



COMP 4161
NICTA Advanced Course

Advanced Topics in Software Verification

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Content

- Intro & motivation, getting started [1]
- Foundations & Principles
 - Lambda Calculus, natural deduction [1,2]
 - Higher Order Logic [3]
 - Term rewriting [4^a]
- Proof & Specification Techniques
 - Inductively defined sets, rule induction [5]
 - Datatypes, recursion, induction [6, 7]
 - Code generation, type classes [7]
 - Hoare logic, proofs about programs, refinement [8^b,9^c,10]
 - Isar, locales [11^d,12]

^aa1 due; ^ba2 due; ^csession break; ^da3 due

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Overview

Automatic Proof and Disproof

- Sledgehammer: automatic proofs
- Quickcheck: counter example by testing
- Nipick: counter example by SAT

Based on slides by Jasmin Blanchette, Lukas Bulwahn, and Tobias Nipkow (TUM).

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Automation

Dramatic improvements in fully automated proofs in the last 2 decades.

- First-order logic (ATP): Otter, Vampire, E, SPASS
- Propositional logic (SAT): MiniSAT, Chaff, RSat
- SAT modulo theory (SMT): CVC3, Yices, Z3

The key:

Efficient reasoning engines, and **restricted logics**.

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Automation in Isabelle



1980s rule applications, write ML code

1990s simplifier, automatic provers (blast, auto), arithmetic

2000s embrace external tools, but don't trust them (ATP/SMT/SAT)

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DEMO: SLEDGEHAMMER



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Sledgehammer

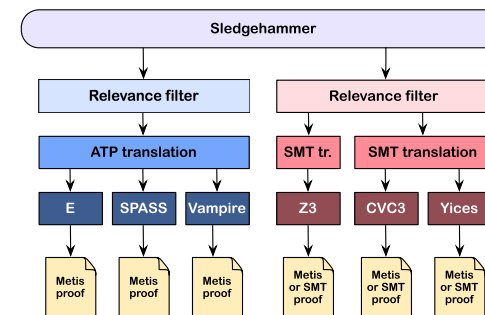


Sledgehammer:

- Connects Isabelle with ATPs and SMT solvers:
[E](#), [SPASS](#), [Vampire](#), [CVC3](#), [Yices](#), [Z3](#)
- Simple invocation:
 - Users don't need to select or know facts
 - or ensure the problem is first-order
 - or know anything about the automated prover
- Exploits local parallelism and remote servers

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Sledgehammer Architecture



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Fact Selection



Provers perform poorly if given 1000s of facts.

- Best number of facts depends on the prover
- Need to take care which facts we give them
- Idea: order facts by relevance, give top n to prover ($n = 250, 1000, \dots$)
- Meng & Paulson method: **lightweight, symbol-based filter**
- Machine learning method:
look at **previous proofs to get a probability of relevance**



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From HOL to FOL



Source: higher-order, polymorphism, type classes
Target: first-order, untyped or simply-typed

- **First-order:**
 - SK combinators, λ -lifting
 - Explicit function application operator
- **Encode types:**
 - Monomorphise (generate multiple instances), or
 - Encode polymorphism on term level

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Reconstruction



We don't want to trust the external provers.

Need to check/reconstruct proof.

- Re-find using Metis
Usually fast and reliable (sometimes too slow)
- Rerun external prover for trusted replay
Used for SMT. **Re-runs prover each time!**
- Recheck stored explicit external representation of proof
Used for SMT, no need to re-run. **Fragile.**
- Recast into structured Isar proof
Fast, **experimental.**

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Judgement Day

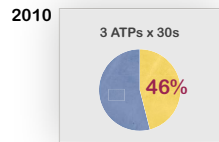


Evaluating Sledgehammer:

- 1240 goals out of 7 existing theories.
- How many can sledgehammer solve?
- **2010:** E, SPASS, Vampire (for 5-120s). 46%
 $ESV \times 5s \approx V \times 120s$
- **2011:** Add E-SInE, CVC2, Yices, Z3 (30s).
 $Z3 > V$
- **2012:** Better integration with SPASS. 64%
SPASS best (small margin)
- **2013:** Machine learning for fact selection. 69%
Improves a few percent across provers.

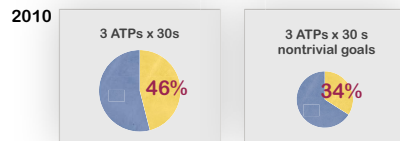
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Evaluation



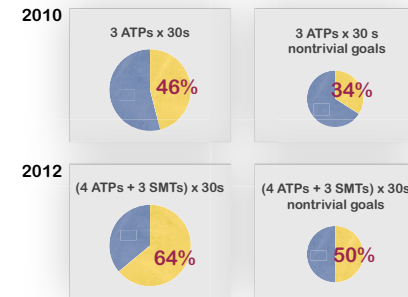
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Evaluation



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Evaluation



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Sledgehammer rules!



Example application:

- Large Isabelle/HOL repository of algebras for modelling imperative programs (Kleene Algebra, Hoare logic, . . . , ≈ 1000 lemmas)
- Intricate refinement and termination theorems
- Sledgehammer and Z3 automate algebraic proofs at textbook level.

"The integration of ATP, SMT, and Nitpick is for our purposes very very helpful." – G. Struth

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DISPROOF

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Theorem proving and testing

Testing can show only the presence of errors, but not their absence. (Dijkstra)

Testing cannot prove theorems, **but it can refute conjectures!**

Sad facts of life:

- Most lemma statements are wrong the first time.
- Theorem proving is expensive as a debugging technique.

Find counter examples automatically!

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Quickcheck



Lightweight validation by testing.

- Motivated by Haskell's QuickCheck
- Uses Isabelle's code generator
- Fast
- Runs in background, proves you wrong as you type.
(current version: PG only, next release also in jEdit)

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Quickcheck



Covers a number of testing approaches:

- Random and exhausting testing.
- Smart test data generators.
- Narrowing-based (symbolic) testing.

Creates test data generators automatically.

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DEMO: QUICKCHECK

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Test generators for datatypes



Fast iteration in continuation-passing-style

datatype α list = Nil | Cons α (α list)

Test function:

$\text{test}_{\alpha \text{ list}} P = P \text{ Nil } \text{andalso } \text{test}_{\alpha} (\lambda x. \text{test}_{\alpha \text{ list}} (\lambda xs. P (\text{Cons } x \text{ xs})))$

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Test generators for predicates



$\text{distinct } xs \implies \text{distinct } (\text{remove1 } x \text{ xs})$

Problem:

Exhaustive testing creates many useless test cases.

Solution:

Use definitions in precondition for smarter generator.

Only generate cases where *distinct xs* is true.

$\text{test-distinct}_{\alpha \text{ list}} P = P \text{ Nil } \text{andalso}$

$\text{test}_{\alpha} (\lambda x. \text{test-distinct}_{\alpha \text{ list}} (\text{if } x \notin xs \text{ then } (\lambda xs. P (\text{Cons } x \text{ xs})) \text{ else True}))$

Use data flow analysis to figure out which variables must be computed and which generated.

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Narrowing



Symbolic execution with demand-driven refinement

→ Test cases can contain variables

→ If execution cannot proceed: instantiate with further symbolic terms

Pays off if large search spaces can be discarded:

$\text{distinct } (\text{Cons } 1 (\text{Cons } 1 \ x))$

False for any x , no further instantiations for x necessary.

Implementation:

Lazy execution with outer refinement loop.

Many re-computations, but fast.

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Quickcheck Limitations



Only **executable** specifications!

- No equality on functions with infinite domain
- No axiomatic specifications

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Nitpick



Finite model finder

- Based on SAT via Kodkod (backend of Alloy prover)
- Soundly approximates infinite types

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NITPICK

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Nitpick Successes



- Algebraic methods
- C++ memory model
- Found soundness bugs in TPS and LEO-II

Fan mail:

"Last night I got stuck on a goal I was sure was a theorem. After 5–10 minutes I gave Nitpick a try, and within a few secs it had found a splendid counterexample—despite the mess of locales and type classes in the context!"

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DEMO: NITPICK

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We have seen today ...



- Proof: Sledgehammer
- Counter examples: Quickcheck
- Counter examples: Nitpick

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