

**COMP 4161**  
NICTA Advanced Course

**Advanced Topics in Software Verification**

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**Content**

- Intro & motivation, getting started [1]
- Foundations & Principles
  - Lambda Calculus, natural deduction [1,2]
  - Higher Order Logic [3<sup>a</sup>]
  - Term rewriting [4]
- Proof & Specification Techniques
  - Inductively defined sets, rule induction [5]
  - Datatypes, recursion, induction [6, 7]
  - Hoare logic, proofs about programs, C verification [8<sup>b</sup>, 9]
  - (mid-semester break)
  - Writing Automated Proof Methods [10]
  - Isar, codegen, typeclasses, locales [11<sup>c</sup>, 12]

<sup>a</sup>a1 due; <sup>b</sup>a2 due; <sup>c</sup>a3 due

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**Overview**

**Automatic Proof and Disproof**

- Sledgehammer: automatic proofs
- Quickcheck: counter example by testing
- Nipick: counter example by SAT

Based on slides by Jasmin Blanchette, Lukas Bulwahn, and Tobias Nipkow (TUM).

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**Automation**

Dramatic improvements in fully automated proofs in the last 2 decades.

- First-order logic (ATP): Otter, Vampire, E, SPASS
- Propositional logic (SAT): MiniSAT, Chaff, RSat
- SAT modulo theory (SMT): CVC3, Yices, Z3

**The key:**

Efficient reasoning engines, and **restricted logics**.

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## Automation in Isabelle



**1980s** rule applications, write ML code

**1990s** simplifier, automatic provers (blast, auto), arithmetic

**2000s** embrace external tools, but don't trust them (ATP/SMT/SAT)

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## DEMO: SLEDGEHAMMER



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## Sledgehammer

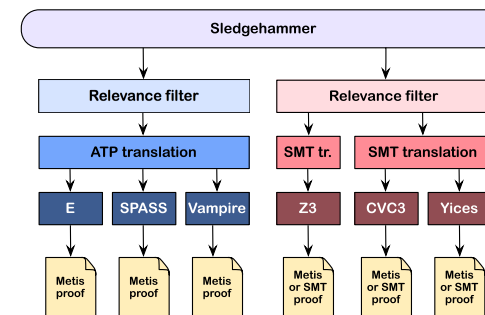


### Sledgehammer:

- Connects Isabelle with ATPs and SMT solvers:  
[E](#), [SPASS](#), [Vampire](#), [CVC3](#), [Yices](#), [Z3](#)
- Simple invocation:
  - Users don't need to select or know facts
  - or ensure the problem is first-order
  - or know anything about the automated prover
- Exploits local parallelism and remote servers

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## Sledgehammer Architecture



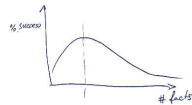
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## Fact Selection



### Provers perform poorly if given 1000s of facts.

- Best number of facts depends on the prover
- Need to take care which facts we give them
- Idea: order facts by relevance, give top  $n$  to prover ( $n = 250, 1000, \dots$ )
- Meng & Paulson method: **lightweight, symbol-based filter**
- Machine learning method:  
look at previous proofs to get a probability of relevance



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## From HOL to FOL



**Source:** higher-order, polymorphism, type classes

**Target:** first-order, untyped or simply-typed

- **First-order:**
  - SK combinators,  $\lambda$ -lifting
  - Explicit function application operator
- **Encode types:**
  - Monomorphise (generate multiple instances), or
  - Encode polymorphism on term level

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## Reconstruction



### We don't want to trust the external provers.

Need to check/reconstruct proof.

- Re-find using Metis  
**Usually fast and reliable** (sometimes too slow)
- Rerun external prover for trusted replay  
Used for SMT. **Re-runs prover each time!**
- Recheck stored explicit external representation of proof  
Used for SMT, no need to re-run. **Fragile.**
- Recast into structured Isar proof  
Fast, **experimental.**

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## Judgement Day

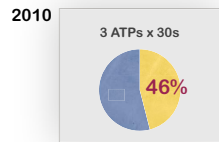


### Evaluating Sledgehammer:

- 1240 goals out of 7 existing theories.
- How many can sledgehammer solve?
- **2010:** E, SPASS, Vampire (for 5-120s). 46%  
 $ESV \times 5s \approx V \times 120s$
- **2011:** Add E-SInE, CVC2, Yices, Z3 (30s).  
 $Z3 > V$
- **2012:** Better integration with SPASS. 64%  
**SPASS best (small margin)**
- **2013:** Machine learning for fact selection. 69%  
**Improves a few percent across provers.**

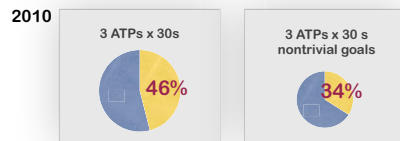
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Evaluation



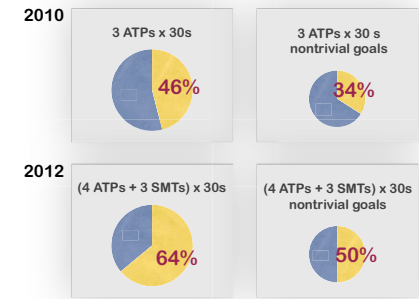
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Evaluation



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Evaluation



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Sledgehammer rules!



**Example application:**

- Large Isabelle/HOL repository of algebras for modelling imperative programs (Kleene Algebra, Hoare logic, . . . , ≈ 1000 lemmas)
- Intricate refinement and termination theorems
- Sledgehammer and Z3 automate algebraic proofs at textbook level.

"The integration of ATP, SMT, and Nitpick is for our purposes very very helpful." – G. Struth

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## DISPROOF

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### Theorem proving and testing

Testing can show only the presence of errors, but not their absence. (Dijkstra)

Testing cannot prove theorems, but it can refute conjectures!

#### Sad facts of life:

- Most lemma statements are wrong the first time.
- Theorem proving is expensive as a debugging technique.

Find counter examples automatically!

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### Quickcheck



#### Lightweight validation by testing.

- Motivated by Haskell's QuickCheck
- Uses Isabelle's code generator
- Fast
- Runs in background, proves you wrong as you type.

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### Quickcheck



#### Covers a number of testing approaches:

- Random and exhausting testing.
- Smart test data generators.
- Narrowing-based (symbolic) testing.

Creates test data generators automatically.

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## DEMO: QUICKCHECK

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## Test generators for datatypes



### Fast iteration in continuation-passing-style

**datatype**  $\alpha$  list = Nil | Cons  $\alpha$  ( $\alpha$  list)

#### Test function:

$\text{test}_{\alpha \text{ list}} P = P \text{ Nil } \text{andalso } \text{test}_{\alpha} (\lambda x. \text{test}_{\alpha \text{ list}} (\lambda xs. P (\text{Cons } x \text{ xs})))$

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## Test generators for predicates



$\text{distinct } xs \implies \text{distinct } (\text{remove1 } x \text{ xs})$

#### Problem:

Exhaustive testing creates many useless test cases.

#### Solution:

Use definitions in precondition for smarter generator.

Only generate cases where *distinct xs* is true.

$\text{test-distinct}_{\alpha \text{ list}} P = P \text{ Nil } \text{andalso}$

$\text{test}_{\alpha} (\lambda x. \text{test-distinct}_{\alpha \text{ list}} (\text{if } x \notin xs \text{ then } (\lambda xs. P (\text{Cons } x \text{ xs})) \text{ else True}))$

Use data flow analysis to figure out which variables must be computed and which generated.

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## Narrowing



### Symbolic execution with demand-driven refinement

→ Test cases can contain variables

→ If execution cannot proceed: instantiate with further symbolic terms

### Pays off if large search spaces can be discarded:

$\text{distinct } (\text{Cons } 1 (\text{Cons } 1 \ x))$

False for any  $x$ , no further instantiations for  $x$  necessary.

#### Implementation:

Lazy execution with outer refinement loop.

Many re-computations, but fast.

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## Quickcheck Limitations

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### Only **executable** specifications!

- No equality on functions with infinite domain
- No axiomatic specifications

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## NITPICK

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## Nitpick

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### Finite model finder

- Based on SAT via Kodkod (backend of Alloy prover)
- Soundly approximates infinite types

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## Nitpick Successes

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- Algebraic methods
- C++ memory model
- Found soundness bugs in TPS and LEO-II

### Fan mail:

"Last night I got stuck on a goal I was sure was a theorem. After 5–10 minutes I gave Nitpick a try, and within a few secs it had found a splendid counterexample—despite the mess of locales and type classes in the context!"

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## DEMO: NITPICK

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We have seen today ...



- Proof: Sledgehammer
- Counter examples: Quickcheck
- Counter examples: Nitpick

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