COMP 4161
NICTA Advanced Course

Advanced Topics in Software Verification

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C

Last Time

- ➜ Verifying C by translating into Simpl
- ➜ Expressions
- ➜ C control flow
- ➜ Exceptions with Hoare logic rules
- ➜ C functions and procedures with Hoare logic rules

Content

- ➜ Intro & motivation, getting started [1]
- ➜ Foundations & Principles
  - Lambda Calculus, natural deduction [1,2]
  - Higher Order Logic [3[a]]
  - Term rewriting [4]
- ➜ Proof & Specification Techniques
  - Isar [5]
  - Inductively defined sets, rule induction [6[b]]
  - Datatypes, recursion, induction [7[c], 8]
  - Calculational reasoning, code generation [9]
  - Hoare logic, proofs about programs [10[d],11,12]

[a] a1 due; [b] a2 due; [c] session break; [d] a3 due

C

Main new problems in verifying C programs:

- ➜ expressions with side effects
- ➜ more control flow (do/while, for, break, continue, return)
- ➜ local variables and blocks
- ➜ functions & procedures
- ➜ **prevent undefined execution**
- ➜ **concrete C data types**
- ➜ **C memory model and C pointers**

Undefined Execution

In C, we're not allowed to:

- ➜ divide by zero
- ➜ shift more than <architecture defined> bits
- ➜ dereference a Null pointer
- ➜ access outside array bounds
- ➜ access unallocated memory
- ➜ free unallocated memory
- ➜ . . .

Their absence should become proof obligations.

Simpl Guards

Syntax:

`Guard 'f "'s bexp" "('s,'p,'f) com"`

Semantics:

$$[\![s \in g;\ \Gamma \vdash (c, \text{Normal } s) \Rightarrow t]\!] \Longrightarrow \Gamma \vdash (\text{Guard } f\ g\ c, \text{Normal } s) \Rightarrow t$$

$$s \notin g \Longrightarrow \Gamma \vdash (\text{Guard } f\ g\ c, \text{Normal } s) \Rightarrow \text{Fault } f$$

Hoare rules:

$$\frac{\Gamma \vdash_F \{g \wedge P\}\ c\ \{Q\}}{\Gamma \vdash_F \{g \wedge P\}\ \text{Guard } f\ g\ c\ \{Q\}} \qquad \frac{f \in F \quad \Gamma \vdash_F \{g \wedge P\}\ c\ \{Q\}}{\Gamma \vdash_F \{P\}\ \text{Guard } f\ g\ c\ \{Q\}}$$

Simpl Guards: Why two Hoare rules?

Why two Hoare rules?

So we can separate out verification of guards.

F controls which guards are currently assumed and which are proved.

Example:

Do automated verification of array guards separately
⇒ get to assume array guards "for free" in the rest.

Simpl Guards: Why two Hoare rules?

Use Guards for:
Every time an expression or statement does something potentially undefined, add a guard in the translation.

Example:

`x = a / b` ⇒ Guard DivByZero (b $\neq$ 0) (x :== a / b)

DEMO: GUARDS

C data types

Next problem: C data types

C has the following types:

- ➜ basic: int (long/short, signed/unsigned), char, void, float, double, long double
- ➜ enum types
- ➜ pointers: type*
- ➜ array types: type[n], type[n][m], type[]
- ➜ struct types: like records, but can use recursion for pointers
- ➜ unions: multiple interpretations of same memory content
- ➜ function pointers

Size of basic types is architecture dependent.
Encoding in memory partially compiler dependent.

Basic types

- ➜ float/double ⇒ IEEE floating point numbers, no Isabelle formalisation yet. (Any takers?)
- ➜ void ⇒ unit type in Isabelle
- ➜ integer types ⇒ finite machine words (x mod 2^{32} etc)

Why bother with finite words? Why not nat/real?

Want to model overflow precisely.

Depending on application, could work with nat and guards instead.

Binary Search (`java.util.Arrays`)

```
1:    public static int binarySearch(int[] a, int key) {
2:        int low = 0;
3:        int high = a.length - 1;
4:
5:        while (low <= high) {
6:            int mid = (low + high) / 2;
7:            int midVal = a[mid];
8:
9:            if (midVal < key)
10:                low = mid + 1
11:            else if (midVal > key)
12:                high = mid - 1;
13:            else
14:                return mid; // key found
15:        }
16:        return -(low + 1);  // key not found.
17:    }
```

```
6:            int mid = (low + high) / 2;
```

http://googleresearch.blogspot.com/2006/06/extra-extra-read-all-about-it-nearly.html

Machine Words

Goal: want to write things like

$$x \text{ AND } y = 0 \Longrightarrow x + y = x \text{ OR } y$$

$$(x << n) \text{ !! } m = x \text{ !! } (n + m)$$

$$x << 2 = 4 * x \qquad \text{ucast } (y + \text{0xFF21}) = (x - \text{0b01001011})$$

$$\text{unat } x + \text{unat } y < 2\hat{}\ \text{word\_size} \Longrightarrow \text{unat } (x + y) = \text{unat } x + \text{unat } y$$

$$x :: 32 \text{ word} \qquad y :: 8 \text{ word} \qquad z :: n \text{ word}$$

AND bitwise and, OR bitwise or, !! test bit at position n, $<<$ shift left, ”ucast” cast between word sizes, ”unat” convert words to nat

Formalisation Idea

Goal:
Create an Isabelle type that captures machine words of length n

Problem:
The parameter n is not a type, but a value.
This is called a **dependent type**.
Isabelle does not support dependent types.

Solutions: make a type ’a word, encode length in type ’a

- either implicitly as number of elements in ’a,
- or explicitly via type class function

Formalisation in Isabelle

Type class used in HOL/Word/Word.thy:

- ’a must be class len
- class len has function len_of :: ’a itself ⇒ nat
- to implement class len, a type must provide that function

’a itself:

- ’a itself is a type with one element of type ’a
- the one element is written TYPE(’a)

Numeric types in Library/Numeral_Type.thy:

- create types written as numbers (type 1, 16, etc)
- have 1, 16, etc elements
- the numbers are syntax for type constructors encoding 0, 1, 2*n, 2*n+1

Representation (no taxation)

Now can encode length. How do we represent words?

Options:

- nat mod 2ˆ n
- int mod 2ˆ n
- bool lists of length n
- test-bit functions nat ⇒ bool

All of these are equivalent. Actual definition in Isabelle is int mod 2ˆ n.

All others are provided as well as simulated type defs.

Operators

Rest is standard (see HOL/Word/Word.thy + HOL/Word/Examples/):

- define standard arithmetic and bit-wise operators with syntax
- prove lemmas connecting to known type representations
- determine abstract structure:
  commutative ring with 1, partial order, boolean algebra for bitwise ops, etc
- prove library with characteristic properties
- provide some automation: smt connection, auto cast to nat
- . . .
- profit

DEMO: WORD

C Data types

Can now represent all C types apart from float.

(Making explicit architecture assumptions on size etc.)

- integer types (incl enum): word
- pointers: **datatype 'a ptr = 32 word**
- arrays: pointers or array types in Isabelle
- structs: records or data types
- unions: separate struct types with conversions
- function pointers: word

Missing: modelling C memory

C Memory Model

Heap models so far:

- addr ⇒ obj option
- separate heaps by type
- separate heaps by record field

C is more ugly:

- pointer arithmetic and casting breaks type safety
- objects could overlap
- objects can be access under different types (union)
- systems programmers might rely on data layout (device access)
- could have pointers into stack (reference to local var)

Our model solves all but the last one.

(Can also solve that one, but it gets even more ugly.)

C Memory Model

The Memory Model:
Heap = function ”32 word ⇒ 8 word”

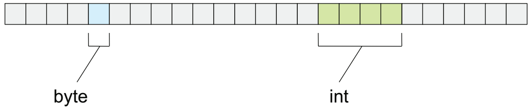
That’s it.
Ok, not quite: It’s the basis. We build a whole machinery on top.

Basic idea:

- 32 word ⇒ 8 word is the information that C runtime has
- we store additional type information for proofs (ghost state)
- use that type information to automatically get abstract Isabelle objects from heap
- if we stay in type-safe fragment of C, can reason like in separate heaps.

C Memory Model Diagram (1)

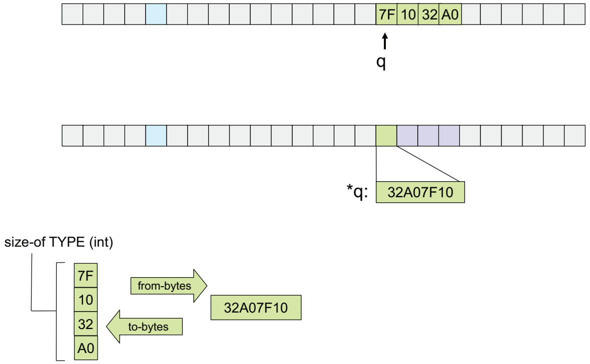
- basic function ”32 word ⇒ 8 word”
- additional type information for regions of memory



byte

int

C Memory Model Diagram (2)



7F 10 32 A0

q

*q: 32A07F10

size-of TYPE (int)

7F
10
32
A0

from-bytes

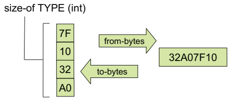
to-bytes

32A07F10

Encoding Type Information

Another type class:

- for Isabelle types ’a that represent C types
- from-bytes :: 8 word list ⇒ ’a option
- to-bytes :: ’a ⇒ 8 word list
- size-of :: ’a itself ⇒ nat
- tag :: ’a itself ⇒ typ-tag



size-of TYPE (int)

7F
10
32
A0

from-bytes

to-bytes

32A07F10

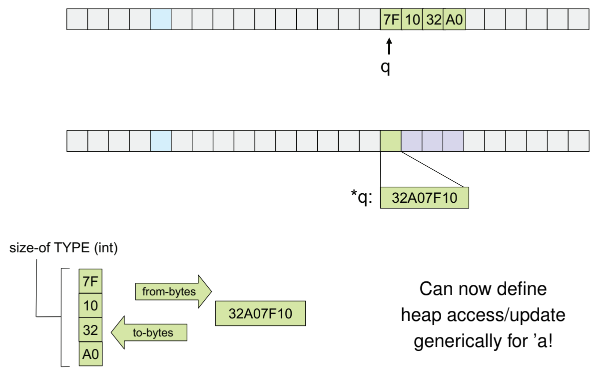
Laws:

- from-bytes (to-bytes v) = Some v
- length (to-bytes (v::’a)) = size-of TYPE(’a)

Example picture unsigned int = 32 word (depending on architecture):

- from-bytes/to-bytes = big/little endian encoding (depending on architecture)
- size-of = 4
- tag = ”32 word”

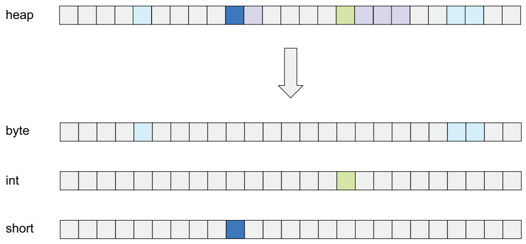
Encoding Type Information



Can now define
heap access/update
generically for 'a!

C Memory Model Diagram (3)

Goal:

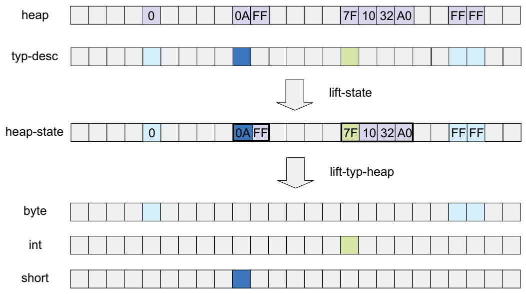


Separate Heaps

Plan:

- ➜ combine type info and real heap into one object typed-hp
- ➜ write 'view' function lift :: typed-hp ⇒ ('a ptr ⇒ 'a option)
- ➜ models type-safe heap access
- ➜ returns None if request type 'a does not match type in memory

C Memory Model Diagram (4)



Separate Heaps Properties

Lemmas about lift and heap-update:

If lift hp (p :: 'a ptr) $\neq$ None, then

- ➜ lift$_{'a}$ (heap-update p v hp) = (lift$_{'a}$ hp) (p $\mapsto$ v)
- ➜ TYPE('a) $\perp$ TYPE('b) $\Longrightarrow$ lift$_{'b}$ (heap-update p v hp) = lift $_{'b}$

where TYPE('a) $\perp$ TYPE('b) = the two types are disjoint.

This means 'lift' works like a separate heap for each type!

DEMO: POINTERS

DEMO: C PROGRAM TRANSLATION

We have seen today ...

- ➜ preventing undefined execution
- ➜ finite machine words
- ➜ concrete C data types
- ➜ C memory model and pointers