$ sucks as well

Problem: program variables are functions, not values

Solution: distinguish program variables syntactically

Choices:

- ➜ declare program variables with each Hoare triple
  - nice, usual syntax
  - works well if you state full program and only use vcg
- ➜ separate program variables from Hoare triple (use extensible records), indicate usage as function syntactically
  - more syntactic overhead
  - program pieces compose nicely

Slide 8

DEMO

Slide 9

Arrays

Depending on language, model arrays as functions:

- Array access = function application:
  a[i] = a i
- Array update = function update:
  a[i] :== v = a :== a(i:= v)

Use lists to express length:

- Array access = nth:
  a[i] = a ! i
- Array update = list update:
  a[i] :== v = a :== a[i:= v]
- Array length = list length:
  a.length = length a

Slide 10

Pointers

Choice 1

datatype ref = Ref int | Null
types heap = int ⇒ val
datatype val = Int int | Bool bool | Struct_x int int bool | . . .

- hp :: heap, p :: ref
- Pointer access: *p = the_Int (hp (the_addr p))
- Pointer update: *p :== v = hp :== hp ((the_addr p) := v)

- a bit klunky
- gets even worse with structs
- lots of value extraction (the_Int) in spec and program

Slide 11

Pointers

Choice 2 (Burstall '72, Bornat '00)

struct with next pointer and element

datatype ref = Ref int | Null
types next_hp = int ⇒ ref
types elem_hp = int ⇒ int

- next :: next_hp, elem :: elem_hp, p :: ref
- Pointer access: p→next = next (the_addr p)
- Pointer update: p→next :== v = next :== next ((the_addr p) := v)

- a separate heap for each struct field
- buys you p→next ≠ p→elem automatically (aliasing)
- still assumes type safe language

Slide 12

DEMO

Slide 13

We have seen today ...

- ➜ Weakest precondition
- ➜ Verification conditions
- ➜ Example program proofs
- ➜ Arrays, pointers

Slide 14