COMP 4161
NICTA Advanced Course

Advanced Topics in Software Verification

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Content

- ➜ Intro & motivation, getting started [1]
- ➜ Foundations & Principles
  - Lambda Calculus, natural deduction [1,2]
  - Higher Order Logic [3[a]]
  - Term rewriting [4]
- ➜ Proof & Specification Techniques
  - Isar [5]
  - Inductively defined sets, rule induction [6[b]]
  - Datatypes, recursion, induction [7[c], 8]
  - Calculational reasoning, code generation [9]
  - Hoare logic, proofs about programs [10[d],11,12]

[a] a1 due; [b] a2 due; [c] session break; [d] a3 due

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Last Time

- ➜ More Isar
- ➜ Fix/Obtain
- ➜ Moreover/Ultimately
- ➜ Mixing Proof Styles

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SPECIFICATION TECHNIQUES: SETS

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Sets in Isabelle

Type **'a set**: sets over type 'a

- $\{\}$, $\{e_1, \ldots, e_n\}$, $\{x.\ P\ x\}$
- $e \in A$, $A \subseteq B$
- $A \cup B$, $A \cap B$, $A - B$, $-A$
- $\bigcup x \in A.\ B\ x$, $\bigcap x \in A.\ B\ x$, $\bigcap\ A$, $\bigcup A$
- $\{i..j\}$
- insert :: $\alpha \Rightarrow \alpha$ set $\Rightarrow \alpha$ set
- $f`A \equiv \{y.\ \exists x \in A.\ y = f\ x\}$
- $\ldots$

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Proofs about Sets

Natural deduction proofs:

- equalityI: $[\![A \subseteq B;\ B \subseteq A]\!] \Longrightarrow A = B$
- subsetI: $(\bigwedge x.\ x \in A \Longrightarrow x \in B) \Longrightarrow A \subseteq B$
- $\ldots$ (see Tutorial)

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Bounded Quantifiers

- $\forall x \in A.\ P\ x \equiv \forall x.\ x \in A \longrightarrow P\ x$
- $\exists x \in A.\ P\ x \equiv \exists x.\ x \in A \wedge P\ x$

- ballI: $(\bigwedge x.\ x \in A \Longrightarrow P\ x) \Longrightarrow \forall x \in A.\ P\ x$
- bspec: $[\![\forall x \in A.\ P\ x; x \in A]\!] \Longrightarrow P\ x$

- bexI: $[\![P\ x; x \in A]\!] \Longrightarrow \exists x \in A.\ P\ x$
- bexE: $[\![\exists x \in A.\ P\ x; \bigwedge x.\ [\![x \in A; P\ x]\!] \Longrightarrow Q]\!] \Longrightarrow Q$

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DEMO: SETS

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The Three Basic Ways of Introducing Types

➜ **typedecl**: by name only

Example: **typedecl** names
Introduces new type *names* without any further assumptions

➜ **type_synonym**: by abbreviation

Example: **type_synonym** α rel = "$\alpha \Rightarrow \alpha \Rightarrow bool$"
Introduces abbreviation *rel* for existing type $\alpha \Rightarrow \alpha \Rightarrow bool$
Type abbreviations are immediately expanded internally

➜ **typedef**: by definiton as a set

Example: **typedef** new_type = "{some set}" <proof>
Introduces a new type as a subset of an existing type.
The proof shows that the set on the rhs in non-empty.

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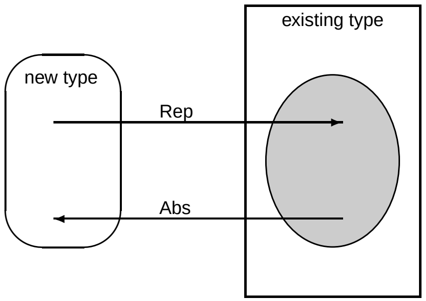
How typedef works



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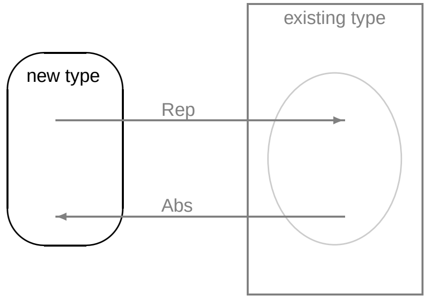
How typedef works



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Example: Pairs

(α, β) Prod

① Pick existing type: $\alpha \Rightarrow \beta \Rightarrow$ bool

② Identify subset:
(α, β) Prod $= \{f.\ \exists a\ b.\ f = \lambda(x :: \alpha)\ (y :: \beta).\ x = a \wedge y = b\}$

③ We get from Isabelle:

- functions Abs_Prod, Rep_Prod
- both injective
- Abs_Prod (Rep_Prod x) $= x$

④ We now can:

- define constants Pair, fst, snd in terms of Abs_Prod and Rep_Prod
- derive all characteristic theorems
- forget about Rep/Abs, use characteristic theorems instead

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DEMO: INTRODUCING NEW TYPES

INDUCTIVE DEFINITIONS

Example

$$\frac{}{\langle \mathsf{skip}, \sigma \rangle \longrightarrow \sigma} \qquad \frac{[\![e]\!]\sigma = v}{\langle \mathsf{x := e}, \sigma \rangle \longrightarrow \sigma[x \mapsto v]}$$

$$\frac{\langle c_1, \sigma \rangle \longrightarrow \sigma' \quad \langle c_2, \sigma' \rangle \longrightarrow \sigma''}{\langle c_1; c_2, \sigma \rangle \longrightarrow \sigma''}$$

$$\frac{[\![b]\!]\sigma = \mathsf{False}}{\langle \mathsf{while}\ b\ \mathsf{do}\ c, \sigma \rangle \longrightarrow \sigma}$$

$$\frac{[\![b]\!]\sigma = \mathsf{True} \quad \langle c, \sigma \rangle \longrightarrow \sigma' \quad \langle \mathsf{while}\ b\ \mathsf{do}\ c, \sigma' \rangle \longrightarrow \sigma''}{\langle \mathsf{while}\ b\ \mathsf{do}\ c, \sigma \rangle \longrightarrow \sigma''}$$

What does this mean?

- ➜ $\langle c, \sigma \rangle \longrightarrow \sigma'$ fancy syntax for a relation $(c, \sigma, \sigma') \in E$
- ➜ relations are sets: $E :: (\mathsf{com} \times \mathsf{state} \times \mathsf{state})\ \mathsf{set}$
- ➜ the rules define a set inductively

But which set?

Simpler Example

$$\frac{}{0 \in N} \qquad \frac{n \in N}{n+1 \in N}$$

- ➜ N is the set of natural numbers $\mathbb{N}$
- ➜ But why not the set of real numbers? $0 \in \mathbb{R}$, $n \in \mathbb{R} \Longrightarrow n+1 \in \mathbb{R}$
- ➜ $\mathbb{N}$ is the **smallest** set that is **consistent** with the rules.

Why the smallest set?

- ➜ Objective: **no junk**. Only what must be in X shall be in X.
- ➜ Gives rise to a nice proof principle (rule induction)
- ➜ Alternative (greatest set) occasionally also useful: coinduction

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Rule Induction

$$\frac{}{0 \in N} \qquad \frac{n \in N}{n+1 \in N}$$

induces induction principle

$$[\![P\ 0;\ \bigwedge n.\ P\ n \Longrightarrow P\ (n+1)]\!] \Longrightarrow \forall x \in X.\ P\ x$$

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DEMO: INDUCTIVE DEFINITONS

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We have learned today ...

- ➜ Sets
- ➜ Type Definitions
- ➜ Inductive Definitions

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