

#### **COMP 4161**

NICTA Advanced Course

# **Advanced Topics in Software Verification**

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# more Isar

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 $<sup>^</sup>a$ a1 due;  $^b$ a2 due;  $^c$ session break;  $^d$ a3 due

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Last time ... Isar!



- → syntax: proof, qed, assume, from, show, have, next
- → modes: prove, state, chain
- → backward/forward reasoning
- → fix, obtain
- → abbreviations: this, then, thus, hence, with, ?thesis
- → moreover, ultimately
- → case distinction

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Today



- → Datatypes in Isar
- → Calculational reasoning



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# **DATATYPES IN ISAR**

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# Slide 6

#### Structural induction for type nat



```
\begin{array}{lll} \textbf{show} \ P \ n \\  & \textbf{proof} \ (\textbf{induct} \ n) \\  & \textbf{case} \ 0 \\ & \cdots \\ & \textbf{show} \ ?case \\  & \textbf{next} \\ & \textbf{case} \ (\textbf{Suc} \ n) \\ & \cdots \\ & \textbf{show} \ ?case \\ & \textbf{et} \ ?case = P \ (\textbf{Suc} \ n) \\ & \cdots \\ & \textbf{show} \ ?case \\ & \textbf{qed} \end{array}
```

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# Structural induction with $\Longrightarrow$ and $\land$



```
tion with \Longrightarrow and \bigwedge NICTA
```

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# **DEMO: DATATYPES IN ISAR**

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# **CALCULATIONAL REASONING**

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#### The Goal



Prove:

$$x \cdot x^{-1} = 1$$

using: assoc:  $(x \cdot y) \cdot z = x \cdot (y \cdot z)$ 

 $\begin{array}{ll} \text{left\_inv:} & x^{-1} \cdot x = 1 \\ \text{left\_one:} & 1 \cdot x = x \end{array}$ 

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# The Goal



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#### Prove:

### Can we do this in Isabelle?

- → Simplifier: too eager
- → Manual: difficult in apply style
- → Isar: with the methods we know, too verbose

#### Chains of equations



#### The Problem

$$\begin{array}{rcl}
a & = & b \\
\dots & = & c \\
\dots & = & d
\end{array}$$

shows a = d by transitivity of =

Each step usually nontrivial (requires own subproof)

#### Solution in Isar:

- → Keywords also and finally to delimit steps
- → ...: predefined schematic term variable, refers to right hand side of last expression
- → Automatic use of transitivity rules to connect steps

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#### also/finally



have " $t_0 = t_1$ " [proof]	calculation registe
also	" $t_0 = t_1$ "

have "... =  $t_2$ " [proof]

also "
$$t_0 = t_2$$
"

also " $t_0 = t_{n-1}$ "

have "  $\cdots = t_n$  " [proof]  $t_0 = t_n$ 

show P

— 'finally' pipes fact " $t_0 = t_n$ " into the proof

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# More about also



- → Works for all combinations of =, < and <.
- → Uses all rules declared as [trans].
- → To view all combinations: print\_trans\_rules

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# Designing [trans] Rules



$$\label{eq:lambda} \begin{split} & \mathbf{have} = "l_1 \odot r_1" \, [\mathsf{proof}] \\ & \mathbf{also} \\ & \mathbf{have} \; ". \ldots \odot r_2" \, [\mathsf{proof}] \\ & \mathbf{also} \end{split}$$

# Anatomy of a [trans] rule:

- $\rightarrow$  Usual form: plain transitivity  $[l_1 \odot r_1; r_1 \odot r_2] \Longrightarrow l_1 \odot r_2$
- ightharpoonup More general form:  $\llbracket P\ l_1\ r_1; Q\ r_1\ r_2; A 
  rbracket \Longrightarrow C\ l_1\ r_2$

#### Examples:

- $\rightarrow$  pure transitivity:  $[a=b;b=c] \Longrightarrow a=c$
- ightharpoonup mixed:  $[\![a \leq b; b < c]\!] \Longrightarrow a < c$
- → substitution:  $\llbracket P \ a; a = b \rrbracket \Longrightarrow P \ b$
- $\rightarrow$  antisymmetry:  $[a < b; b < a] \Longrightarrow P$
- → monotonicity:  $\llbracket a = f \ b; b < c; \bigwedge x \ y. \ x < y \Longrightarrow f \ x < f \ y \rrbracket \Longrightarrow a < f \ c$



**CODE GENERATION** 

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# HOL as programming language



#### We have

- → numbers, arithmetic
- → recursive datatypes
- → constant definitions, recursive functions
- → = a functional programming language
- → can be used to get fully verified programs

# Executed using the simplifier. But:

- → slow, heavy-weight
- → does not run stand-alone (without Isabelle)

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# Generating code



Translate HOL functional programming concepts, i.e.

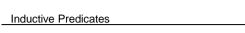
- → datatypes
- → function definitions
- → inductive predicates

# into a stand-alone code in:

- → SML
- → Ocaml
- → Haskell
- → Scala

Syntax	NIGTA	Program Refinement	O •
export_code <definition_names> in SML</definition_names>	NICTA	Aim: choosing appropriate code equations explicitly	NICTA
module_name <module_name> file "<file path="">"</file></module_name>		Syntax:	
export_code <definition_names> in Haskell module_name <module_name> file "<directory path="">"</directory></module_name></definition_names>		lemma [code]: <li><li><li>&lt; f quations on function_name&gt;</li></li></li>	
Takes a space-separated list of constants for which code shall	be generated.	Example: more efficient definition of fibonnacci function	
Anything else needed for those is added implicitly. Generates	ML stucture.		
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Inductive specifications turned into equational ones

# Example:

```
append [] ys ys  \text{append xs ys zs} \Longrightarrow \text{append (x \# xs ) ys (x \# zs )}
```

# Syntax:

code\_pred append .

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We have seen today ...



- → Datatypes in Isar
- → Calculations: also/finally
- → [trans]-rules
- → Code generation

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