COMP 4161

NICTA Advanced Course

Advanced Topics in Software Verification

Gerwin Klein, June Andronick, Toby Murray

Binary Search (`java.util.Arrays`)

```
1:      public static int binarySearch(int[] a, int key) {
2:          int low = 0;
3:          int high = a.length - 1;
4:
5:          while (low <= high) {
6:              int mid = (low + high) / 2;
7:              int midVal = a[mid];
8:
9:              if (midVal < key)
10:                  low = mid + 1
11:              else if (midVal > key)
12:                  high = mid - 1;
13:              else
14:                  return mid; // key found
15:          }
16:          return -(low + 1);  // key not found.
17:     }
```

```
6:        int mid = (low + high) / 2;
```

http://googleresearch.blogspot.com/2006/06/extra-extra-read-all-about-it-nearly.html

Organisatorials

When	Mon	9:00 – 10:30
	Wed	9:00 – 10:30
Where	Mon:	Hut D10, Room G01
	Wed:	Webster 256

`http://www.cse.unsw.edu.au/~cs4161/`

About us

NICTA

Members of the seL4 verification team

- ➜ Functional correctness of a C microkernel
  Isabelle/HOL model ↔ Haskell model ↔ C code
- ➜ 10 000 LOC / 300 000 lines of proof script (!)
- ➜ 25 person years / $6 million

`http://ertos.nicta.com.au/research/l4.verified/`

We are always embarking on exciting new projects.

We offer

- ➜ summer student scholarship projects
- ➜ honours and PhD theses
- ➜ research assistant and verification engineer positions

What you will learn

- ➜ how to use a theorem prover
- ➜ background, how it works
- ➜ how to prove and specify
- ➜ how to reason about programs

Health Warning

Theorem Proving is addictive

Content — Using Theorem Provers

	Rough timeline
➜ Intro & motivation, getting started	[today]
➜ Foundations & Principles	
• Lambda Calculus, natural deduction	[1,2]
• Higher Order Logic	[3[a]]
• Term rewriting	[4]
➜ Proof & Specification Techniques	
• Isar	[5]
• Inductively defined sets, rule induction	[6[b]]
• Datatypes, recursion, induction	[7[c], 8]
• Calculational reasoning, code generation	[9]
• Hoare logic, proofs about programs	[10[d],11,12]

---

[a] a1 due; [b] a2 due; [c] session break; [d] a3 due

What you should do to have a chance at succeeding

- ➜ attend lectures
- ➜ try Isabelle early
- ➜ redo all the demos alone
- ➜ try the exercises/homework we give, when we do give some
- ➜ DO NOT CHEAT
  - Assignments and exams are take-home. This does NOT mean you can work in groups. Each submission is personal.
  - For more info, see Plagiarism Policy[a]

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[a] `http://www.cse.unsw.edu.au/people/studentoffice/policies/yellowform.html#assign`

Credits

some material (in using-theorem-provers part) shamelessly stolen from

Tobias Nipkow, Larry Paulson, Markus Wenzel

David Basin, Burkhardt Wolff

Don’t blame them, errors are mine

What is a proof?

to prove (Merriam-Webster)

- ➜ from Latin probare (test, approve, prove)
- ➜ to learn or find out by experience (archaic)
- ➜ to establish the existence, truth, or validity of
  (by evidence or logic)
  prove a theorem, the charges were never proved in court

pops up everywhere

- ➜ politics (weapons of mass destruction)
- ➜ courts (beyond reasonable doubt)
- ➜ religion (god exists)
- ➜ science (cold fusion works)

What is a mathematical proof?

In mathematics, a proof is a demonstration that, given certain axioms, some statement of interest is necessarily true. (Wikipedia)

Example: $\sqrt{2}$ is not rational.

Proof: assume there is $r \in \mathbb{Q}$ such that $r^2 = 2$.

Hence there are mutually prime p and q with $r = \frac{p}{q}$.

Thus $2q^2 = p^2$, i.e. p^2 is divisible by 2.

2 is prime, hence it also divides p, i.e. $p = 2s$.

Substituting this into $2q^2 = p^2$ and dividing by 2 gives $q^2 = 2s^2$. Hence, q is also divisible by 2. Contradiction. Qed.

Nice, but..

- ➜ still not rigorous enough for some
  - what are the rules?
  - what are the axioms?
  - how big can the steps be?
  - what is obvious or trivial?
- ➜ informal language, easy to get wrong
- ➜ easy to miss something, easy to cheat

Theorem. A cat has nine tails.

Proof. No cat has eight tails. Since one cat has one more tail than no cat, it must have nine tails.

What is a formal proof?

A derivation in a formal calculus

Example: $A \wedge B \longrightarrow B \wedge A$ derivable in the following system

Rules: $\frac{X \in S}{S \vdash X}$ (assumption) $\frac{S \cup \{X\} \vdash Y}{S \vdash X \longrightarrow Y}$ (impI)

$\frac{S \vdash X \quad S \vdash Y}{S \vdash X \wedge Y}$ (conjI) $\frac{S \cup \{X, Y\} \vdash Z}{S \cup \{X \wedge Y\} \vdash Z}$ (conjE)

Proof:

1. $\{A, B\} \vdash B$ (by assumption)
2. $\{A, B\} \vdash A$ (by assumption)
3. $\{A, B\} \vdash B \wedge A$ (by conjI with 1 and 2)
4. $\{A \wedge B\} \vdash B \wedge A$ (by conjE with 3)
5. $\{\} \vdash A \wedge B \longrightarrow B \wedge A$ (by impI with 4)

What is a theorem prover?

Implementation of a formal logic on a computer.

- ➜ fully automated (propositional logic)
- ➜ automated, but not necessarily terminating (first order logic)
- ➜ with automation, but mainly interactive (higher order logic)

- ➜ based on rules and axioms
- ➜ can deliver proofs

There are other (algorithmic) verification tools:

- ➜ model checking, static analysis, ...
- ➜ usually do not deliver proofs

Why theorem proving?

- ➜ Analysing systems/programs thoroughly
- ➜ Finding design and specification errors early
- ➜ High assurance (mathematical, machine checked proof)
- ➜ it’s not always easy
- ➜ it’s fun

Main theorem proving system for this course

Isabelle

- ➜ used here for applications, learning how to prove

What is Isabelle?

A generic interactive proof assistant

- **generic:**
  not specialised to one particular logic
  (two large developments: HOL and ZF, will mainly use HOL)
- **interactive:**
  more than just yes/no, you can interactively guide the system
- **proof assistant:**
  helps to explore, find, and maintain proofs

Why Isabelle?

- ➜ free
- ➜ widely used systems
- ➜ active development
- ➜ high expressiveness and automation
- ➜ reasonably easy to use
- ➜ (and because we know it best ;-))

If I prove it on the computer, it is correct, right?

If I prove it on the computer, it is correct, right?

No, because:

① hardware could be faulty

② operating system could be faulty

③ implementation runtime system could be faulty

④ compiler could be faulty

⑤ implementation could be faulty

⑥ logic could be inconsistent

⑦ theorem could mean something else

If I prove it on the computer, it is correct, right?

No, but:

probability for

- ➜ OS and H/W issues reduced by using different systems
- ➜ runtime/compiler bugs reduced by using different compilers
- ➜ faulty implementation reduced by right architecture
- ➜ inconsistent logic reduced by implementing and analysing it
- ➜ wrong theorem reduced by expressive/intuitive logics

No guarantees, but assurance immensely higher than manual proof

If I prove it on the computer, it is correct, right?

Soundness architectures

careful implementation	PVS
LCF approach, small proof kernel	HOL4
	Isabelle
explicit proofs + proof checker	Coq
	Twelf
	Isabelle
	HOL4

Meta Logic

Meta language:

The language used to talk about another language.

Examples:

English in a Spanish class, English in an English class

Meta logic:

The logic used to formalize another logic

Example:

Mathematics used to formalize derivations in formal logic

Meta Logic – Example

Syntax:

Formulae: $F ::= V \mid F \longrightarrow F \mid F \wedge F \mid False$

$V ::= [A - Z]$

Derivable: $S \vdash X$ X a formula, S a set of formulae

logic / meta logic

$$\frac{X \in S}{S \vdash X} \qquad \frac{S \cup \{X\} \vdash Y}{S \vdash X \longrightarrow Y}$$

$$\frac{S \vdash X \quad S \vdash Y}{S \vdash X \wedge Y} \qquad \frac{S \cup \{X, Y\} \vdash Z}{S \cup \{X \wedge Y\} \vdash Z}$$

Isabelle's Meta Logic

$\bigwedge \qquad \Longrightarrow \qquad \lambda$

$\bigwedge$

Syntax: $\bigwedge x.\ F$ (F another meta level formula)

in ASCII: `!!x. F`

- ➜ universal quantifier on the meta level
- ➜ used to denote parameters
- ➜ example and more later

$\Longrightarrow$

Syntax: $A \Longrightarrow B$ (A, B other meta level formulae)

in ASCII: `A ==> B`

Binds to the right:

$$A \Longrightarrow B \Longrightarrow C \quad = \quad A \Longrightarrow (B \Longrightarrow C)$$

Abbreviation:

$$[\![A; B]\!] \Longrightarrow C \quad = \quad A \Longrightarrow B \Longrightarrow C$$

- ➜ read: A and B implies C
- ➜ used to write down rules, theorems, and proof states

Example: a theorem

mathematics: if $x < 0$ and $y < 0$, then $x + y < 0$

formal logic: $\vdash\ x < 0 \wedge y < 0 \longrightarrow x + y < 0$

variation: $x < 0; y < 0\ \vdash\ x + y < 0$

Isabelle: **lemma** ”$x < 0 \wedge y < 0 \longrightarrow x + y < 0$”

variation: **lemma** ”$[\![x < 0; y < 0]\!] \Longrightarrow x + y < 0$”

variation: **lemma**

assumes ”$x < 0$” and ”$y < 0$” shows ”$x + y < 0$”

Example: a rule

logic: $$\frac{X \quad Y}{X \wedge Y}$$

variation: $$\frac{S \vdash X \quad S \vdash Y}{S \vdash X \wedge Y}$$

Isabelle: $$[\![X; Y]\!] \Longrightarrow X \wedge Y$$

Example: a rule with nested implication

logic:

$$\frac{X \vee Y \quad \begin{array}{c} X \\ \vdots \\ Z \end{array} \quad \begin{array}{c} Y \\ \vdots \\ Z \end{array}}{Z}$$

variation:

$$\frac{S \cup \{X\} \vdash Z \quad S \cup \{Y\} \vdash Z}{S \cup \{X \vee Y\} \vdash Z}$$

Isabelle:

$$[\![X \vee Y; X \Longrightarrow Z; Y \Longrightarrow Z]\!] \Longrightarrow Z$$

λ

Syntax: $\lambda x.\ F$ (F another meta level formula)

in ASCII: `%x. F`

- ➜ lambda abstraction
- ➜ used for functions in object logics
- ➜ used to encode bound variables in object logics
- ➜ more about this in the next lecture

ENOUGH THEORY!

GETTING STARTED WITH ISABELLE

System Architecture

Proof General – user interface

HOL, ZF – object-logics

Isabelle – generic, interactive theorem prover

Standard ML – logic implemented as ADT

User can access all layers!

System Requirements

- **Linux**, **Windows**, or **MacOS X**
- **Standard ML**
  (PolyML fastest, SML/NJ supports more platforms)
- **Emacs** (for ProofGeneral) or **Java** (for jEdit)

Premade packages for Linux, Mac, and Windows + info on:

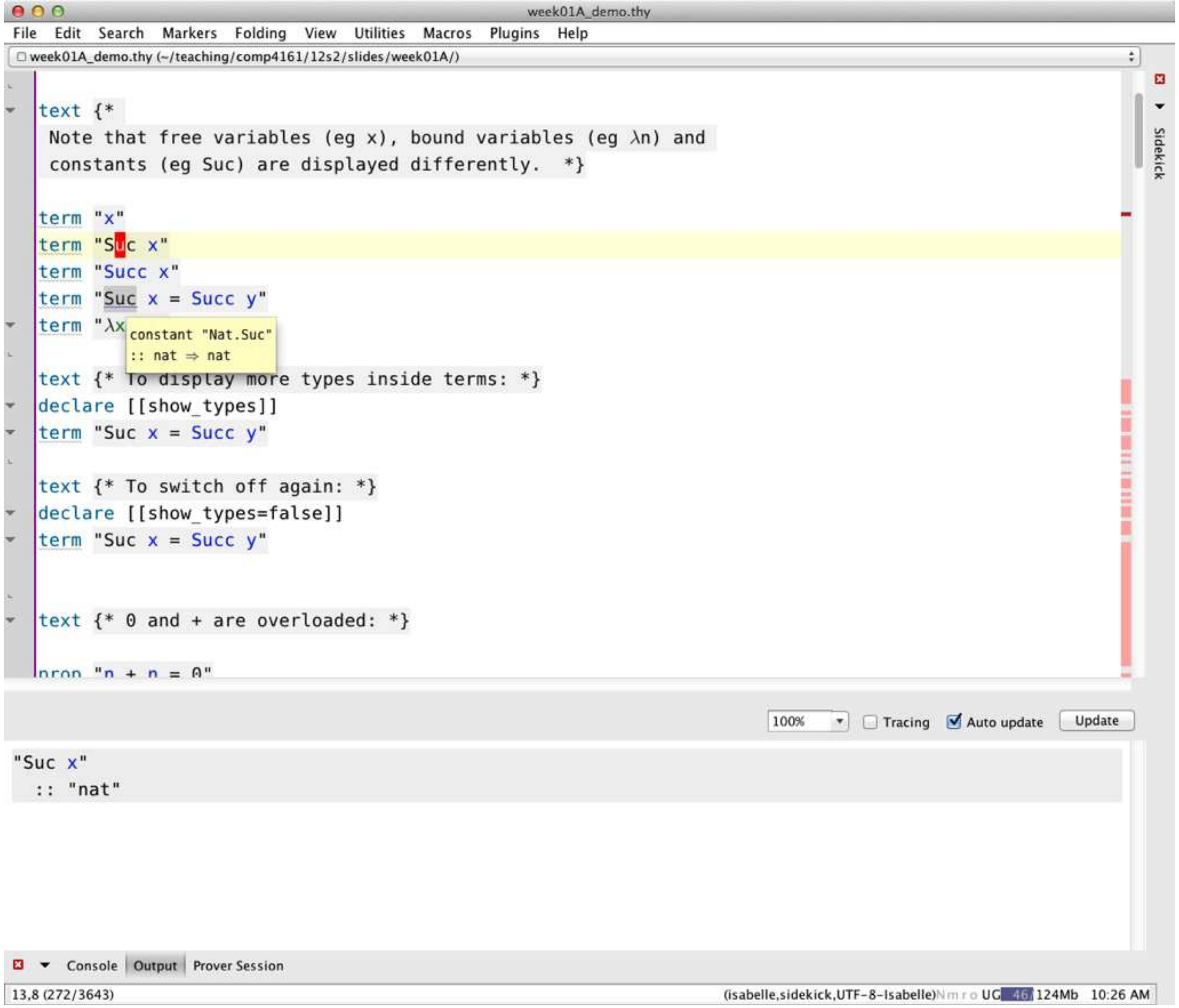
`http://mirror.cse.unsw.edu.au/pub/isabelle/download.html`

Documentation

Available from `http://isabelle.in.tum.de`

- ➜ Learning Isabelle
  - Tutorial on Isabelle/HOL (LNCS 2283)
  - Tutorial on Isar
  - Tutorial on Locales
- ➜ Reference Manuals
  - Isabelle/Isar Reference Manual
  - Isabelle Reference Manual
  - Isabelle System Manual
- ➜ Reference Manuals for Object-Logics

jEdit/PIDE



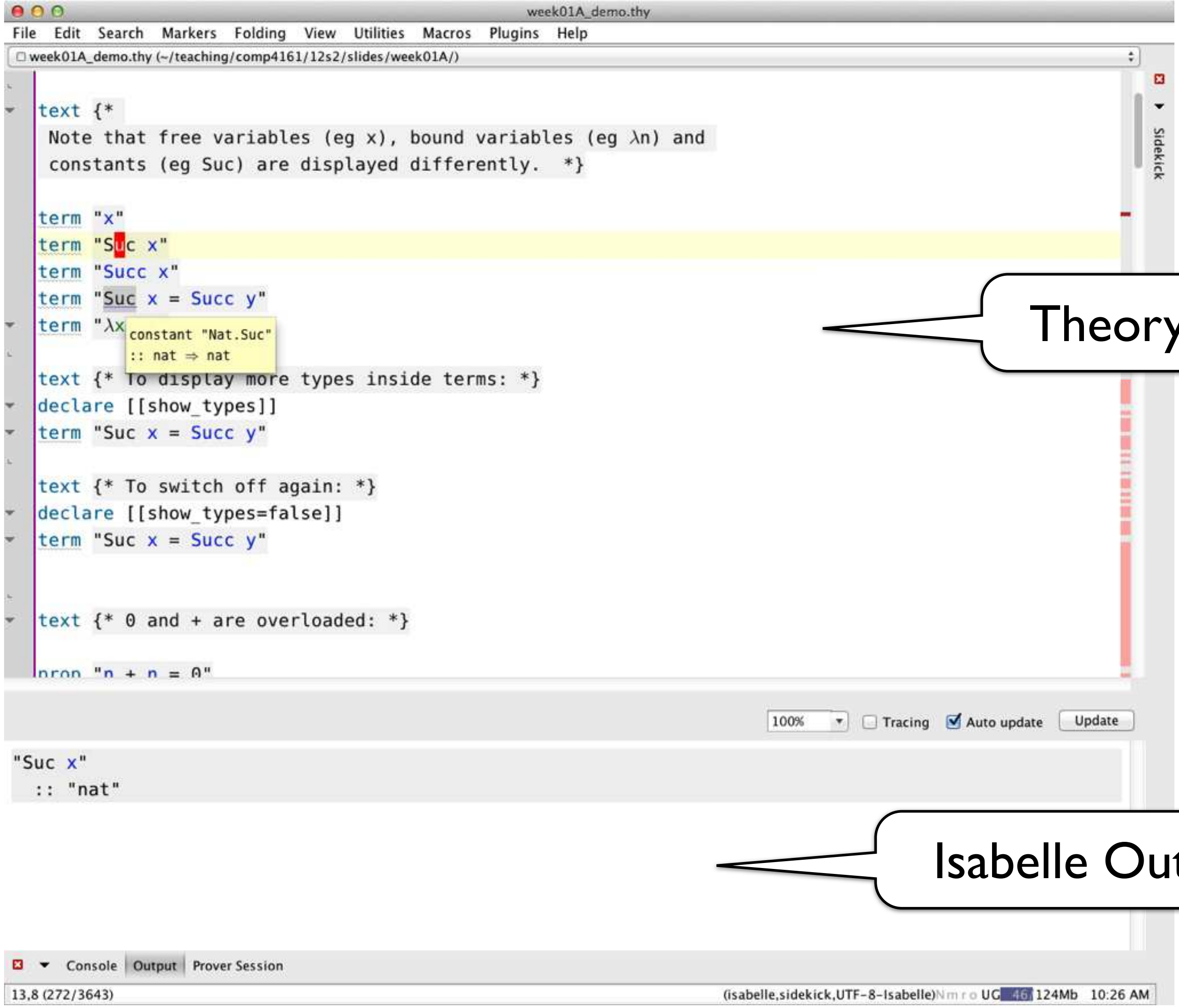
```
text {*
 Note that free variables (eg x), bound variables (eg λn) and
 constants (eg Suc) are displayed differently.  *}

term "x"
term "Suc x"
term "Succ x"
term "Suc x = Succ y"
term "λx

text {* To display more types inside terms: *}
declare [[show_types]]
term "Suc x = Succ y"

text {* To switch off again: *}
declare [[show_types=false]]
term "Suc x = Succ y"


text {* 0 and + are overloaded: *}

prop "n + n = 0"
```

```
constant "Nat.Suc"
:: nat ⇒ nat
```

```
"Suc x"
  :: "nat"
```

```
13,8 (272/3643)          (isabelle,sidekick,UTF-8-Isabelle)Nmro UG 46/ 124Mb   10:26 AM
```

jEdit/PIDE



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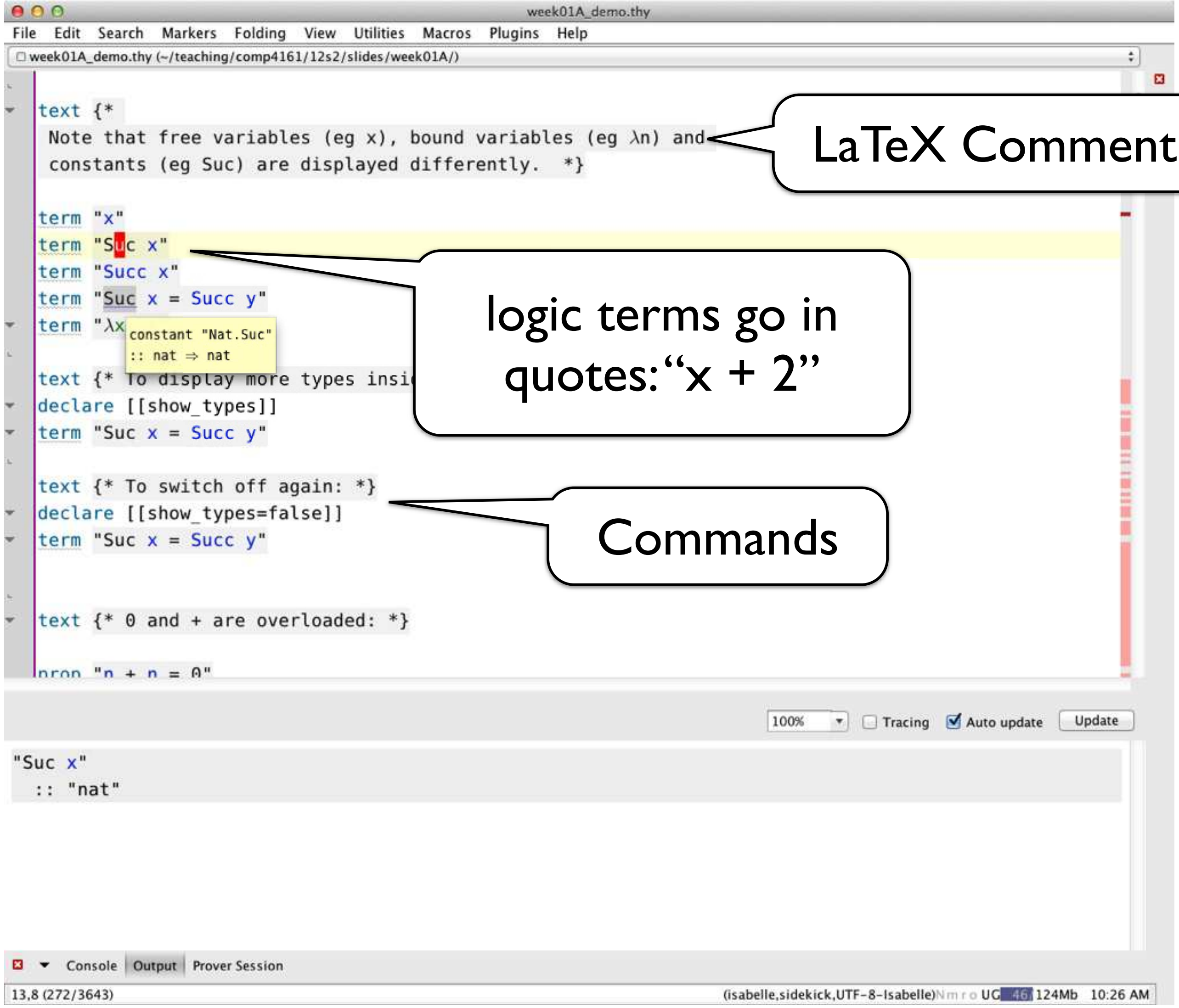
Theory File

Isabelle Output

jEdit/PIDE



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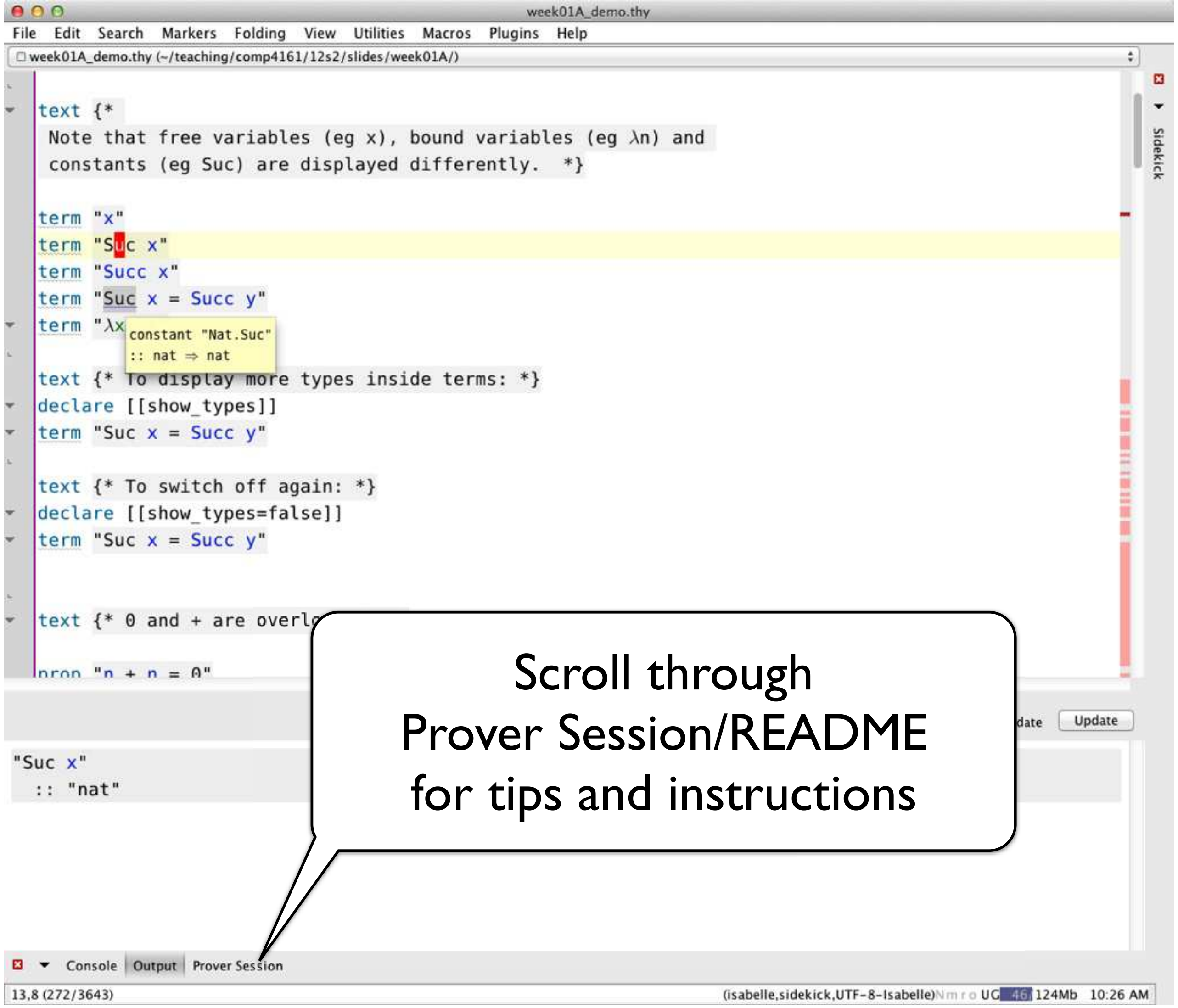
LaTeX Comment

logic terms go in quotes: "x + 2"

Commands

jEdit/PIDE



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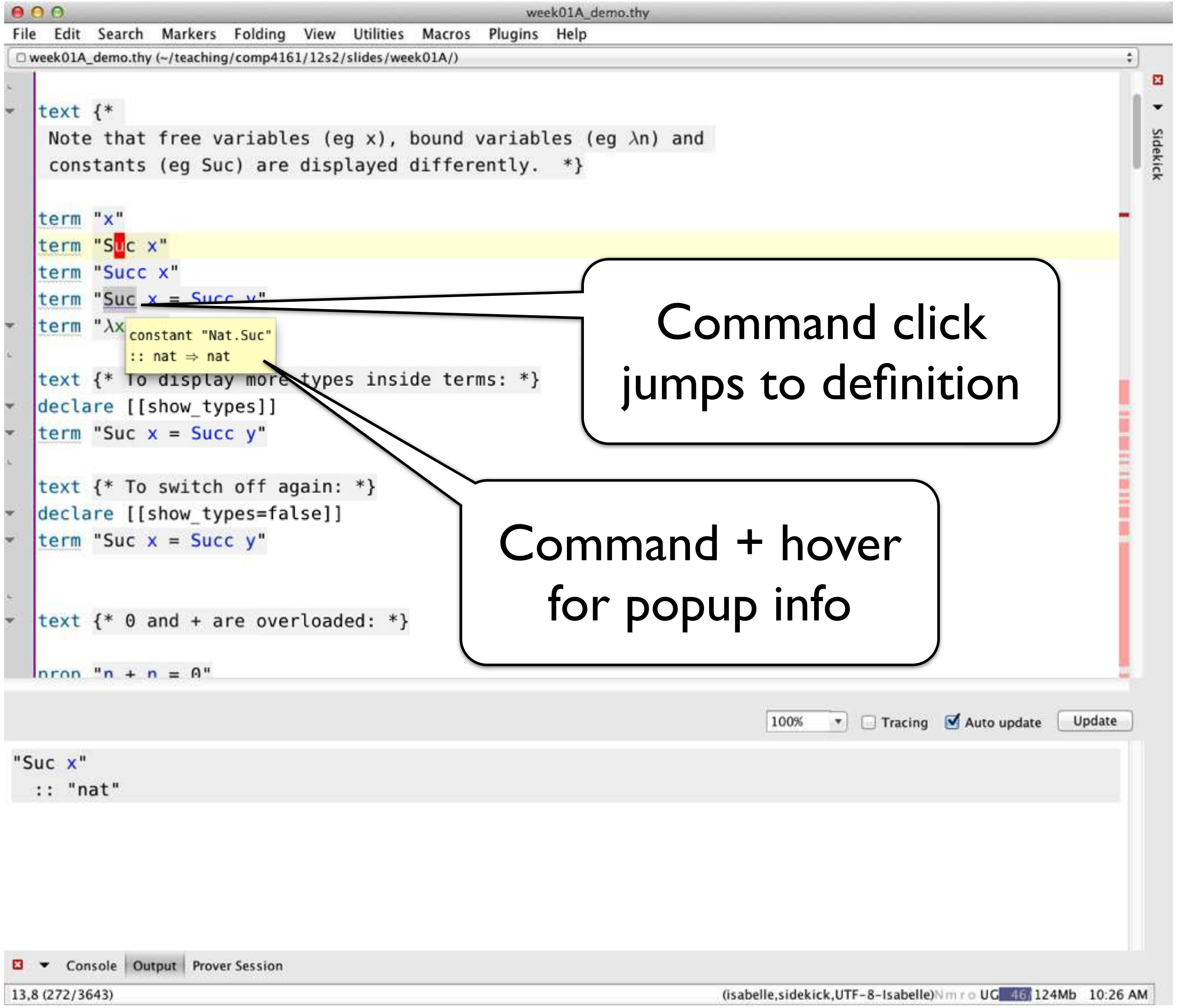
Scroll through
Prover Session/README
for tips and instructions

jEdit/PIDE



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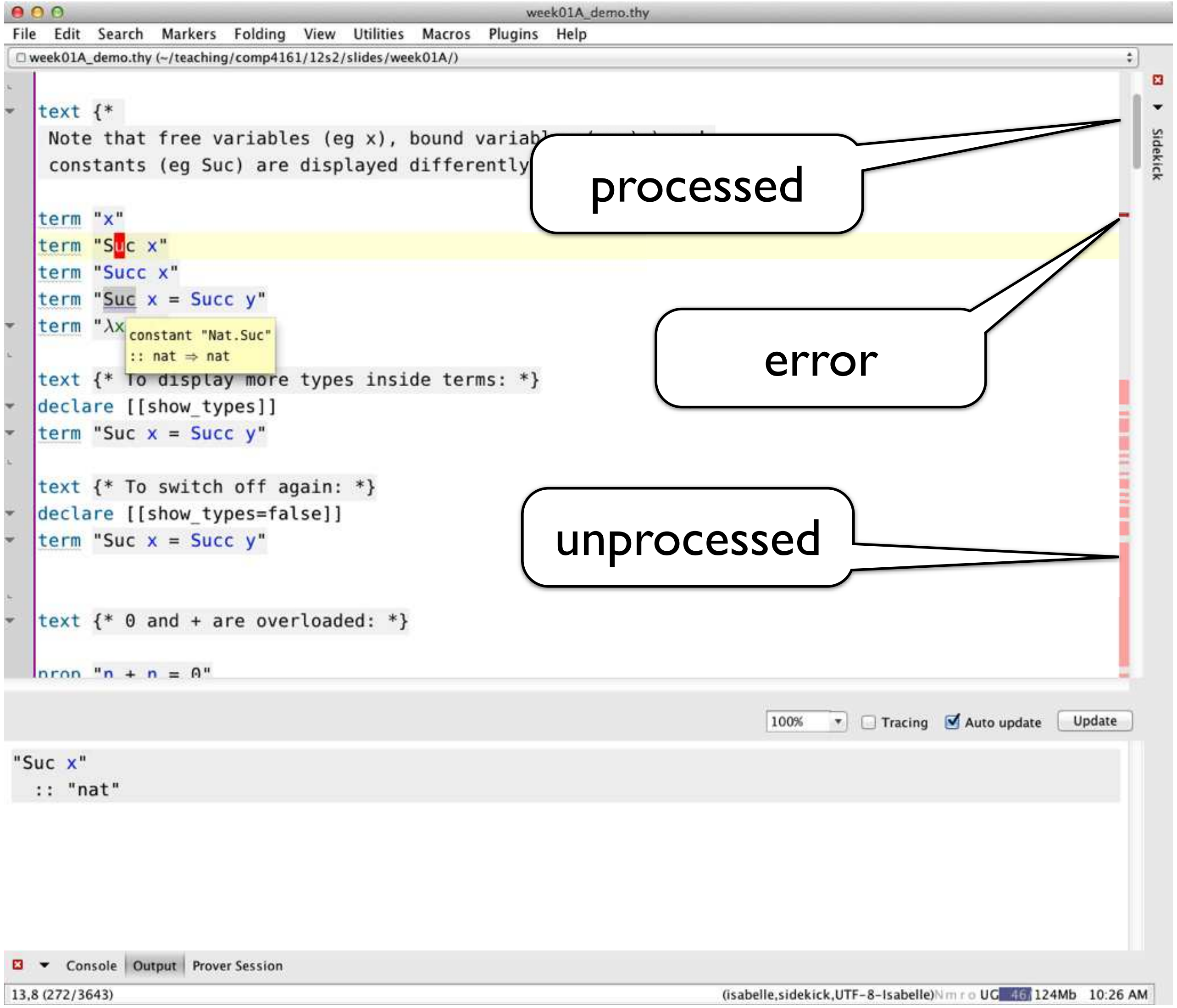
Command click jumps to definition

Command + hover for popup info

jEdit/PIDE



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processed

error

unprocessed

DEMO

Exercises

- ➜ Download and install Isabelle from
  `http://mirror.cse.unsw.edu.au/pub/isabelle/`
- ➜ Step through the demo files from the lecture web page
- ➜ Write your own theory file, look at some theorems in the library, try 'find_theorems'

- ➜ How many theorems can help you if you need to prove something like "Suc(Suc x))"?
- ➜ What is the name of the theorem for associativity of addition of natural numbers in the library?